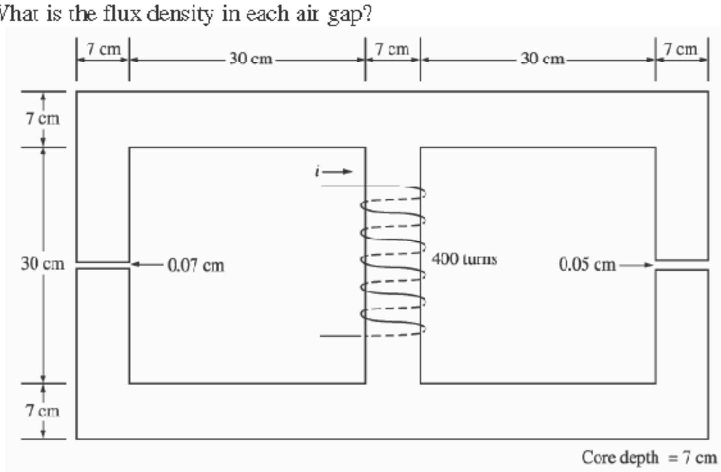
1-6. A ferromagnetic core with a relative permeability of 1500 is shown in Figure P1-3. The dimensions are as shown in the diagram, and the depth of the core is 7 cm. The air gaps on the left and right sides of the core are 0.070 and 0.020 cm, respectively. Because of fringing effects, the effective area of the air gaps is 5 percent larger than their physical size. If there are **400**[1] turns in the coil wrapped around the center leg of the core and if the current in the coil is 1.0 A, what is the flux in each of the left, center, and right legs of the core? What is the flux density in each air gap?



SOLUTION This core can be divided up into five regions. Let $\mathcal{R}_1$ be the reluctance of the left-hand portion of the core, $\mathcal{R}_2$ be the reluctance of the left-hand air gap, $\mathcal{R}_3$ be the reluctance of the right-hand portion of the core, $\mathcal{R}_4$ be the reluctance of the right-hand air gap, and $\mathcal{R}_5$ be the reluctance of the center leg of the core. Then the total reluctance of the core is

$$\mathcal{R}_{TOT} = \mathcal{R}_5 + \frac{(\mathcal{R}_1+\mathcal{R}_2)(\mathcal{R}_3+\mathcal{R}_4)}{\mathcal{R}_1+\mathcal{R}_2+\mathcal{R}_3+\mathcal{R}_4}$$

$$\mathcal{R}_1 = \frac{l_1}{\mu_r\mu_0 A_1} = \frac{1.11\text{ m}}{(2000)(4\pi\times10^{-7}\text{ H/m})(0.07\text{ m})(0.07\text{ m})} = 90.1\text{ kA}\cdot\text{t/Wb}$$

$$\mathcal{R}_2 = \frac{l_2}{\mu_0 A_2} = \frac{0.0007\text{ m}}{(4\pi\times10^{-7}\text{ H/m})(0.07\text{ m})(0.07\text{ m})(1.05)} = 108.3\text{ kA}\cdot\text{t/Wb}$$

$$\mathcal{R}_3 = \frac{l_3}{\mu_r\mu_0 A_3} = \frac{1.11\text{ m}}{(2000)(4\pi\times10^{-7}\text{ H/m})(0.07\text{ m})(0.07\text{ m})} = 90.1\text{ kA}\cdot\text{t/Wb}$$

$$\mathcal{R}_4 = \frac{l_4}{\mu_0 A_4} = \frac{0.0005\text{ m}}{(4\pi\times10^{-7}\text{ H/m})(0.07\text{ m})(0.07\text{ m})(1.05)} = 77.3\text{ kA}\cdot\text{t/Wb}$$

$$\mathcal{R}_5 = \frac{l_5}{\mu_r\mu_0 A_5} = \frac{0.37\text{ m}}{(2000)(4\pi\times10^{-7}\text{ H/m})(0.07\text{ m})(0.07\text{ m})} = 30.0\text{ kA}\cdot\text{t/Wb}$$

The total reluctance is

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_5 + \frac{(\mathcal{R}_1+\mathcal{R}_2)(\mathcal{R}_3+\mathcal{R}_4)}{\mathcal{R}_1+\mathcal{R}_2+\mathcal{R}_3+\mathcal{R}_4} = 30.0 + \frac{(90.1+108.3)(90.1+77.3)}{90.1+108.3+90.1+77.3} = 120.8 \text{ kA}\cdot\text{t/Wb}$$

The total flux in the core is equal to the flux in the center leg:

$$\phi_{\text{center}} = \phi_{\text{TOT}} = \frac{\mathcal{F}}{\mathcal{R}_{\text{TOT}}} = \frac{(400 \text{ t})(1.0 \text{ A})}{120.8 \text{ kA}\cdot\text{t/Wb}} = 0.0033 \text{ Wb}$$

The fluxes in the left and right legs can be found by the "flux divider rule", which is analogous to the current divider rule.

