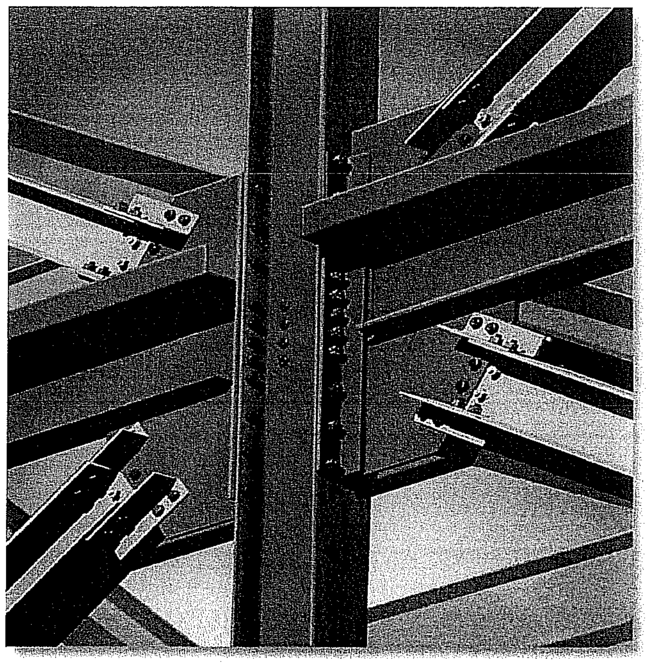
MECH 320-Mechanics of Materials

AMERICAN UNIVERSITY OF BEIRUT



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1h15

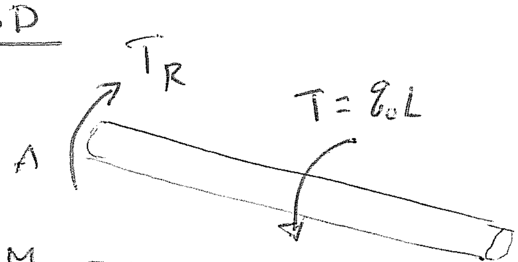
Problem 1: 25 points

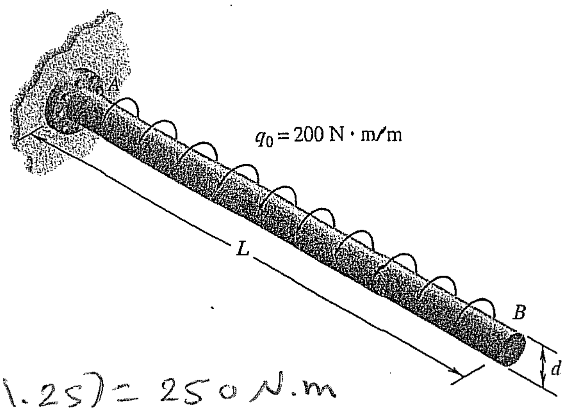
A steel shaft of diameter d=50 mm is loaded by a distributed twisting moment $q_0 = 200$ N.m/m uniform along its length $L = 1.25$ m as shown in the figure below. The shear modulus of steel $G = 80$ GPa . Determine:

1- The torque and angle of twist distribution along the shaft
2- The angle of twist at the middle point of the shaft

1-

* FBD

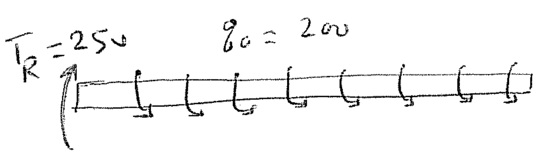




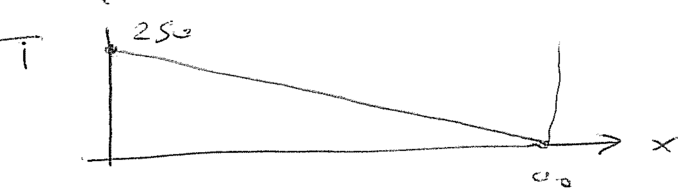
$\Sigma M_A = 0$

$\therefore\ T_R - q_0 L = 0 \Rightarrow T_R = (200)(1.25) = 250$ N.m

* Torque Diagram.



