Problem 1-

SOLUTION

We draw radial lines from the center to the tangent points A and B, as shown in Figure 2.11. The radial lines O_1A and O_2B must be perpendicular to the belt AB, hence both lines are parallel and at the same angle θ with the horizontal. We can draw a line parallel to AB through point O_2 to get line CO_2. Noting that CA is equal to O_2B, we can obtain CO_1 as the difference between the two radii.

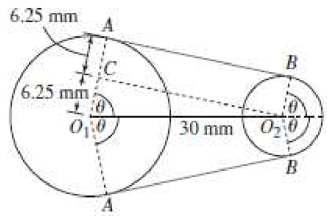


Figure 2.11 Analysis of geometry.

Triangle O_1CO_2 in Figure 2.11 is a right triangle, so we can find side CO_2 and the angle θ as:

$$AB = CO_2 = \sqrt{(30\text{ mm})^2 - (6.25\text{ mm})^2} = 29.342\text{ mm} \tag{E1}$$

$$\cos\theta = \frac{CO_1}{O_1O_2} = \frac{6.25\text{ mm}}{30\text{ mm}} \quad or \quad \theta = \cos^{-1}(0.2083) = 1.3609\text{ rad} \tag{E2}$$

The deformed length L_f of the belt is the sum of arcs AA and BB and twice the length AB:

$$AA = (12.5\text{ mm})(2\pi - 2\theta) = 44.517\text{ mm} \tag{E3}$$

$$BB = (6.25\text{ mm})(2\pi - 2\theta) = 22.258\text{ mm} \tag{E4}$$

$$L_f = 2(AB) + AA + BB = 125.46\text{ mm} \tag{E5}$$

We are given that $0.019 \le \varepsilon \le 0.034$. From Equation (2.1) we obtain the limits on the original length:

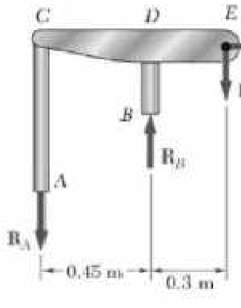
$$\varepsilon = \frac{L_f - L_0}{L_0} \le 0.034 \quad \text{or} \quad L_0 \ge \frac{125.46}{1 + 0.034}\text{ mm} \quad \text{or} \quad L_0 \ge 121.33\text{ mm} \tag{E6}$$

$$\varepsilon = \frac{L_f - L_0}{L_0} \ge 0.019 \quad or \quad L_0 \le \frac{125.46}{1 + 0.019}\text{ mm} \quad or \quad L_0 \le 123.1\text{ mm} \tag{E7}$$

To satisfy Equations (E6) and (E7) to the nearest millimeter, we obtain the following limits on the original length L_0:

ANS. $122\text{ mm} \le L_0 \le 123\text{ mm}$

Problem 2- part 1



SOLUTION

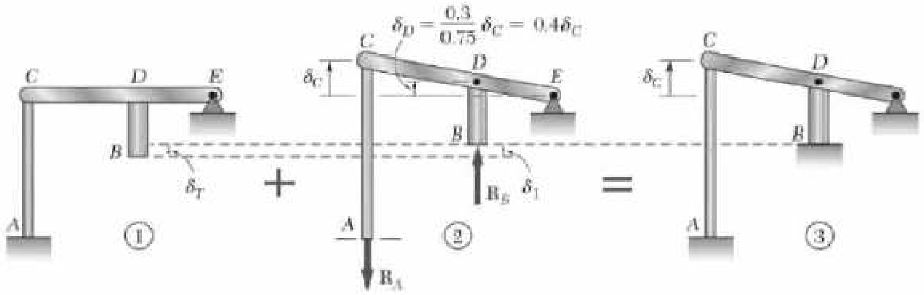
Statics. Considering the free body of the entire assembly, we write

$+\circlearrowleft \Sigma M_E = 0$: $\quad R_A(0.75 \text{ m}) - R_B(0.3 \text{ m}) = 0 \qquad R_A = 0.4R_B \qquad (1)$

Deformations. We use the method of superposition, considering $\mathbf{R}_B$ as redundant. With the support at B removed, the temperature rise of the cylinder causes point B to move down through δ_T. The reaction $\mathbf{R}_B$ must cause a deflection δ_1 equal to δ_T so that the final deflection of B will be zero (Fig. 3).