$$\phi_{\text{left}} = \frac{(\mathcal{R}_3+\mathcal{R}_4)}{\mathcal{R}_1+\mathcal{R}_2+\mathcal{R}_3+\mathcal{R}_4}\phi_{\text{TOT}} = \frac{(90.1+77.3)}{90.1+108.3+90.1+77.3}(0.0033 \text{ Wb}) = 0.00193 \text{ Wb}$$

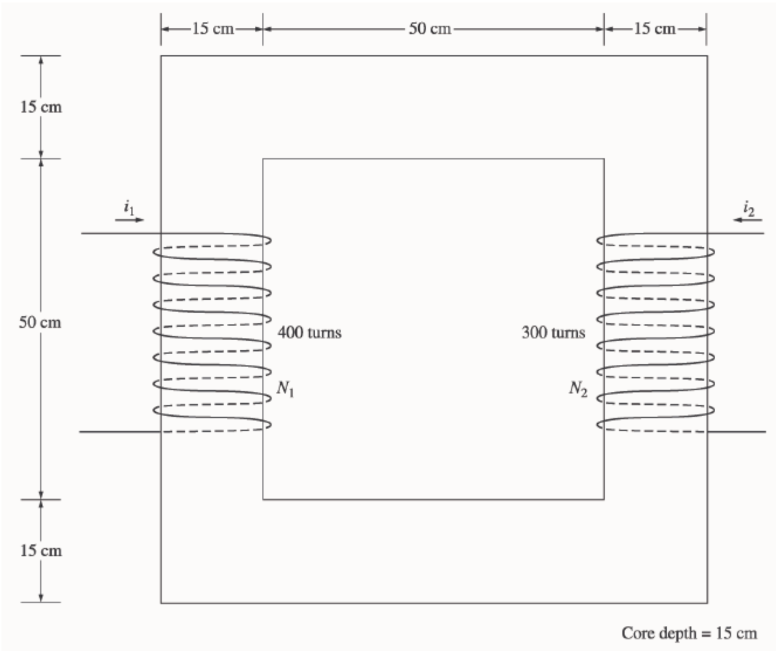
$$\phi_{\text{right}} = \frac{(\mathcal{R}_1+\mathcal{R}_2)}{\mathcal{R}_1+\mathcal{R}_2+\mathcal{R}_3+\mathcal{R}_4}\phi_{\text{TOT}} = \frac{(90.1+108.3)}{90.1+108.3+90.1+77.3}(0.0033 \text{ Wb}) = 0.00229 \text{ Wb}$$

The flux density in the air gaps can be determined from the equation $\phi = BA$:

$$B_{\text{left}} = \frac{\phi_{\text{left}}}{A_{\text{eff}}} = \frac{0.00193 \text{ Wb}}{(0.07 \text{ cm})(0.07 \text{ cm})(1.05)} = 0.375 \text{ T}$$

$$B_{\text{right}} = \frac{\phi_{\text{right}}}{A_{\text{eff}}} = \frac{0.00229 \text{ Wb}}{(0.07 \text{ cm})(0.07 \text{ cm})(1.05)} = 0.445 \text{ T}$$

1-7. A two-legged core is shown in Figure P1-4. The winding on the left leg of the core (N_1) has 400 turns, and the winding on the right (N_2) has 300 turns. The coils are wound in the directions shown in the figure. If the dimensions are as shown, then what flux would be produced by currents $i_1 = 0.5$ A and $i_2 = 0.75$ A? Assume $\mu_r = 1000$ and constant.



SOLUTION The two coils on this core are would so that their magnetomotive forces are additive, so the total magnetomotive force on this core is

$$\mathfrak{F}_{\mathrm{TOT}} = N_1 i_1 + N_2 i_2 = (400\ \mathrm{t})(0.5\ \mathrm{A}) + (300\ \mathrm{t})(0.75\ \mathrm{A}) = 425\ \mathrm{A \cdot t}$$

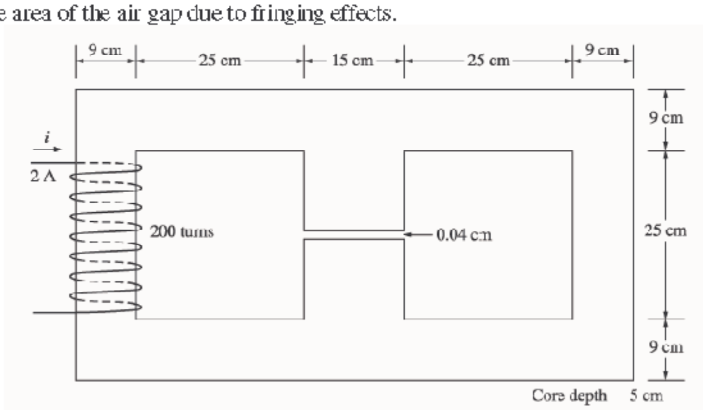
The total reluctance in the core is

$$\mathcal{R}_{\mathrm{TOT}} = \frac{l}{\mu_r \mu_0 A} = \frac{2.60\ \mathrm{m}}{(1000)(4\pi \times 10^{-7}\ \mathrm{H/m})(0.15\ \mathrm{m})(0.15\ \mathrm{m})} = 92.0\ \mathrm{kA \cdot t/Wb}$$

and the flux in the core is:

$$\phi = \frac{\mathfrak{F}_{\mathrm{TOT}}}{\mathcal{R}_{\mathrm{TOT}}} = \frac{425\ \mathrm{A \cdot t}}{92.0\ \mathrm{kA \cdot t/Wb}} = 0.00462\ \mathrm{Wb}$$

