EECE 370 USE FOR ANSWER Final Exam January 30, 2008

1.

a) $V_{B1} = 11000V$ $S_B = 10000\ VA$

$V_{B2} = 380V$

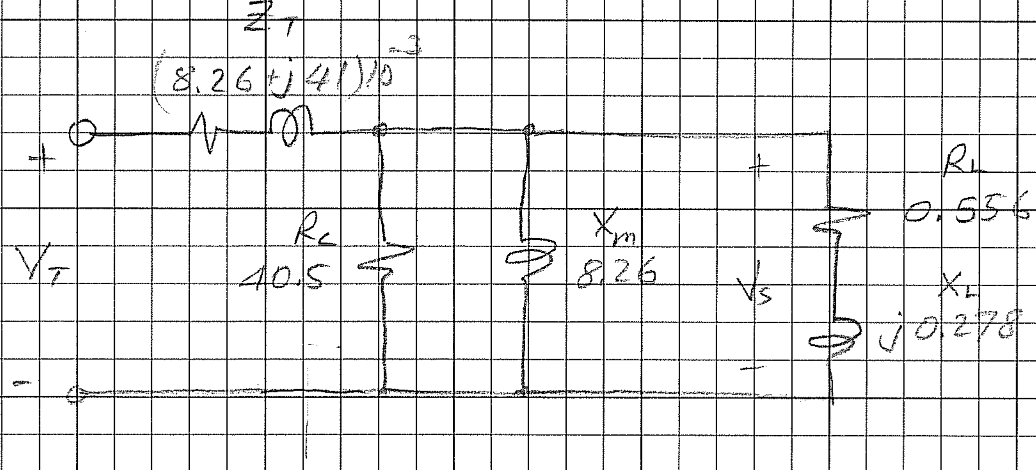
$I_{B1} = \frac{10000}{11000\sqrt{3}} = 0.525A$ $I_{B2} = \frac{10000}{380\sqrt{3}} = \underline{15.2A}$

$Z_{B1} = \frac{V_{B1}^2}{S_B} = \frac{(11000)^2}{10000} = 12100\ \Omega$ $Z_{B2} = \frac{(380)^2}{10000} = \underline{14.4\ \Omega}$

$Z_T = (100 + j500)/12100 = (8.26 + 41) \times 10^{-3}$ per unit

$R_c = \frac{490}{12.1} = \underline{40.5\ p.u.}$ $X_m = \frac{100}{12.1} = \underline{8.26\ p.u.}$

$Z_L = (8 + j4)/14.4 = \underline{0.556 + j0.278\ p.u.}$



b) $V_T = 1.04\angle 0$ p.u.

The equivalent impedance seen by the source is:

$Z_{eq} = Z_T + R_c \parallel X_m \parallel Z_L$

$$Z_{eq} = (8.26 + j41) \times 10^{-3} + \overset{(1.464 - j0.84)^{-1}}{0.514 + j0.295}$$

$$= 0.522 + j0.336 \ \text{p.u.} = \underline{0.621 \angle 32.77^\circ}$$

$$I_T = 1.04 / (0.522 + j0.336) = 1.409 - j0.907 \ \text{p.u.}$$

$$= \underline{1.675 \angle -32.77^\circ} \ \text{p.u}$$

$$P + jQ = V_T I_T^* = 1.04 \times (1.409 + j0.907)$$

$$= 1.465 + j0.943 \ \text{p.u.}$$

in real quantities:

$$I_T = (1.675 \angle -32.77) \times 0.525$$

$$= \underline{0.879 \angle -32.77 \ \text{A}}.$$

$$P + jQ = \underline{14\,650} + \underline{j\,9\,430} \ \text{VA}$$

Secondary Voltage:

$$V_S = V_T - Z_T I_T$$

$$= 1.04 - (0.00826 + j0.041) \times (1.409 - j0.907)$$

$$= 0.991 - j0.0503 = \underline{0.992 \angle -2.9^\circ} \ \text{p.u.}$$

$$= (0.992 \angle -2.9) \times (380) \cong \underline{376.96 \angle -2.9} \ \text{V}$$

c) with the capacitor: $Z_c = -j5/14.4 = -j0.347$ p.u.