$$\frac{dT}{dx} = -q$$



$T(x) - T(0) = -q(x - 0)$

$T(x) = -qx + T(0)$

$\therefore\ \boxed{T(x) = -200x + 250}$ Ans

$$d\phi = \frac{T(x)\,dx}{GJ}, \quad GJ = 80 \times 10^9 \times \frac{\pi}{2}(25)^4 \times 10^{-12}$$

$\therefore\ GJ = 49087.4$ N.m²

$$\phi(x) = \frac{1}{GJ}\int(-200x + 250)\,dx$$

$$\therefore\ \phi(x) = \frac{1}{49087.4}\left[-100x^2 + 250x\right] + C$$

$\phi(0) = 0 \Rightarrow C = 0$

$$\therefore\ \phi(x) = \frac{1}{49087.4}(-100x^2 + 250x)$$

(2-)

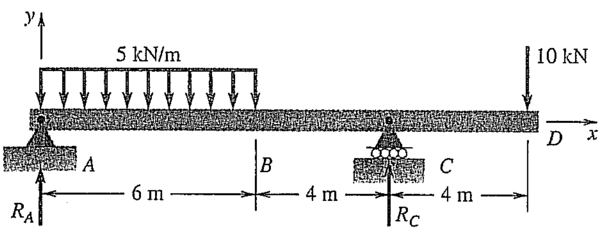
$\therefore \quad \phi(x) = \frac{1}{49087.4} \left(-100x^2 + 250x \right)$

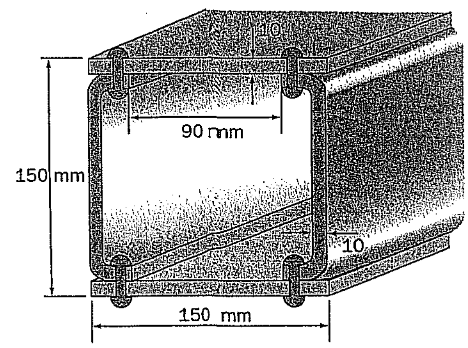
$\phi\left(\frac{L}{2}\right) = \phi(0.625) = \frac{1}{49087.4} \left(-100\,(0.625)^2 + 250(0.625) \right)$

$\therefore \quad \phi \Big|_{x=0.625} = 0.00238 .$ Ans

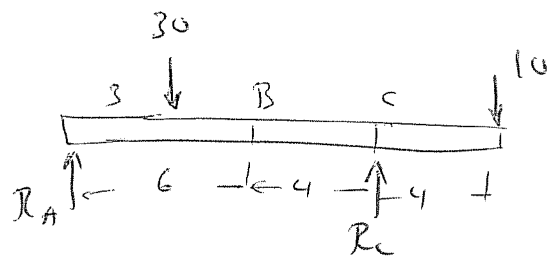
Problem 2: 25 points

A box beam is built up of plate material riveted together as shown by the cross section below. The beam is loaded by a constant distributed load over a segment AB and a concentrated load at its right end. Estimate a suitable rivet diameter if the rivets are to be pitched at about 100 mm intervals. The shear stress is not to exceed 50 MPa in each rivet.