1-8. A core with three legs is shown in Figure P1-5. Its depth is 5 cm, and there are 200 turns on the leftmost leg. The relative permeability of the core can be assumed to be 1500 and constant. What flux exists in each of the three legs of the core? What is the flux density in each of the legs? Assume a 4% increase in the effective area of the air gap due to fringing effects.



SOLUTION This core can be divided up into four regions. Let $\mathcal{R}_1$ be the reluctance of the left-hand portion of the core, $\mathcal{R}_2$ be the reluctance of the center leg of the core, $\mathcal{R}_3$ be the reluctance of the center air gap, and $\mathcal{R}_4$ be the reluctance of the right-hand portion of the core. Then the total reluctance of the core is

$$\mathcal{R}_{\mathrm{TOT}} = \mathcal{R}_1 + \frac{(\mathcal{R}_2 + \mathcal{R}_3)\mathcal{R}_4}{\mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}$$

$$\mathcal{R}_1 = \frac{l_1}{\mu_r \mu_0 A_1} = \frac{1.08\ \mathrm{m}}{(1500)(4\pi \times 10^{-7}\ \mathrm{H/m})(0.09\ \mathrm{m})(0.05\ \mathrm{m})} = 127.3\ \mathrm{kA \cdot t/Wb}$$

$$\mathcal{R}_2 = \frac{l_2}{\mu_r \mu_0 A_2} = \frac{0.34\ \mathrm{m}}{(1500)(4\pi \times 10^{-7}\ \mathrm{H/m})(0.15\ \mathrm{m})(0.05\ \mathrm{m})} = 24.0\ \mathrm{kA \cdot t/Wb}$$

$$\mathcal{R}_3 = \frac{l_3}{\mu_0 A_3} = \frac{0.0004\ \mathrm{m}}{(4\pi \times 10^{-7}\ \mathrm{H/m})(0.15\ \mathrm{m})(0.05\ \mathrm{m})(1.04)} = 40.8\ \mathrm{kA \cdot t/Wb}$$

$$\mathcal{R}_4 = \frac{l_4}{\mu_r \mu_0 A_4} = \frac{1.08\ \mathrm{m}}{(1500)(4\pi \times 10^{-7}\ \mathrm{H/m})(0.09\ \mathrm{m})(0.05\ \mathrm{m})} = 127.3\ \mathrm{kA \cdot t/Wb}$$

The total reluctance is

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_1 + \frac{(\mathcal{R}_2 + \mathcal{R}_3)\mathcal{R}_4}{\mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4} = 127.3 + \frac{(24.0 + 40.8)127.3}{24.0 + 40.8 + 127.3} = 170.2 \text{ kA} \cdot \text{t/Wb}$$

The total flux in the core is equal to the flux in the left leg:

$$\phi_{\text{left}} = \phi_{\text{TOT}} = \frac{\mathcal{F}}{\mathcal{R}_{\text{TOT}}} = \frac{(200 \text{ t})(2.0 \text{ A})}{170.2 \text{ kA} \cdot \text{t/Wb}} = 0.00235 \text{ Wb}$$

The fluxes in the center and right legs can be found by the "flux divider rule", which is analogous to the current divider rule.

$$\phi_{\text{center}} = \frac{\mathcal{R}_4}{\mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}\phi_{\text{TOT}} = \frac{127.3}{24.0 + 40.8 + 127.3}(0.00235 \text{ Wb}) = 0.00156 \text{ Wb}$$

$$\phi_{\text{right}} = \frac{\mathcal{R}_2 + \mathcal{R}_3}{\mathcal{R}_2 + \mathcal{R}_3 + \mathcal{R}_4}\phi_{\text{TOT}} = \frac{24.0 + 40.8}{24.0 + 40.8 + 127.3}(0.00235 \text{ Wb}) = 0.00079 \text{ Wb}$$