$$Z'_{eq} = (0.00826 + j0.041) + \overset{(1.463 + j2.042)^{-1}}{(0.232 - j0.324)} \checkmark$$

$$= 0.240 - j0.283 = 0.371 \angle -49.6^\circ = 0.371 \angle -49.6^\circ$$

$$I_T' = 1.04/(0.240 - j0.283) = 1.815 + j2.132 = 2.80 \angle 49.6$$

$$P + jQ = V_T' I_T'^{*} = \underline{1.89 - j2.22}$$

$$V_S' = V_T' - Z_T I_T' = 1.04 - (0.00826 + j0.041) \times (1.815 - j2.132)$$
$$= 1.112 - j0.092$$
$$= \underline{1.116 \angle -4.73^{\circ}}$$

The addition of the capacitor reduced the amount of reactive power supplied from the source. The magnitude of the current increased from 1.675 to 2.8. The voltage on the secondary increased from 0.992 to 1.116 per unit. This is a case of over compensation.

USE FOR ANSWER

2.

a)

$$Z_B = \frac{380^2}{500000} = 0.289\,\Omega$$

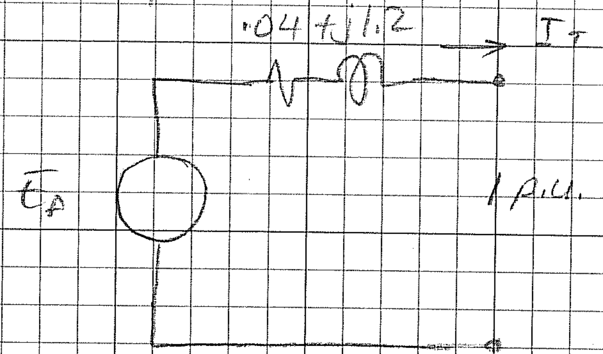
$$P_G + jQ_G = S_G(\cos\phi + j\sin\phi) \qquad I_B = \frac{500000}{\sqrt{3}\,380} = 759.7$$

$S = 500\,kVA$, $\cos\phi = 0.85$, $\sin\phi = 0.527$

$$\therefore \quad P_G + jQ_G = 500(0.85 + j0.527) = \underline{425 + j263.5\ kVA}$$

The internal voltage is obtained from the following equivalent circuit.

$$R_A + jX_s = (0.04 + j1.2)\frac{380^2}{500\times10^3}$$
$$= 0.0116 + j0.346\,\Omega$$



Note:
$V = 1$ p.u. $\Rightarrow$
$I_{p.u.} = S^*_{p.u.}$

The current in per unit is:

$$I = \frac{S^*_{(p.u.)}}{V_{(p.u.)}} = (425 - j263.5)/500$$

$$= 0.85 - j0.527 = 1\angle{-31.8°}\ p.u. = 759.7\angle{-31.8}$$

$$E_A = 1 + (0.85 - j0.527)\times(0.04 + j1.2)$$

$$= 1 + 0.666 + j0.999 = 1.67 + j1.0\ p.u.$$

$$E_A = (1.67 + j1.0) = 1.94\angle{30.9°}\ p.u.$$

$$|E_A^{ll}| = \underline{738.2\ V} \qquad |E_A^{ln}| = 426.2\ V$$

From graph $\underline{i_F = 0.38\ A}$.

b) The complex power delivered is given by:

$$S = VI^* = V(E_A - V)^* / (R_A + jX_s)^*$$

So

$$(0.85 + j0.527) \times 0.75 \times (0.04 - j1.2) = VE_A - |V|^2$$

$$0.5 - j0.75 = E_A|V|(\cos\delta - j\sin\delta) - |V|^2$$

where δ is angle of V with respect to E_A

Equating real and imaginary parts:

$$0.5 = E_A|V|\cos\delta - |V|^2 \quad (1)$$

$$-0.75 = -E_A|V|\sin\delta \quad (2)$$

$$(1) \Rightarrow |V|\cos\delta = \frac{0.5 + |V|^2}{1.94}$$

$$(2) \Rightarrow |V|\sin\delta = \frac{0.75}{1.94}$$

Take the sum of the squares:

$$|V|^2 = \left(\frac{0.5 + |V|^2}{1.94}\right)^2 + \left(\frac{0.75}{1.94}\right)^2$$

$$3.76|V|^2 = 0.25 + |V|^4 + |V|^2 + 0.5625 \Rightarrow$$

$$|V|^4 - 2.76|V|^2 + 0.8125 = 0$$

$$|V|^2 = \frac{2.76 \pm \sqrt{(2.76)^2 - 4 \times 0.8125}}{2} = \frac{2.76 \pm 2.09}{2}$$

