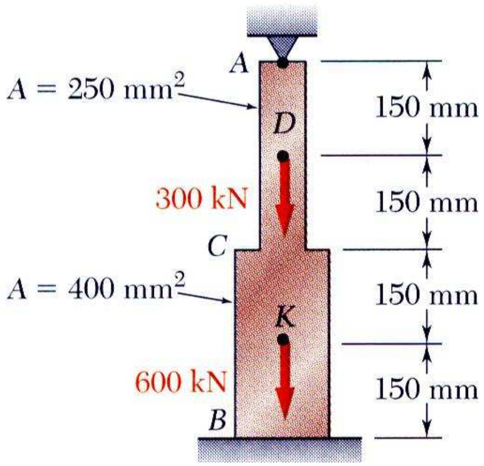
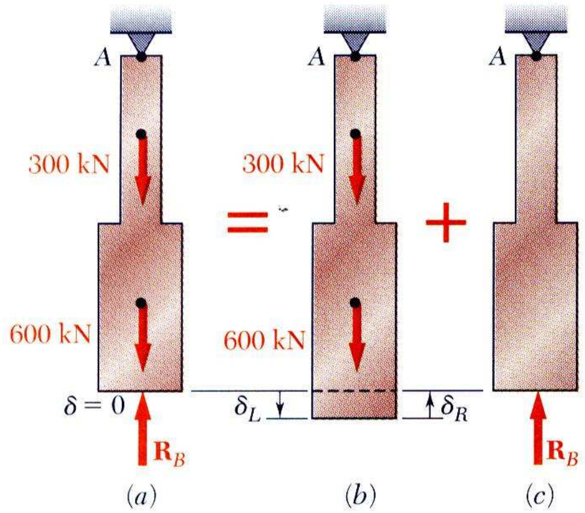
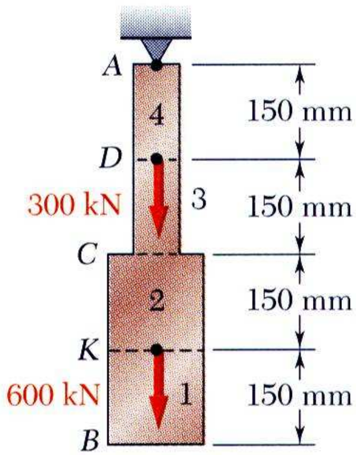


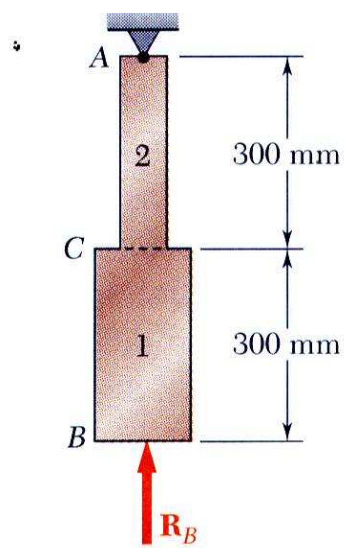


Determine the reactions at *A* and *B* for the steel bar and loading shown, assuming a close fit at both supports before the loads are applied.

SOLUTION:

- Consider the reaction at *B* as redundant, release the bar from that support, and solve for the displacement at *B* due to the applied loads.
- Solve for the displacement at *B* due to the redundant reaction at *B.*
- Require that the displacements due to the loads and due to the redundant reaction be compatible, i.e., require that their sum be zero.
- Solve for the reaction at *A* due to applied loads and the reaction found at *B*.





SOLUTION:

- Solve for the displacement at *B* due to the applied loads with the redundant constraint released,

$$P_1 = 0 \quad P_2 = P_3 = 600\times10^3\,\mathrm{N} \quad P_4 = 900\times10^3\,\mathrm{N}$$

$$A_1 = A_2 = 400\times10^{-6}\,\mathrm{m}^2 \quad A_3 = A_4 = 250\times10^{-6}\,\mathrm{m}^2$$