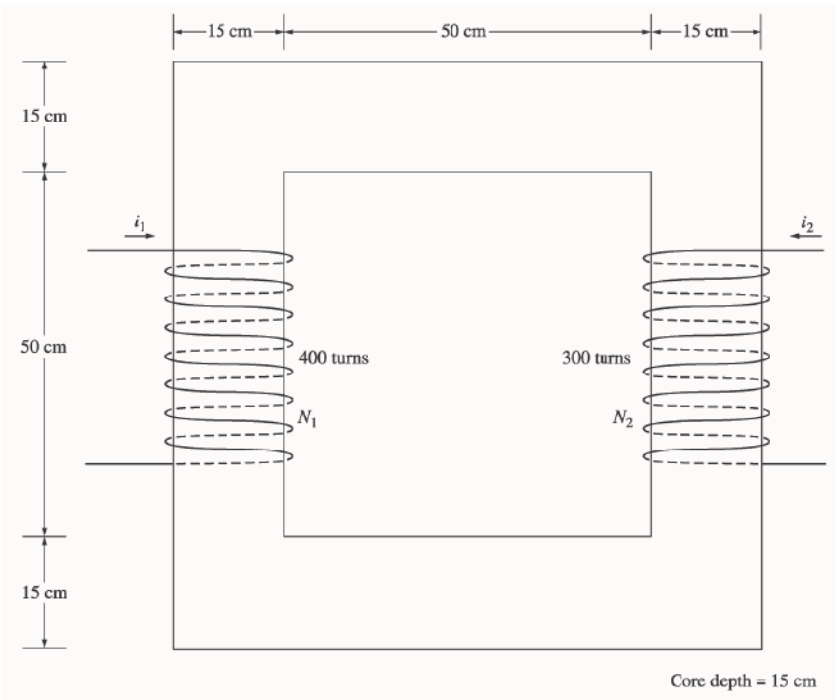
The flux density in the legs can be determined from the equation $\phi = BA$:

$$B_{\text{left}} = \frac{\phi_{\text{left}}}{A} = \frac{0.00235 \text{ Wb}}{(0.09 \text{ cm})(0.05 \text{ cm})} = 0.522 \text{ T}$$

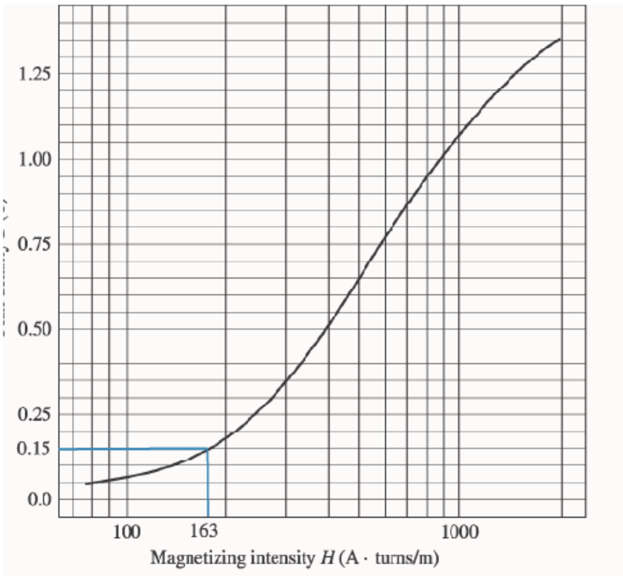
$$B_{\text{center}} = \frac{\phi_{\text{center}}}{A} = \frac{0.00156 \text{ Wb}}{(0.15 \text{ cm})(0.05 \text{ cm})} = 0.208 \text{ T}$$

$$B_{\text{right}} = \frac{\phi_{\text{left}}}{A} = \frac{0.00079 \text{ Wb}}{(0.09 \text{ cm})(0.05 \text{ cm})} = 0.176 \text{ T}$$

1-12. The core shown in Figure P1-4 is made of a steel whose magnetization curve is shown in Figure P1-9. Repeat Problem 1-7, but this time do *not* assume a constant value of μ_r. How much flux is produced in the core by the currents specified? What is the relative permeability of this core under these conditions? Was the assumption in Problem 1-7 that the relative permeability was equal to 1000 a good assumption for these conditions? Is it a good assumption in general?



SOLUTION The magnetization curve for this core is shown below:



The two coils on this core are wound so that their magnetomotive forces are additive, so the total magnetomotive force on this core is

$$\mathcal{F}_{TOT} = N_1 i_1 + N_2 i_2 = (400\text{ t})(0.5\text{ A}) + (300\text{ t})(0.75\text{ A}) = 425\text{ A}\cdot\text{t}$$

Therefore, the magnetizing intensity H is

$$H = \frac{\mathcal{F}}{l_c} = \frac{425\text{ A}\cdot\text{t}}{2.60\text{ m}} = 163\text{ A}\cdot\text{t/m}$$

From the magnetization curve,

$$B = 0.15\text{ T}$$

and the total flux in the core is

$$\phi_{TOT} = BA = (0.15\text{ T})(0.15\text{ m})(0.15\text{ m}) = 0.0033\text{ Wb}$$

The relative permeability of the core can be found from the reluctance as follows:

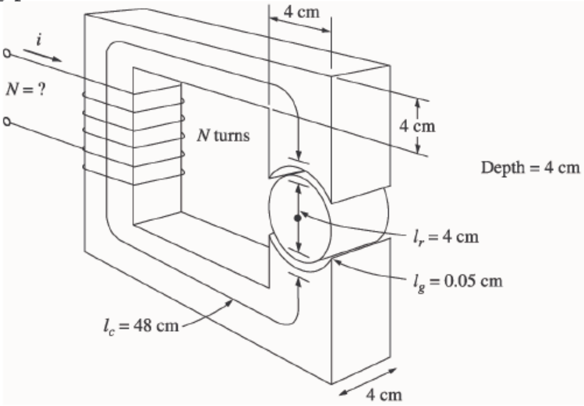
$$\mathcal{R} = \frac{\mathcal{F}_{TOT}}{\phi_{TOT}} = \frac{l}{\mu_r \mu_0 A}$$