$\tau_{all} = 50$ MPa



* Find R_A, R_C

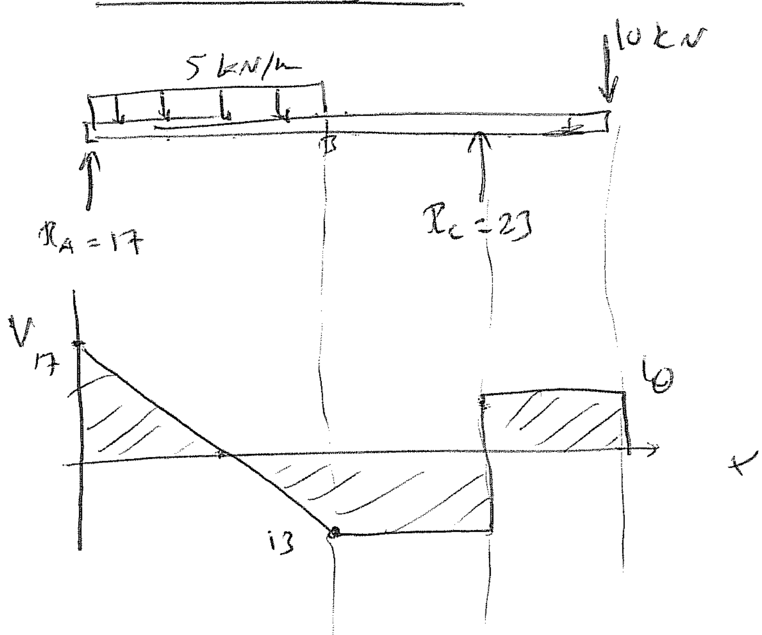
$\Sigma F = 0$

$R_A + R_C = 40$

$\Sigma M_A = 0 \Rightarrow 10(14) + 30(3) - 10 R_C = 0$

$\therefore R_C = 23 \Rightarrow R_A = 17$

* shear diagram



$V_{max} = 17$ kN

$q_{max} = \frac{V_{max} \, Q_{max}}{I}$

→

$$I \simeq \frac{1}{12}(0.15 \times 0.15^3) - \frac{0.13 \times 0.11^3}{12} - \frac{1}{12}(0.09 \times 0.01^3) \times 2$$

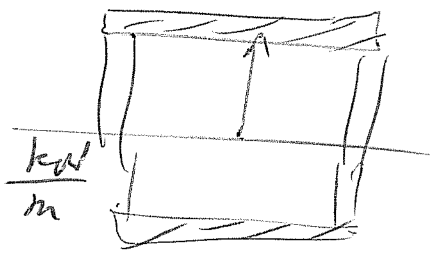
$$- 2 \times (0.09 \times 0.01)(0.06)^2$$

$$I \simeq 0.211 \times 10^{-4}\ m^4$$

$$Q_{max} = A\bar{y} = (\overbrace{150 \times 10}^{A}) \times 70 \simeq 105 \times 10^3\ mm^3$$

↳ at the ~~interface~~ interface between the outer plates and the side plate

$$\therefore q = \frac{(17000)(105) \times 10^{-6}}{21.1 \times 10^{-6}} = 84.56\ \frac{kN}{m}$$



$$\tau = \frac{F}{A}, \quad F = qs$$

Now for the rivet, F will be shared between 2 rivets

$$\therefore \tau_{all} = \frac{qs/2}{A_{rivet}} \Rightarrow A_{rivet} = \frac{(qs)/2}{\tau_{all}}$$

$$\frac{\pi d^2}{4} = \frac{\left(\frac{84.56}{2}\right)(0.1) \times 10^3}{50 \times 10^6}$$

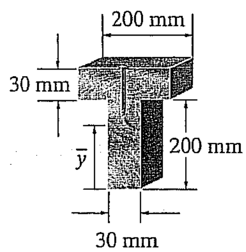
$$\therefore d \simeq 1.037 \times 10^{-2}\ m$$

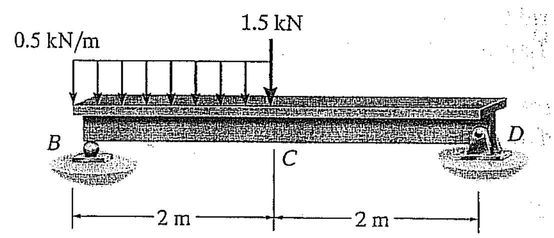
$$\boxed{d \simeq 10.4\ mm}$$

Problem 3: 25 points

The wooden T-beam shown in the figure below is made from 200×30 mm boards. Determine the following:

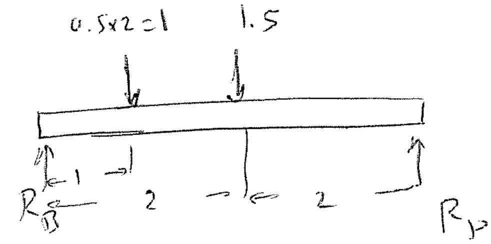
1) The maximum tensile and the compressive stresses in the beam
2) If the allowable shear stress of the board is 0.80 MPa, will the beam support the loading safely?
3) If each nail can support 1.5 KN in shear, what is the minimum number of nails that can be used?