USE FOR ANSWER

$|V|^2 = 2.425 \Rightarrow |V| = 1.557 \text{ p.u.} = 592\text{V } l\text{-}l.$

From (1) $\Rightarrow \cos\delta = \frac{0.5 + |V|^2}{1.94|V|} = \frac{0.5 + 2.425}{1.94 \times 1.557}$

$= \frac{2.925}{3.021} = 0.9684 \Rightarrow$

$\delta = -14.45$

It is lagging because there is power transfer from E_A to V:

$V = 1.557 \angle -14.45^\circ =$

$I = \frac{1.94 - 1.557 \angle -14.45^\circ}{0.04 + j1.2} = 0.335 - j0.35 \text{ p.u.}$

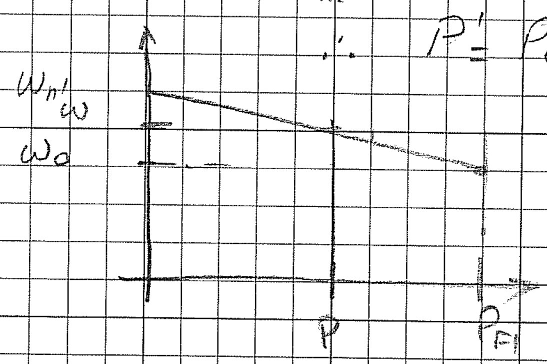
$= 0.484 \angle -46.25$

c) For Part a): $T = \frac{P_{out} + P_{loss}}{\omega} = \frac{425 + 34}{314} = 1.46 \text{ kNm}$

For Part b) the new power is:

$P'_{out} = 0.75 \times 425 = 318.75 \text{ kW}$

$\therefore P' = P'_{out} + P'_{loss} = 318.75 + 18.7 = 337.5 \text{ kW}$



The speed is calculated from the droop characteristic:

$$\frac{P_{fl} - 0}{\omega_0 - \omega_{nl}} = \frac{P_{fl} - P}{\omega_0 - \omega} \quad (3)$$

$P = 0.75 P_{F1}$

But first we need to determine ω_{n1} from:

$$\frac{\omega_{n1} - \omega_0}{\omega_0} = 0.05 \Rightarrow \quad \omega_{n1} = \omega_0 (1 + 0.05)$$

$$= 314 \times 1.05 = 329.7 \text{ rad}$$

$$= 52.5 \text{ Hz}$$

From (3) we obtain:

$$P_{F1} (\omega_0 - \omega) = (P_F - 0.75 P_{F1}) (\omega_0 - \omega_{n1}) \Rightarrow$$

$$\omega_0 - \omega = 0.25 (\omega_0 - \omega_{n1}) \Rightarrow$$

$$\omega = \omega_0 - 0.25 (\omega_0 - \omega_{n1})$$

$$= 314 - 0.25 (314 - 329.7)$$

$$= 317.9 \text{ rad/s}$$

$$\therefore \quad f = 50.6 \text{ Hz}$$

+ P_{loss} for Part a):

$$P_{loss} = \underbrace{6 + 7 + 1 + 3 \times (759.7)^2 \times 0.0116 \times 10^{-3}}_{20} = 34 \text{ kW}$$

P_{loss} for Part b)

$$P'_{loss} = \underbrace{6 + 7 + 1 + 3 \cdot (0.484 \times 759.7)^2 \times 0.0116 \times 10^{-3}}_{4.7} = 18.7$$

Torque in Part b):

$$T' = \frac{337.5}{317.9} = \underline{1.06 \text{ kNm}}$$

3.

a)

Locked Rotor Test:

$$PF = \frac{5586}{\sqrt{3}\, 98.7 \times 70} = 0.467 \Rightarrow \sin\theta_{LR} = 0.884$$

$$|Z_{LR}| = \frac{98.7}{\sqrt{3} \times 70} = 0.814\ \Omega$$

$$Z_{LR} = 0.814\,(0.467 + j0.884) = 0.380 + j0.72$$

$$R_{LR} = R_1 + R_2 = 0.380\ \Omega$$

$$X_{LR} \simeq X_1 + X_2 = 0.72\ \Omega$$

Class B: $\boxed{X_1 =} 0.72 \times 0.4 = \boxed{0.288\ \Omega}$ $\boxed{X_2 = 0.432\ \Omega}$