Solving for μ_r yields

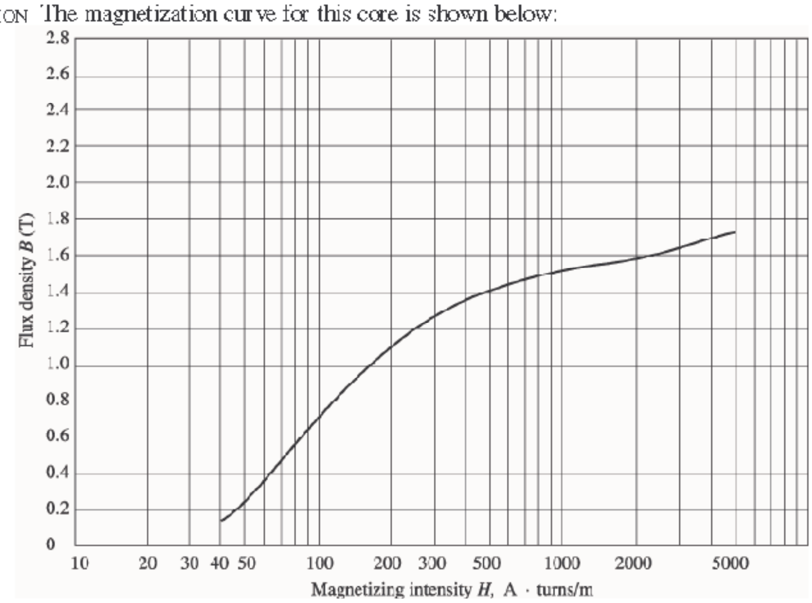
$$\mu_r = \frac{\phi_{TOT}\ l}{\mathcal{F}_{TOT}\ \mu_0\ A} = \frac{(0.0033\text{ Wb})(2.6\text{ m})}{(425\text{ A}\cdot\text{t})(4\pi\times10^{-7}\text{ H/m})(0.15\text{ m})(0.15\text{ m})} = 714$$

The assumption that μ_r = 1000 is not very good here. It is not very good in general.

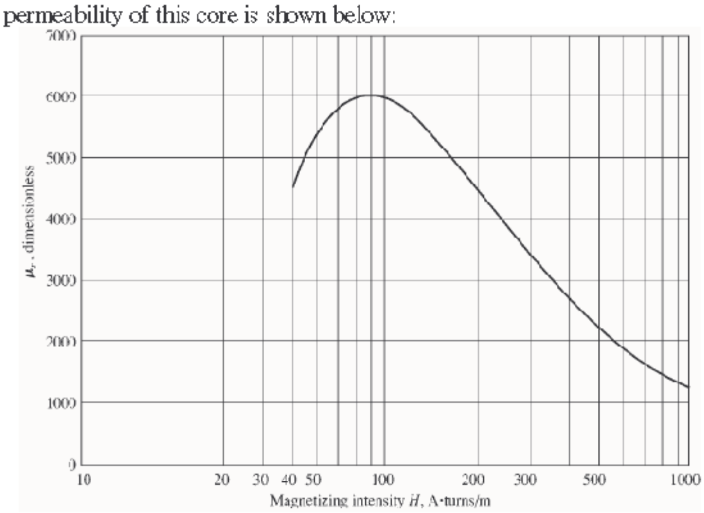
1-17. Figure P1-13 shows the core of a simple dc motor. The magnetization curve for the metal in this core is given by Figure 1-10*c* and *d*. Assume that the cross-sectional area of each air gap is 18 cm^2 and that the width of each air gap is 0.05 cm. The effective diameter of the rotor core is 4 cm.



SOLUTION The magnetization curve for this core is shown below:



The relative permeability of this core is shown below:



Note: This is a design problem, and the answer presented here is not unique. Other values could be selected for the flux density in part *(a)*, and other numbers of turns could be selected in part *(c)*. These other answers are also correct if the proper steps were followed, and if the choices were reasonable.

(a) From Figure 1-10*c*, a reasonable maximum flux density would be about 1.2 T. Notice that the saturation effects become significant for higher flux densities.