$$L_1 = L_2 = L_3 = L_4 = 0.150\,\mathrm{m}$$

$$\delta_L = \sum_i \frac{P_i L_i}{A_i E_i} = \frac{1.125\times10^9}{E}$$

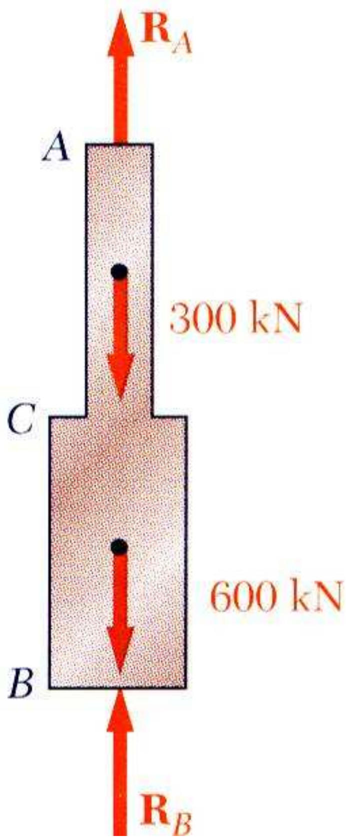
- Solve for the displacement at *B* due to the redundant constraint,

$$P_1 = P_2 = -R_B$$

$$A_1 = 400\times10^{-6}\,\mathrm{m}^2 \quad A_2 = 250\times10^{-6}\,\mathrm{m}^2$$

$$L_1 = L_2 = 0.300\,\mathrm{m}$$

$$\delta_R = \sum_i \frac{P_i L_i}{A_i E_i} = -\frac{\left(1.95\times10^3\right) R_B}{E}$$



- Require that the displacements due to the loads and due to the redundant reaction be compatible,

$$\delta = \delta_L + \delta_R = 0$$

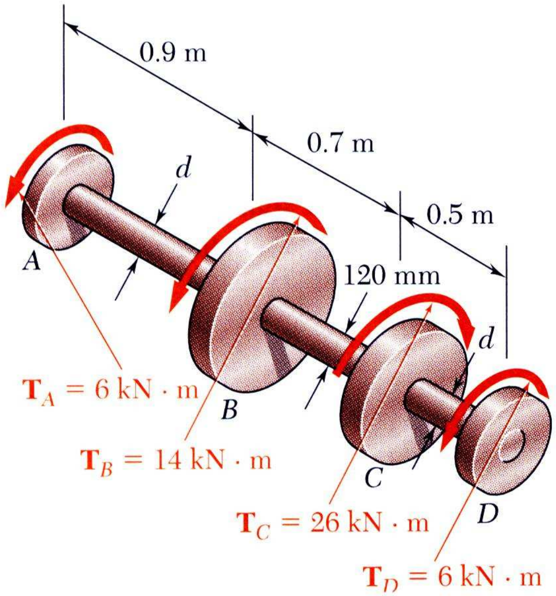
$$\delta = \frac{1.125 \times 10^9}{E} - \frac{\left(1.95 \times 10^3\right) R_B}{E} = 0$$

$$R_B = 577 \times 10^3 \,\text{N} = 577 \,\text{kN}$$

- Find the reaction at *A* due to the loads and the reaction at *B* $\sum F_y = 0 = R_A - 300\,\text{kN} - 600\,\text{kN} + 577\,\text{kN}$

$$R_A = 323\,\text{kN}$$

$$R_A = 323\,\text{kN}$$
$$R_B = 577\,\text{kN}$$



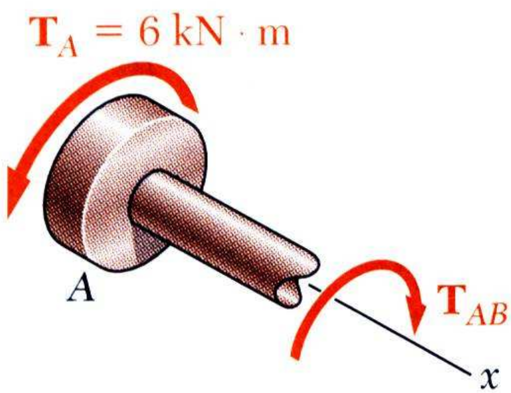
Shaft *BC* is hollow with inner and outer diameters of 90 mm and 120 mm, respectively. Shafts *AB* and *CD* are solid of diameter *d*. For the loading shown, determine (*a*) the minimum and maximum shearing stress in shaft *BC*, (*b*) the required diameter *d* of shafts *AB* and *CD* if the allowable shearing stress in these shafts is 65 MPa.