DC Resistance Test: $R_1 = \dfrac{V_{dc}}{2 I_{dc}} = \dfrac{36.4}{2 \times 70} = 0.257\ \Omega$

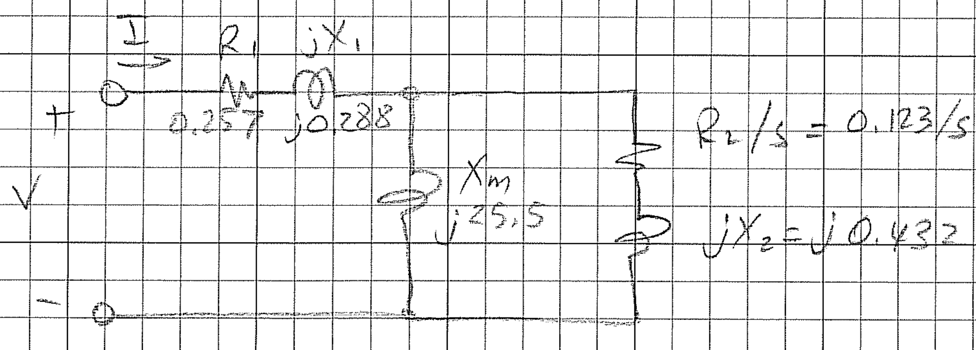
$\boxed{R_2 = 0.123\ \Omega}$ $\boxed{R_1 = 0.257}$

No-Load Test:

$$\frac{V}{\sqrt{3}\, I} \cong X_1 + X_m = \frac{380}{\sqrt{3}\, 8.6} = 25.5\ \Omega$$

$$\boxed{X_m = 25.2\ \Omega}$$

$$P_{rot} = 2148 - 3 \times 8.6^2 \times 0.257 = 2091\ W.$$



b)

$P_{out} = 50 \times 0.746 = 37.3$ kW

$P_{conv} = P_{out} + P_{rot} = 37.3 + 2.1 = 39.4$ kW

The current at rated output is usually given in the locked rotor test!

So: $P_{conv} = 3 R_2 I_2^2 \left(\frac{1-s}{s}\right) \Rightarrow$

$$39.4 \times 10^3 = 3 \times 0.123 \times 70^2 \left(\frac{1-s}{s}\right) \Rightarrow$$

$$21.8 = \left(\frac{1-s}{s}\right) \Rightarrow s = \frac{1}{22.8} = 0.044$$

The equivalent motor impedance:

$$Z_{eq} = R_1 + jX_1 + \left[\frac{1}{jX_m} + \frac{1}{R_2/s + jX_2}\right]^{-1}$$

$$= 0.257 + j0.288 + \left(\frac{1}{j25.2} + \frac{1}{\frac{0.123}{0.044} + j0.432}\right)^{-1}$$

$$= 0.257 + j0.288 + \quad 2.67 + j0.716$$

$$= 2.927 + j1.004$$

$$I = \frac{380}{\sqrt{3} \times (2.927 + j1.004)} = 67.06 - j23.0 = 70.9 \angle -18.93^\circ$$

$$P + jQ = \sqrt{3}\, 380 \times (67.06 + j23.0)$$

$$= 44.14 + j15.14 \quad \text{KVA}$$

c) At starting $s=1$ and the equivalent impedance is given by:

$$Z_{eq(s)} = R_1 + jX_1 + \left((jX_m)^{-1} + (R_2 + jX_2)^{-1} \right)^{-1}$$
$$= 0.257 + j0.288 + \left((j25.2)^{-1} + (0.123 + j0.432)^{-1} \right)^{-1}$$
$$= 0.257 + j0.288 + (0.1189 + j0.4253)$$
$$= 0.3759 + j0.7133$$

$$I_s = \frac{380}{\sqrt{3}\,(0.3759 + j0.7133)} = 126.86 + j240.73 \text{ A}$$

$$= 272.1 \angle -62.2^\circ$$

The air gap power is the input power minus the stator copper losses:

$$P_{AG} = \sqrt{3}\, V I \cos\phi - 3 R_1 I_s^2$$

$$= \sqrt{3}\, 380 \times 272.1 \times \cos 62.2 - 3 \times 0.257 \times 272.1^2$$
$$= 83.523 - 57.084$$
$$= 26.439 \text{ kW}$$

$$\tau_s = \frac{P_{AG}}{\omega_s} = \frac{26.439}{157} = 0.168 \text{ kN.m}$$

The expression for P_{AG} may also be used with $s=1$ and the same answer is obtained!