(b) At a flux density of 1.2 T, the total flux in the core would be

$$\phi = BA = (1.2\text{ T})(0.04\text{ m})(0.04\text{ m}) = 0.00192\text{ Wb}$$

(c) The total reluctance of the core is:

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_{\text{stator}} + \mathcal{R}_{\text{air gap 1}} + \mathcal{R}_{\text{rotor}} + \mathcal{R}_{\text{air gap 2}}$$

At a flux density of 1.2 T, the relative permeability μ_r of the stator is about 3800, so the stator reluctance is

$$\mathcal{R}_{\text{stator}} = \frac{l_{\text{stator}}}{\mu_{\text{stator}} A_{\text{stator}}} = \frac{0.48\text{ m}}{(3800)(4\pi\times10^{-7}\text{ H/m})(0.04\text{ m})(0.04\text{ m})} = 62.8\text{ kA}\cdot\text{t/Wb}$$

At a flux density of 1.2 T, the relative permeability μ_r of the rotor is 3800, so the rotor reluctance is

$$\mathcal{R}_{\text{rotor}} = \frac{l_{\text{rotor}}}{\mu_{\text{stator}} A_{\text{rotor}}} = \frac{0.04\text{ m}}{(3800)(4\pi\times10^{-7}\text{ H/m})(0.04\text{ m})(0.04\text{ m})} = 5.2\text{ kA}\cdot\text{t/Wb}$$

The reluctance of both air gap 1 and air gap 2 is

$$\mathcal{R}_{\text{air gap 1}} = \mathcal{R}_{\text{air gap 2}} = \frac{l_{\text{air gap}}}{\mu_{\text{air gap}} A_{\text{air gap}}} = \frac{0.0005\text{ m}}{(4\pi\times10^{-7}\text{ H/m})(0.0018\text{ m}^2)} = 221\text{ kA}\cdot\text{t/Wb}$$

Therefore, the total reluctance of the core is

$$\mathcal{R}_{\text{TOT}} = \mathcal{R}_{\text{stator}} + \mathcal{R}_{\text{air gap 1}} + \mathcal{R}_{\text{rotor}} + \mathcal{R}_{\text{air gap 2}}$$

$$\mathcal{R}_{\text{TOT}} = 62.8 + 221 + 5.2 + 221 = 510\text{ kA}\cdot\text{t/Wb}$$

The required MMF is

$$\mathcal{F}_{\text{TOT}} = \phi\mathcal{R}_{\text{TOT}} = (0.00192\text{ Wb})(510\text{ kA}\cdot\text{t/Wb}) = 979\text{ A}\cdot\text{t}$$

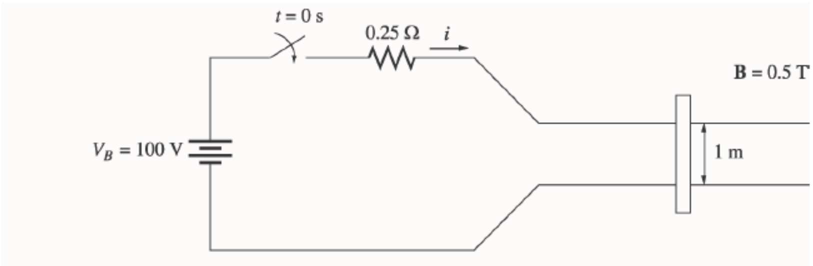
Since $\mathcal{F} = Ni$, and the current is limited to 1 A, one possible choice for the number of turns is $N = 1000$.

1-21. A linear machine has a magnetic flux density of 0.5 T directed into the page, a resistance of 0.25 Ω, a bar length l = 1.0 m, and a battery voltage of 100 V.

(a) What is the initial force on the bar at starting? What is the initial current flow?

(b) What is the no-load steady-state speed of the bar?

(c) If the bar is loaded with a force of 25 N opposite to the direction of motion, what is the new steady-state speed? What is the efficiency of the machine under these circumstances?



SOLUTION

(a) The current in the bar at starting is

$$i = \frac{V_B}{R} = \frac{100\text{ V}}{0.25\ \Omega} = 400\text{ A}$$

Therefore, the force on the bar at starting is

$$\mathbf{F} = i(\mathbf{l} \times \mathbf{B}) = (400\text{ A})(1\text{ m})(0.5\text{ T}) = 200\text{ N, to the right}$$

(b) The no-load steady-state speed of this bar can be found from the equation

$$V_B = e_{\text{ind}} = vBl$$

$$v = \frac{V_B}{Bl} = \frac{100\text{ V}}{(0.5\text{ T})(1\text{ m})} = 200\text{ m/s}$$

(c) With a load of 25 N opposite to the direction of motion, the steady-state current flow in the bar will be given by

$$F_{\text{app}} = F_{\text{ind}} = ilB$$

$$i = \frac{F_{\text{app}}}{Bl} = \frac{25\text{ N}}{(0.5\text{ T})(1\text{ m})} = 50\text{ A}$$

The induced voltage in the bar will be

$$e_{\text{ind}} = V_B - iR = 100\text{ V - }(50\text{ A})(0.25\ \Omega) = 87.5\text{ V}$$

and the velocity of the bar will be

$$v = \frac{V_B}{Bl} = \frac{87.5\text{ V}}{(0.5\text{ T})(1\text{ m})} = 175\text{ m/s}$$