SOLUTION:

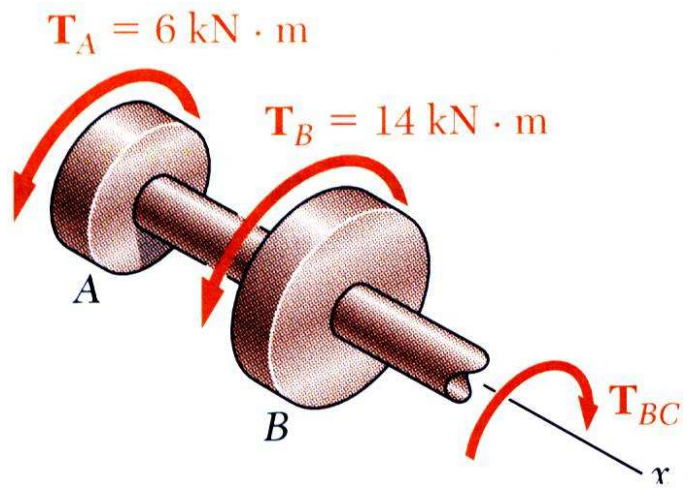
- Cut sections through shafts *AB* and *BC* and perform static equilibrium analysis to find torque loadings
- Apply elastic torsion formulas to find minimum and maximum stress on shaft *BC*
- Given allowable shearing stress and applied torque, invert the elastic torsion formula to find the required diameter

SOLUTION:

- Cut sections through shafts *AB* and *BC* and perform static equilibrium analysis to find torque loadings

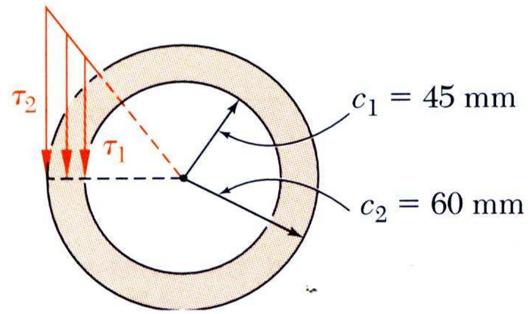


$$\sum M_x = 0 = (6\,\mathrm{kN \cdot m}) - T_{AB}$$
$$T_{AB} = 6\,\mathrm{kN \cdot m} = T_{CD}$$



$$\sum M_x = 0 = (6\,\mathrm{kN \cdot m}) + (14\,\mathrm{kN \cdot m}) - T_{BC}$$
$$T_{BC} = 20\,\mathrm{kN \cdot m}$$

- Apply elastic torsion formulas to find minimum and maximum stress on shaft *BC*



$$J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right) = \frac{\pi}{2}\left[(0.060)^4 - (0.045)^4\right]$$

$$= 13.92 \times 10^{-6}\,\text{m}^4$$

$$\tau_{max} = \tau_2 = \frac{T_{BC}c_2}{J} = \frac{(20\,\text{kN}\cdot\text{m})(0.060\,\text{m})}{13.92\times 10^{-6}\,\text{m}^4}$$

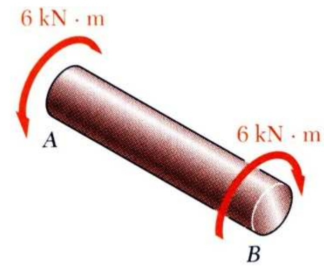
$$= 86.2\,\text{MPa}$$

$$\frac{\tau_{min}}{\tau_{max}} = \frac{c_1}{c_2} \qquad \frac{\tau_{min}}{86.2\,\text{MPa}} = \frac{45\,\text{mm}}{60\,\text{mm}}$$

$$\tau_{min} = 64.7\,\text{MPa}$$

$$\boxed{\begin{aligned}\tau_{max} &= 86.2\,\text{MPa}\\ \tau_{min} &= 64.7\,\text{MPa}\end{aligned}}$$

- Given allowable shearing stress and applied torque, invert the elastic torsion formula to find the required diameter



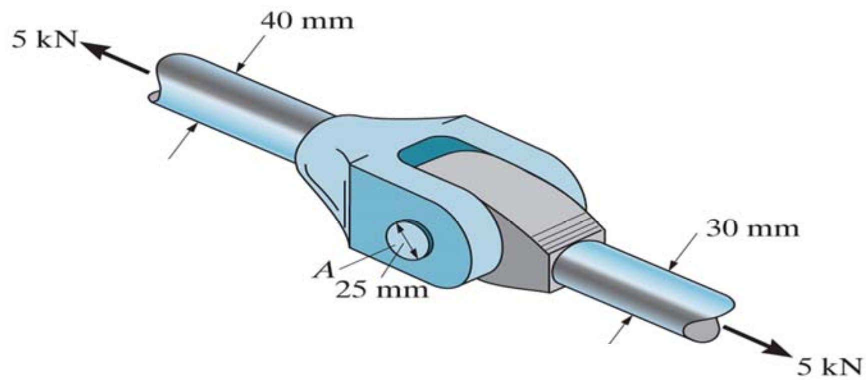
$$\tau_{max} = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} \qquad 65MPa = \frac{6\,\text{kN}\cdot\text{m}}{\frac{\pi}{2}c^3}$$

$$c = 38.9\times 10^{-3}\,\text{m}$$

$$\boxed{d = 2c = 77.8\,\text{mm}}$$

Problem1: 25 points

The yoke and rod connection is subjected to a tensile force of 5 kN. Determine the average normal stress in each rod and the average stress in the pin at A. Assuming that the rods are made of steel having a yield stress of 200 MPa, what is the maximum load that can be applied so that the rod will not yield?