FE 1

4.

a)

$$V_T = E_A + I_A R_A \qquad (1)$$

$$I_A = I_T - \frac{V_T}{R_F + R_j} = 50 - 2.2 = 47.8\ A$$

$$(1) \Rightarrow \quad E_A = V_T - I_A R_A = 220 - 47.8 \times 0.4 = 201\ V$$

$$\frac{E_{A0}}{n_0} = \frac{E_A}{n} \Rightarrow \quad n = \frac{E_A}{E_{A0}} n_0 = \frac{201}{220} 1200 \Rightarrow$$

$$\boxed{n = 1096\ r/min.}$$

b)

$$P_{ind} = E_A I_A$$

$$P_{out} = E_A I_A - P_{rot} = 201 \times 47.8 - 1000 = \underline{8602\ W}$$

$$\omega = 2\pi \times 1096/60 = 114.8\ rad/s$$

$$T_{out} = \frac{8602}{114.8} = \underline{74.9\ N.m}$$

$$\eta = \frac{P_{out}}{P_{in}} =$$

$P_{out} = 8602\ W$ and $P_{in} = V_T I_T = 220 \times 50 = 11000\ W$

$$\therefore \quad \eta = \frac{8602}{11000} = \underline{78.2\ \%}$$

c)

The new field current is:

$$I_f' = \frac{220}{75} = 2.93A$$

Since the magnetization curve is nearly linear then the flux will change in a proportional manner:

At $R_j = 50\,\Omega$: $T = K\phi I_A = K\phi \times I_A$

At $R_j = 25\,\Omega$: $T' = K\phi' I_A' = K\phi' \times I_A'$

For constant torque: $T' = T \Rightarrow$

$$\phi \times I_A = \phi' I_A' \Rightarrow I_A' = I_A \frac{\phi}{\phi'} = I_A \frac{I_f}{I_f'} \Rightarrow$$

$$I_A' = 47.8 \frac{2.2}{2.93} = \underline{35.9A}$$

The new back emf is given by:

$$E_A' = V_T - I_A' R_A = 220 - 35.9 \times 0.4 = \underline{205.6V}$$

The new speed is calculated from:

$$E_A = K\phi n \quad \text{and} \quad E_A' = K\phi' n'$$

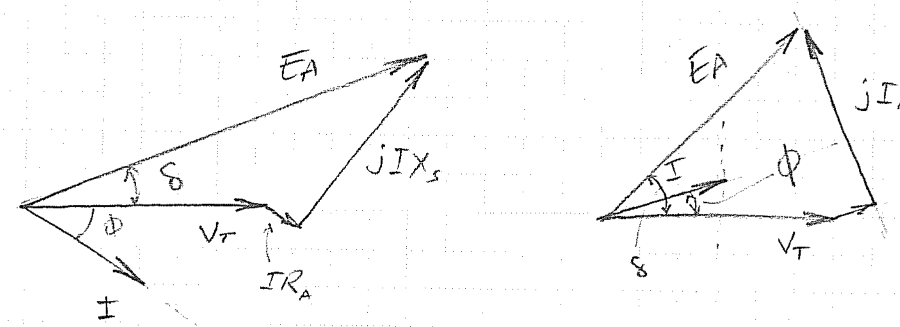
So $$\frac{E_A}{E_A'} = \frac{\phi n}{\phi' n'} \Rightarrow n' = \frac{E_A'}{E_A} \frac{\phi}{\phi'} n \Rightarrow$$

$$n' = \frac{205.6}{201} \times \frac{2.2}{2.93} \times 1096 = \underline{842\ r/min}$$

5.

a) Class A is the standard motor design with a low slip, normal starting torque, but high starting current (5 to 8 per unit). They have a very high pull out torque (3 per unit). Class B motor also have relatively low slip (< than 5%). The starting current is about 25% less for the same starting torque. Their pull out torque is less than that of class A but still appreciable at 2 per unit. Class B is preferred over class A because of its low starting current requirement.