The *input* power to the linear machine under these conditions is

$$P_{\text{in}} = V_B i = (100\text{ V})(50\text{ A}) = 5000\text{ W}$$

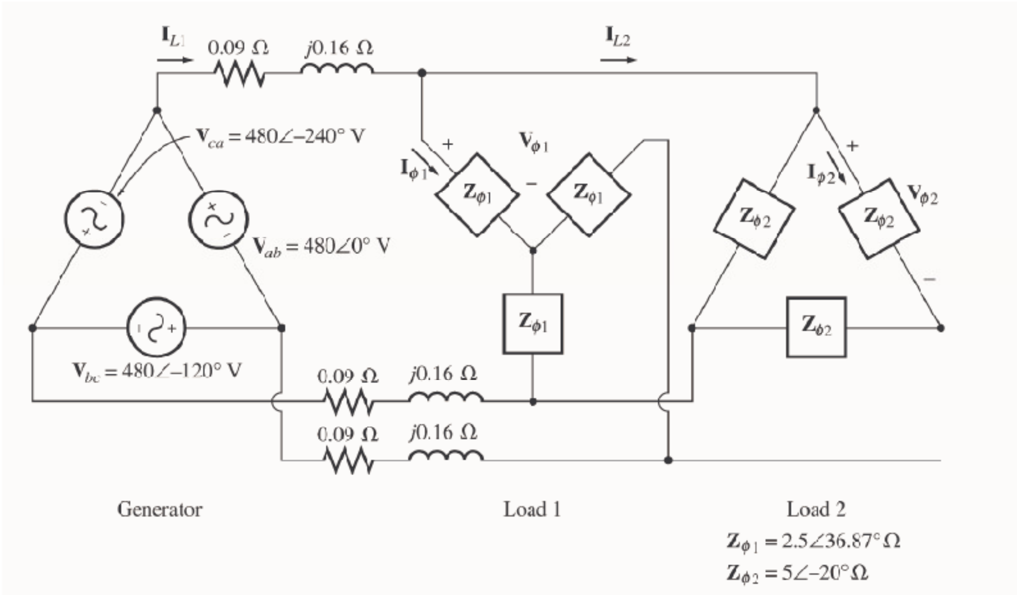
The *output* power from the linear machine under these conditions is

$$P_{\text{out}} = V_B i = (87.5\text{ V})(50\text{ A}) = 4375\text{ W}$$

Therefore, the efficiency of the machine under these conditions is

$$\eta = \frac{P_{\text{out}}}{P_{\text{in}}} \times 100\% = \frac{4375\text{ W}}{5000\text{ W}} \times 100\% = 87.5\%$$

A-2. Figure PA-1 shows a three-phase power system with two loads. The Δ-connected generator is producing a line voltage of 480 V, and the line impedance is 0.09 + j0.16 Ω. Load 1 is Y connected, with a phase impedance of 2.5∠36.87° Ω and load 2 is Δ-connected, with a phase impedance of 5∠-20° Ω.



(a) What is the line voltage of the two loads?

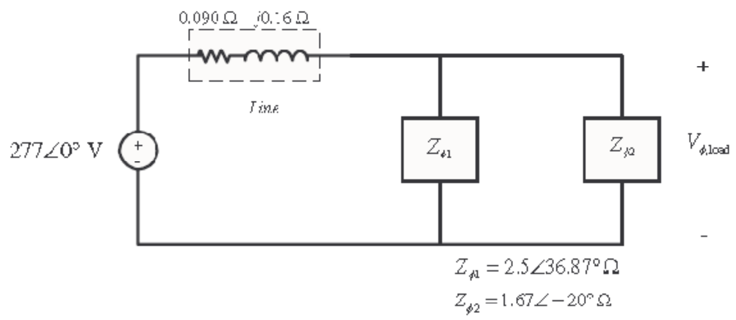
(b) What is the voltage drop on the transmission lines?

(c) Find the real and reactive powers supplied to each load.

(d) Find the real and reactive power losses in the transmission line.

(e) Find the real power, reactive power, and power factor supplied by the generator.

SOLUTION To solve this problem, first convert the two deltas to equivalent wyes, and get the per-phase equivalent circuit.



(a) The phase voltage of the equivalent Y-loads can be found by nodal analysis.

$$\frac{\mathbf{V}_{\phi,\text{load}} - 277\angle 0^\circ \text{ V}}{0.09 + j0.16\ \Omega} + \frac{\mathbf{V}_{\phi,\text{load}}}{2.5\angle 36.87^\circ\ \Omega} + \frac{\mathbf{V}_{\phi,\text{load}}}{1.67\angle -20^\circ\ \Omega} = 0$$

$$(5.443\angle -60.6^\circ)\ \left(\mathbf{V}_{\phi,\text{load}} - 277\angle 0^\circ \text{ V}\right) + (0.4\angle -36.87^\circ)\mathbf{V}_{\phi,\text{load}} + (0.6\angle 20^\circ)\mathbf{V}_{\phi,\text{load}} = 0$$

$$(5.955\angle -53.34^\circ)\ \mathbf{V}_{\phi,\text{load}} = 1508\angle -60.6^\circ$$

$$\mathbf{V}_{\phi,\text{load}} = 253.2\angle -7.3^\circ \text{ V}$$

Therefore, the line voltage at the loads is $V_L \sqrt{3}\ V_\phi = 439$ V .