Average Normal Stresses

1- In 30 mm and 40 mm diameter rods

$$\sigma_1 = \frac{P}{A_1} = \frac{5\times10^3}{\frac{\pi}{4}(30\times10^{-3})^2} = 7.07\text{MPa}$$

$$\sigma 2 = \frac{P}{A_2} = \frac{5\times10^3}{\frac{\pi}{4}(40\times10^{-3})^2} = 3.98\text{MPa}$$

2-Shearing of pin at A is a double shear loading., therefore

$$\tau_p = \frac{P}{2A_p} = \frac{5\times10^3}{2\times\frac{\pi}{4}\times(25\times10^{-3})^2} = 5.09MPa$$

3- Maximum load without yielding (For Rods)

$$P = \sigma_y A = 200\times10^6 \times\frac{\pi}{4}\times(30\times10^{-3})^2 = 141.4kN$$

We also need to check for the yielding in the pins. The yielding shear stress is equal to ½ yielding normal stress. For double shear, each pin takes ½ P

$$\frac{P}{2} = \tau_y \times A_p = 100\times10^6 \times\frac{\pi}{4}\times(25\times10^{-3})^2 = 49.1kN$$

$$P = 98.2kN$$

Problem 3: 25 points

The drill pipe shown in the figure below is designed as a hollow shaft with an outer diameter of 80mm, inner diameter of 70 mm , weight of 800 N/m and a length of 12 m. The pipe is made of steel alloy with a modulus of elasticity of 200 GPa. If the pipe is subjected to an axial load P of 7500 N and it is turning at a speed of 60 rev/min driven by a 7500 Watt diesel engine, determine the following:

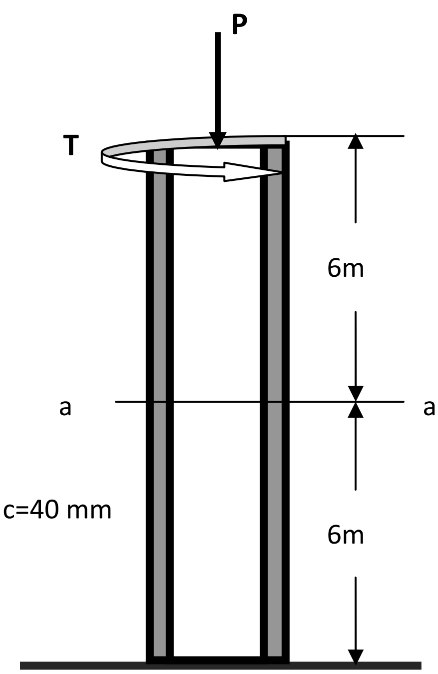
a) The average bearing stress.

b) The shear stress induced due to the torque on the outer surface of the pipe.

c) The normal compressive stress developed at section a-a located at half the pipe's length.

a-The average bearing stress

The contact load between the hollow shaft and the ground is the sum of the shaft's weight and the external load P.



$$W_{tot} = w \times L = 800 \times 12 = 9600N$$

$$\sigma_b = \frac{P + W_{tot}}{A_b} = \frac{7500 + 9600}{\frac{\pi}{4}(80^2 - 70^2) \times 10^{-6}} = 14.5MPa$$

b- The maximum shear stress is induced at the outer surface of the shaft. i.e c=40 mm

$$\omega = 2 \times \pi \times 60 / 60 = 6.28 rad / s$$

$$T = \frac{power}{\omega} = \frac{7500}{6.28} = 1193.7 N.m$$

$$\tau_o = \frac{Tc_o}{J} = \frac{1193.7 \times 40 \times 10^{-3}}{\frac{\pi}{2}(40^4 - 35^4) \times 10^{-12}} = 28.7MPa$$

c-At section a-a, the axial compressive load is the sum of the external load P and the weight of upper part of the shaft form the top to the middle section

$$W_a = w \times L = 800 \times 6 = 4800N$$

$$\sigma_{a-a} = \frac{P + W_a}{A} = \frac{7500 + 6 \times 800}{\frac{\pi}{4}(80^2 - 70^2) \times 10^{-6}} = 10.4MPa$$