Deflection δ_T. Because of a temperature rise of $50° - 20° = 30°\text{C}$, the length of the brass cylinder increases by δ_T.

$$\delta_T = L(\Delta T)\alpha = (0.3 \text{ m})(30°\text{C})(20.9 \times 10^{-6}/°\text{C}) = 188.1 \times 10^{-6} \text{ m} \downarrow$$



Deflection δ_1. We note that $\delta_D = 0.4\delta_C$ and $\delta_1 = \delta_D + \delta_{B/D}$.

$$\delta_C = \frac{R_A L}{AE} = \frac{R_A(0.9 \text{ m})}{\frac{1}{4}\pi(0.022 \text{ m})^2(200 \text{ GPa})} = 11.84 \times 10^{-9} R_A \uparrow$$

$$\delta_D = 0.40\delta_C = 0.4(11.84 \times 10^{-9} R_A) = 4.74 \times 10^{-9} R_A \uparrow$$

$$\delta_{B/D} = \frac{R_B L}{AE} = \frac{R_B(0.3 \text{ m})}{\frac{1}{4}\pi(0.03 \text{ m})^2(105 \text{ GPa})} = 4.04 \times 10^{-9} R_B \uparrow$$

We recall from (1) that $R_A = 0.4R_B$ and write

$$\delta_1 = \delta_D + \delta_{B/D} = [4.74(0.4R_B) + 4.04R_B]10^{-9} = 5.94 \times 10^{-9} R_B \uparrow$$

But $\delta_T = \delta_1$: $\quad 188.1 \times 10^{-6} \text{ m} = 5.94 \times 10^{-9} R_B \qquad R_B = 31.7 \text{ kN}$

Stress in Cylinder: $\quad \sigma_B = \frac{R_B}{A} = \frac{31.7 \text{ kN}}{\frac{1}{4}\pi(0.03)^2} \qquad \sigma_B = 44.8 \text{ MPa} \blacktriangleleft$

Problem 2-Part 2

A 60-mm cube is made from layers of graphite epoxy with fibers aligned in the x direction. The cube is subjected to a compressive load of 140 kN in the x direction. The properties of the composite material are: $E_x = 155.0$ GPa, $E_y = 12.10$ GPa, $E_z = 12.10$ GPa, $\nu_{xy} = 0.248$, $\nu_{xz} = 0.248$, and $\nu_{yz} = 0.458$. Determine the changes in the cube dimensions, knowing that (*a*) the cube is free to expand in the y and z directions (Fig. 2.56); (*b*) the cube is free to expand in the z direction, but is restrained from expanding in the y direction by two fixed frictionless plates (Fig. 2.57).

(a) Free in y and z Directions. We first determine the stress σ_x in the direction of loading. We have

$$\sigma_x = \frac{P}{A} = \frac{-140 \times 10^3 \text{ N}}{(0.060 \text{ m})(0.060 \text{ m})} = -38.89 \text{ MPa}$$

Since the cube is not loaded or restrained in the y and z directions, we have $\sigma_y = \sigma_z = 0$. Thus, the right-hand members of Eqs. (2.45) reduce to their first terms. Substituting the given data into these equations, we write

$$\epsilon_x = \frac{\sigma_x}{E_x} = \frac{-38.89 \text{ MPa}}{155.0 \text{ GPa}} = -250.9 \times 10^{-6}$$

$$\epsilon_y = -\frac{\nu_{xy}\sigma_x}{E_x} = -\frac{(0.248)(-38.89 \text{ MPa})}{155.0 \text{ GPa}} = +62.22 \times 10^{-6}$$

$$\epsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} = -\frac{(0.248)(-38.89 \text{ MPa})}{155.0 \text{ GPa}} = +62.22 \times 10^{-6}$$

The changes in the cube dimensions are obtained by multiplying the corresponding strains by the length $L = 0.060$ m of the side of the cube:

$$\delta_x = \epsilon_x L = (-250.9 \times 10^{-6})(0.060 \text{ m}) = -15.05\ \mu\text{m}$$
$$\delta_y = \epsilon_y L = (+62.2 \times 10^{-6})(0.060 \text{ m}) = +3.73\ \mu\text{m}$$
$$\delta_z = \epsilon_z L = (+62.2 \times 10^{-6})(0.060 \text{ m}) = +3.73\ \mu\text{m}$$

(b) Free in z Direction, Restrained in y Direction. The stress in the x direction is the same as in part a, namely, $\sigma_x = -38.89$ MPa. Since the cube is free to expand in the z direction as in part a, we again have $\sigma_z = 0$. But since the cube is now restrained in the y direction, we should expect a stress σ_y different from zero. On the other hand, since the cube cannot expand in the y direction, we must have $\delta_y = 0$ and, thus, $\epsilon_y = \delta_y/L = 0$. Making $\sigma_z = 0$ and $\epsilon_y = 0$ in the second of Eqs. (2.45), solving that equation for σ_y, and substituting the given data, we have

$$\sigma_y = \left(\frac{E_y}{E_x}\right)\nu_{xy}\sigma_x = \left(\frac{12.10}{155.0}\right)(0.248)(-38.89 \text{ MPa})$$
$$= -752.9 \text{ kPa}$$