Part I

Ⓐ Find R_B and R_D

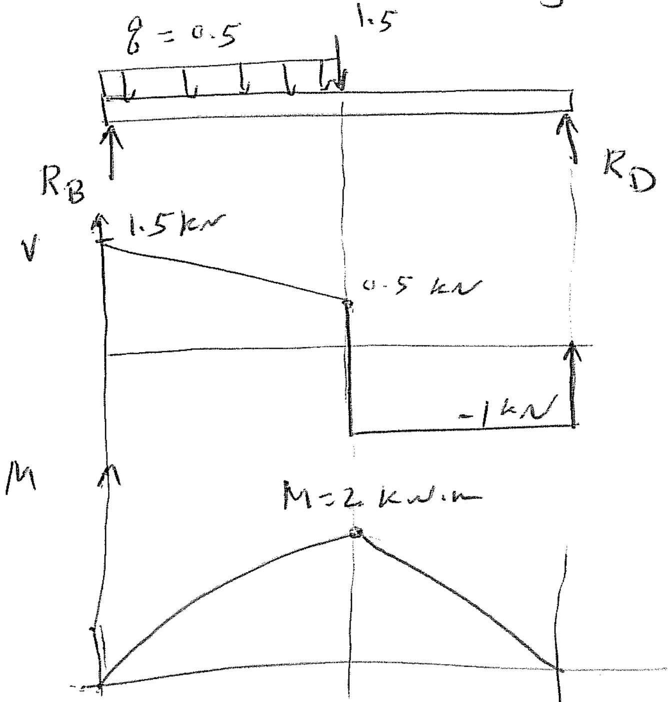


$R_B + R_D = 1 + 1.5 = 2.5$ kN

$\Sigma M_B = 0 \Rightarrow 1\times1 + 1.5\times2 - 4R_D = 0$

$\boxed{R_D = 1.0 \text{ kN}}$, $\boxed{R_c = 1.5 \text{ kN}}$

Ⓑ Shear and Moment Diagrams

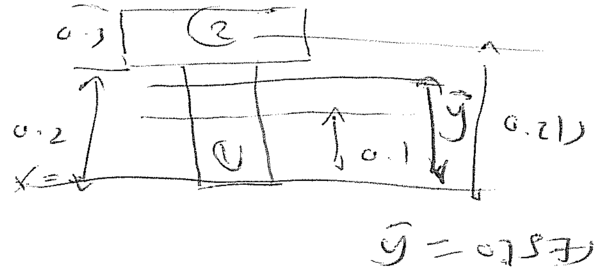


$\Rightarrow V_{max} = 1.5$ $\Big|$ $0 \le x \le 2$ m

$V = -1$ kN $\Big)$ $2 < x < 4$

$M_{max} = 2$ kN.m

Find $\bar{y}$



$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A}$$

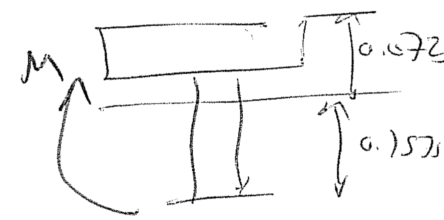
$$\therefore \bar{y} = \frac{(0.1)(0.03)(0.2) + 0.215(0.03)(0.2)}{(0.03)(0.2) + (0.03)(0.2)} = 0.1575 \text{ m}$$

$$I = \frac{1}{12}(0.03)(0.2)^3 + \overbrace{(0.03)(0.2)}^{A_1}\overbrace{(0.1575 - 0.1)^2}^{d_1^2}$$

$$+ \frac{1}{12}(0.2)(0.03)^3 + \overbrace{(0.03)(0.2)}^{A_2}\overbrace{(0.215 - 0.1575)^2}^{d_2^2}$$

$$\therefore I = 60.125 \times 10^{-6} \text{ m}^4$$

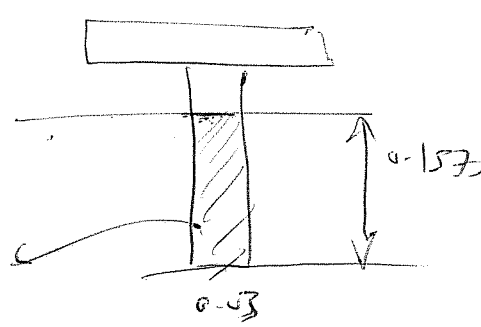
$$\therefore \sigma_{max} = \frac{Mc}{I} = \frac{2 \times 10^3 \times 0.1575}{60.125 \times 10^{-6}} = 33.26 \text{ MPa}$$



$$(\sigma_{min})_{Comp} = \left(\frac{0.0725}{0.1575}\right) 33.26 = 15.3 \text{ MPa}$$

$$(\sigma_{max})_{Tension} = 33.26 \text{ MPa}, \quad (\sigma_{max})_{Compression} = 15.3 \text{ MPa}$$