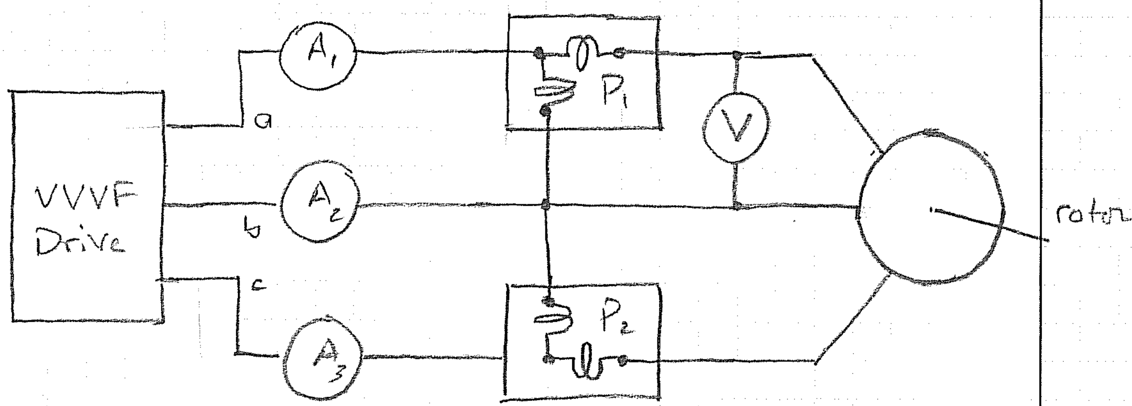
b) lagging leading



Note that $I\cos\phi$ is constant since P is the same. The magnitude of $\bar{E}_A$ is smaller in case of the leading current but its angle δ is much higher than in the lagging case.

The reduction of E_A and increase in δ will make the system less stable.

c)



In the no-load test the rotor is allowed to rotate freely and full load voltage is applied. The total active power is measured ($P_1 + P_2$) along with current at rated voltage and frequency. This test gives the no-load rotational losses (i.e. $P_{core} + P_{FW}$) as the measured power minus stator copper losses:

$$P_{core} + P_{FW} = P_{nl} - 3 I_{nl}^2 R_1$$

R_1 is measured a shown below. The magnetization reactance is also obtained from this test:

$$X_1 + X_m \simeq \frac{V_{nl}}{\sqrt{3}\, I_{nl}}$$

X_1 is determined from the locked rotor test.

In the locked rotor test the rotor is not allowed to rotate and voltage is applied (at rated frequency) until rated current is observed. This test gives the total series impedance of the motor:

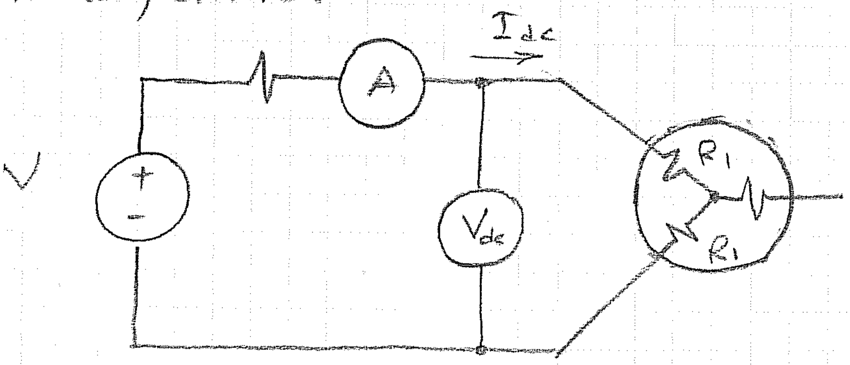
$$(R_1+R_2)+j(X_1+X_2) = |Z_{LR}|(\cos\theta + j\sin\theta)$$

where:

$$|Z_{LR}| = \frac{V_{LR}}{\sqrt{3}\, I_{LR}}$$

$$\theta = \cos^{-1} PF = \cos^{-1} \frac{P_1+P_2}{\sqrt{3}\, V_{LR} I_{LR}}$$

The resistance R_1 is measured in a dc resistance test as follows:



$$2R_1 = \frac{V_{dc}}{I_{dc}} \Rightarrow$$