Now that the three components of stress have been determined, we can use the first and last of Eqs. (2.45) to compute the strain components ϵ_x and ϵ_z. But the first of these equations contains

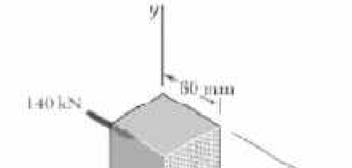


Fig. 2.56

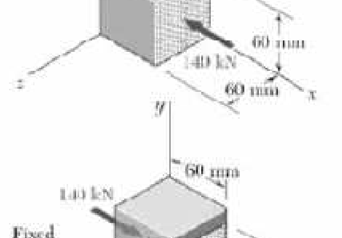


Fig. 2.57

Poisson's ratio ν_{yx} and, as we saw earlier, this ratio *is not equal* to the ratio ν_{xy} which was among the given data. To find ν_{yx} we use the first of Eqs. (2.46) and write

$$\nu_{yx} = \left(\frac{E_y}{E_x}\right)\nu_{xy} = \left(\frac{12.10}{155.0}\right)(0.248) = 0.01936$$

Making $\sigma_z = 0$ in the first and third of Eqs. (2.45) and substituting in these equations the given values of E_x, E_y, ν_{xz} and ν_{yz}, as well as the values obtained for σ_x, σ_y, and ν_{yx}, we have

$$\epsilon_x = \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y} = \frac{-38.89 \text{ MPa}}{155.0 \text{ GPa}} - \frac{(0.01936)(-752.9 \text{ kPa})}{12.10 \text{ GPa}} = -249.7 \times 10^{-6}$$

$$\epsilon_z = -\frac{\nu_{xz}\sigma_x}{E_x} - \frac{\nu_{yz}\sigma_y}{E_y} = -\frac{(0.248)(-38.89 \text{ MPa})}{155.0 \text{ GPa}} - \frac{(0.458)(-752.9 \text{ kPa})}{12.10 \text{ GPa}} = +90.72 \times 10^{-6}$$

The changes in the cube dimensions are obtained by multiplying the corresponding strains by the length $L = 0.060$ m of the side of the cube:

$$\delta_x = \epsilon_x L = (-249.7 \times 10^{-6})(0.060 \text{ m}) = -14.98\ \mu\text{m}$$
$$\delta_y = \epsilon_y L = (0)(0.060 \text{ m}) = 0$$
$$\delta_z = \epsilon_z L = (+90.72 \times 10^{-6})(0.060 \text{ m}) = +5.44\ \mu\text{m}$$

Comparing the results of parts a and b, we note that the difference between the values obtained for the deformation δ_x in the direction of the fibers is negligible. However, the difference between the values obtained for the lateral deformation δ_z is not negligible. This deformation is clearly larger when the cube is restrained from deforming in the y direction.

Problem 3

Equilibrium Condition: $\sigma_x = a + by + cz$

$$0 = \int_A \sigma_x \, dA$$

$$0 = \int_A (a + by + cz) \, dA$$

$$0 = a\int_A dA + b\int_A y\, dA + c\int_A z\, dA \qquad \textbf{[1]}$$

$$M_y = \int_A z\,\sigma_x \, dA$$

$$= \int_A z(a + by + cz)\, dA$$

$$= a\int_A z\, dA + b\int_A yz\, dA + c\int_A z^2\, dA \qquad \textbf{[2]}$$

$$M_z = \int_A -y\,\sigma_x \, dA$$

$$= \int_A -y(a + by + cz)\, dA$$

$$= -a\int_A y dA - b\int_A y^2\, dA - c\int_A yz\, dA \qquad \textbf{[3]}$$

Section Properties: The integrals are defined in Appendix A. Note that $\int_A y\, dA = \int_A z\, dA = 0$. Thus,

From Eq. [1] $\quad Aa = 0$

From Eq. [2] $\quad M_y = bI_{yz} + cI_y$

From Eq. [3] $\quad M_z = -bI_z - cI_{yz}$

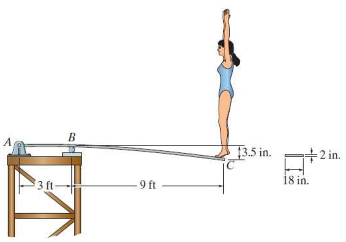
Solving for a, b, c:

$a = 0$ (Since $A \neq 0$)

$$b = -\left(\frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right) \qquad c = \frac{M_y I_z + M_z I_{yz}}{I_y I_z - I_{yz}^2}$$

Thus, $$\sigma_x = -\left(\frac{M_z I_y + M_y I_{yz}}{I_y I_z - I_{yz}^2}\right)y + \left(\frac{M_y I_y + M_z I_{yz}}{I_y I_z - I_{yz}^2}\right)z \qquad \textbf{(Q.E.D.)}$$

Problem 4



Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Functions. Referring to the free-body diagrams of the diving board's cut segments, Fig. *b*, $M(x_1)$ is

$$\circlearrowleft + \Sigma M_O = 0; \qquad M(x_1) + 3Wx_1 = 0 \qquad M(x_1) = -3Wx_1$$

and $M(x_2)$ is

$$\circlearrowleft + \Sigma M_O = 0; \qquad -M(x_2) - Wx_2 = 0 \qquad M(x_2) = -Wx_2$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1:

$$EI\frac{d^2v_1}{dx_1^2} = -3Wx_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{3}{2}Wx_1^2 + C_1 \tag{1}$$

$$EIv_1 = -\frac{1}{2}Wx_1^3 + C_1x_1 + C_2 \tag{2}$$

For coordinate x_2

$$EI\frac{d^2v_2}{dx_2^2} = -Wx_2$$

$$EI\frac{dv_2}{dx_2} = -\frac{1}{2}Wx_2^2 + C_3 \tag{3}$$

$$EIv_2 = -\frac{1}{6}Wx_2^3 + C_3x_2 + C_4 \tag{4}$$

Boundary Conditions. At $x_1 = 0$, $v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{1}{2}W(0^3) + C_1(0) + C_2 \qquad C_2 = 0$$

At $x_1 = 3$ ft, $v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{1}{2}W(3^3) + C_1(3) + 0 \qquad C_1 = 4.5W$$

At $x_2 = 9$ ft, $v_2 = 0$. Then, Eq. (4) gives

$$EI(0) = -\frac{1}{6}W\left(9^3\right) + C_3(9) + C_4$$

$$9C_3 + C_4 = 121.5W \tag{5}$$

Continuity Conditions. At $x_1 = 3$ ft and $x_2 = 9$ ft, $\frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$. Thus, Eqs. (1) and (3) give

$$\frac{3}{2}W\left(3^2\right) + 4.5W = \left[\frac{1}{2}W\left(9^2\right) + C_3\right] \qquad C_3 = 49.5W$$

Substituting the value of C_3 into Eq. (5),

$$C_4 = -324W$$

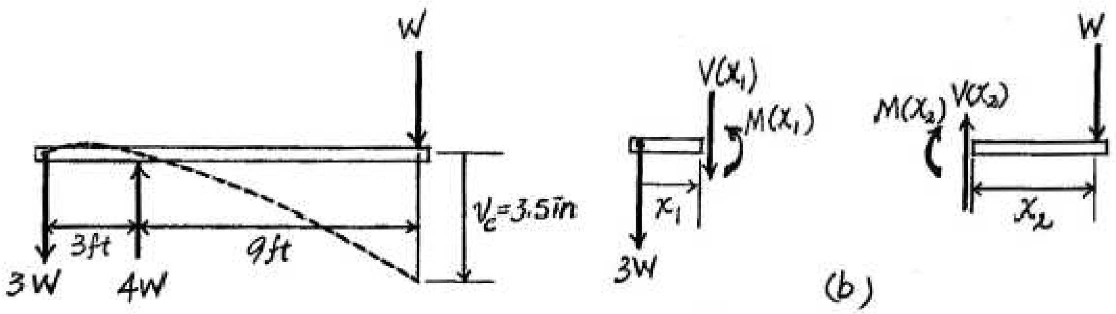
Substituting the values of C_3 and C_4 into Eq. (4),

$$v_2 = \frac{1}{EI}\left(-\frac{1}{6}Wx_2{}^3 + 49.5Wx_2 - 324W\right)$$

At $x_2 = 0$, $v_2 = -3.5$ in. Then,

$$-3.5 = \frac{-324W(1728)}{1.5\left(10^6\right)\left[\frac{1}{12}(18)\left(2^3\right)\right]}$$

$$W = 112.53 \text{ lb} = 113 \text{ lb}$$ **Ans.**



(a) (b)