Part 2

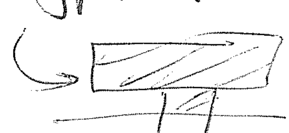


$$\tau_{all} = 0.8 \text{ MPa}$$

$$\tau = \frac{V_{max} Q_{max}}{It}, \quad Q_{max} = A\bar{y}$$

$$\therefore Q_{max} = (0.1575)(0.03)\left(\frac{0.1575}{2}\right) = 372.09 \times 10^{-6} \text{ m}^3$$

You can get the same answer if you take the upper area



$t = 0.03, \quad V_{max} = 1.5 \text{ kN}$

$$\tau_{max} = \frac{(1.5\times10^{3})(372.09)\times10^{-6}}{(60.125\times10^{-6})(0.03)} \simeq 0.309 \text{ MPa}$$

$\tau_{max} < 0.8$ MPa $\Rightarrow$ The beam can support the load

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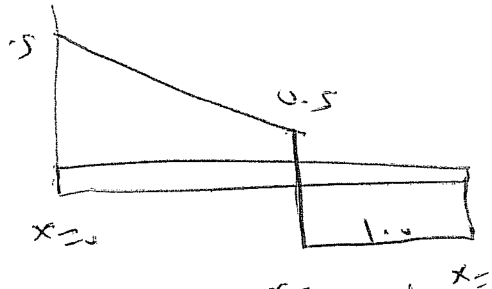
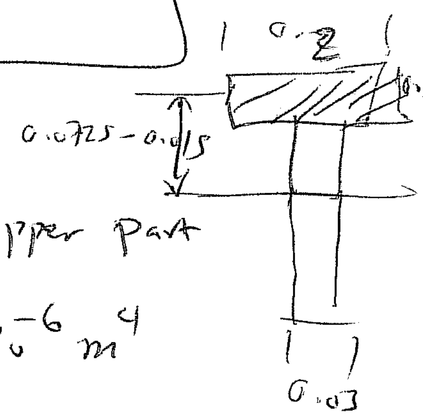
Part 3

Recall the shear diagram



$0 < x < 2 \Rightarrow$ you can use $V = 1.5$ kN or even $V = \frac{1.5+0.5}{2} = 1.0$ kN

$2 < x < 4 \Rightarrow V_{max} = 1.0$ kN



$q_1 = \frac{VQ}{I}$, $Q = \bar{y}A$ (for the upper part

$Q = (0.0725 - 0.015)(0.03)(0.2) = 345\times10^{-6}\ m^4$

$$q_1 = \frac{(1.5\times10^{3})(345)\times10^{-6}}{60.125\times10^{-6}} \simeq 8607.1\ N = 8.607\ kN$$

$q_1 s_1 = 1.50$ kN $\Rightarrow s_1 = \frac{1.5}{8.607} \simeq 0.174$ m

$L_1 = 2$ m

$n_1 = 2/0.174 = 11.47$ nails

$$q_2 = \frac{(1.0\times10^{3})(345)\times10^{-6}}{60.125\times10^{-6}} \simeq 5.735\ kN$$

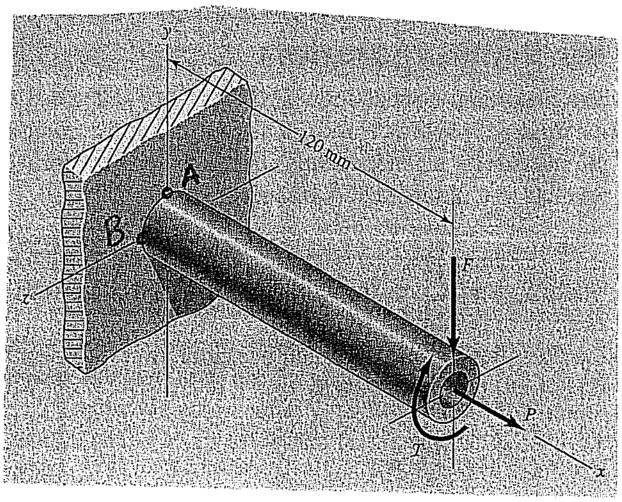
$S_2 = 1.5/5.735 = 0.261$m

$n_2 = \frac{2}{0.261} = 7.6$ nails

Problem 4: 25 points

The cantilever tube shown in the figure below is made of 2014 aluminum alloy that has an outer diameter of 30 mm, inner diameter of 25 mm, and yield strength of 276 MPa. The bending force F= 1.75 kN, the axial tension P= 9.0 kN and the torsion T=72 N.m are applied as shown. determine:

1- The normal and shear stresses at point A.
2- The normal and shear stresses at point B.
3- Knowing that the equivalent stress for this combined loading is expressed by :
$\sigma_e = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$. Which location is more critical A or B?



at Point A

$$\sigma_A = \frac{P}{A} + \frac{Mc_o}{I} \quad , \quad M = FL = (1.75)(120) = 210 \text{ N.m}$$

$$\therefore \sigma_A = \frac{9 \times 10^3}{\frac{\pi}{4}(30^2 - 25^2) \times 10^{-6}} + \frac{(210)(15 \times 10^{-3})}{\frac{\pi}{4}(15^4 - 12.5^4) \times 10^{-12}}$$

$$= \cancel{153} \; 41.67 \times 10^6 + 153 \times 10^6$$

$$\therefore \boxed{\sigma_A = 194.67 \text{ MPa}}$$

$$\tau_A = \frac{Tr}{J} + \frac{VQ}{It} \quad , \quad Q = 0.0$$

$$\therefore \tau_A = \frac{Tr}{J}$$

$$\therefore \tau_A = \frac{(72)(15\times10^{-3})}{\frac{\pi}{2}(15^4-12.5^4)\times10^{-12}}$$

$$\therefore \tau_A = 26.23 \text{ MPa}$$

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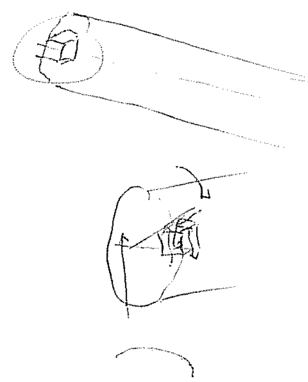
at Point B

$$\sigma_B = \frac{P}{A} + \frac{Mc}{I}, \quad c = 0$$ because B is at the neutral axis

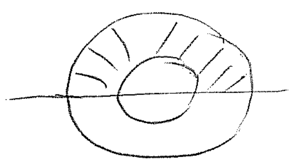
$$\sigma_B = \frac{P}{A} = 41.67 \text{ MPa}$$

$$\tau_B = \frac{Tr}{J} - \frac{VQ}{It}$$



$$= \frac{(72)(15\times10^{-3})}{\frac{\pi}{2}(15^4-12.5^4)\times10^{-12}} - \frac{VQ}{It}$$

Find Q



$$Q = \bar{y}_1 A_1 - \bar{y}_2 A_2$$

$$\frac{4}{3\pi}(15)\left(\frac{\pi 15^2}{2}\right) - \frac{4}{3\pi}(12.5)\left(\frac{\pi(12.5)^2}{2}\right) = 947.9 \text{ mm}^3$$

$$t = (30-25) = 5 \text{ mm}$$

$$I = \frac{\pi}{4}(15^4-12.5^4)\times10^{-12}$$

$$\therefore \tau_B = \frac{72\times15\times10^{-3}}{\frac{\pi}{2}(15^4-12.5^4)\times10^{-12}} - \frac{1.75\times10^3\times947.9\times10^{-9}}{\frac{\pi}{4}(15^4-12.5^4)\times10^{-12}\times5\times10^{-3}}$$

$$= 26.3 - 16.12 \approx 10.1 \text{ MPa}$$

$$\sigma_e = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

$$(\sigma_e)_A = \sqrt{194.67^2 + 3(26.3)^2} \cong 200 \text{ MPa}$$

$$(\sigma_e)_B = \sqrt{41.6^2 + 3(10.1)^2} \simeq 45.1 \text{ MPa}$$

$\therefore$ Point A is more critical

* $\phi = \int_0^L \frac{T(x)\,dx}{JG}$: Angle of twist

* $\sigma = \frac{-My}{I}$: bending stress

* $\tau = \frac{VQ}{It}$: Shear formula in beams

* $q = \frac{VQ}{I}$: Shear flow

* $\tau_{Torque} = \frac{T\rho}{J}$: shear stress due to torque