(b) The voltage drop in the transmission lines is

$$\Delta \mathbf{V}_{\text{line}} = \mathbf{V}_{\phi,\text{gen}} - \mathbf{V}_{\phi,\text{load}} = 277\angle 0° \text{ V} \; - \; 253.2\angle\text{-}7.3° \; = \; 41.3\angle 52° \text{ V}$$

(c) The real and reactive power of each load is

$$P_1 = 3\frac{V_\phi^{\,2}}{Z}\cos\theta = 3\frac{(253.2 \text{ V})^2}{2.5\,\Omega}\cos\, 36.87° = 61.6 \text{ kW}$$

$$Q_1 = 3\frac{V_\phi^{\,2}}{Z}\sin\theta = 3\frac{(253.2 \text{ V})^2}{2.5\,\Omega}\sin\, 36.87° = 46.2 \text{ kvar}$$

$$P_2 = 3\frac{V_\phi^{\,2}}{Z}\cos\theta = 3\frac{(253.2 \text{ V})^2}{1.67\,\Omega}\cos\,(\text{-}20°) = 108.4 \text{ kW}$$

$$Q_2 = 3\frac{V_\phi^{\,2}}{Z}\sin\theta = 3\frac{(253.2 \text{ V})^2}{1.67\,\Omega}\sin\,(\text{-}20°) = -39.5 \text{ kvar}$$

(d) The line current is

$$\mathbf{I}_{\text{line}} = \frac{\Delta \mathbf{V}_{\text{line}}}{Z_{\text{line}}} = \frac{41.3\angle 52° \text{ V}}{0.09 + j0.16\,\Omega} = 225\angle -8.6° \text{ A}$$

Therefore, the loses in the transmission line are

$$P_{\text{line}} = 3I_{\text{line}}^{\;2}R_{\text{line}} = 3\,(225 \text{ A})^2(0.09\,\Omega) = 13.7 \text{ kW}$$

$$Q_{\text{line}} = 3I_{\text{line}}^{\;2}X_{\text{line}} = 3\,(225 \text{ A})^2(0.16\,\Omega) = 24.3 \text{ kvar}$$

(e) The real and reactive power supplied by the generator is

$$P_{\text{gen}} = P_{\text{line}} + P_1 + P_2 = 13.7 \text{ kW} + 61.6 \text{ kW} \; + \; 108.4 \text{ kW} = 183.7 \text{ kW}$$

$$Q_{\text{gen}} = Q_{\text{line}} + Q_1 + Q_2 = 24.3 \text{ kvar} + 46.2 \text{ kvar} \; - \; 39.5 \text{ kvar} = 31 \text{ kvar}$$

The power factor of the generator is

$$\text{PF} = \cos\ \tan^{-1}\frac{Q_{\text{gen}}}{P_{\text{gen}}} = \cos\ \tan^{-1}\frac{31 \text{ kvar}}{183.7 \text{ kW}} = 0.986 \text{ lagging}$$

A-3. Figure PA-2 shows a one-line diagram of a simple power system containing a single 480 V generator and three loads. Assume that the transmission lines in this power system are lossless, and answer the following questions.

(a) Assume that Load 1 is Y-connected. What are the phase voltage and currents in that load?

(b) Assume that Load 2 is Δ-connected. What are the phase voltage and currents in that load?

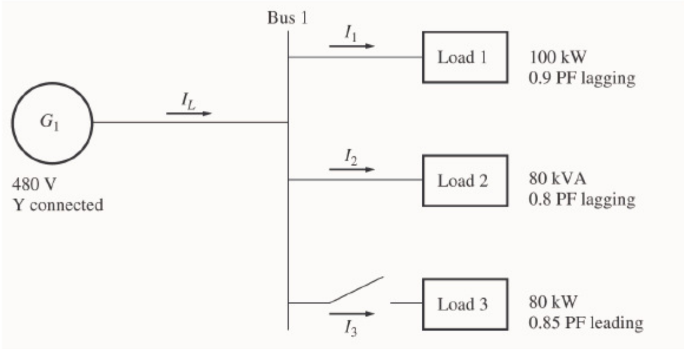
(c) What real, reactive, and apparent power does the generator supply when the switch is open?

(d) What is the total line current I_L when the switch is open?

(e) What real, reactive, and apparent power does the generator supply when the switch is closed?

(f) What is the total line current I_L when the switch is closed?

(g) How does the total line current I_L compare to the sum of the three individual currents $I_1 + I_2 + I_3$? If they are not equal, why not?



SOLUTION Since the transmission lines are lossless in this power system, the full voltage generated by G_1 will be present at each of the loads.

(a) Since this load is Y-connected, the phase voltage is

$$V_{\phi 1} = \frac{480 \text{ V}}{\sqrt{3}} = 277 \text{ V}$$

The phase current can be derived from the equation $P = 3V_\phi I_\phi \cos\theta$ as follows:

$$I_{\phi 1} = \frac{P}{3V_\phi \cos\theta} = \frac{100 \text{ kW}}{3(277 \text{ V})(0.9)} = 133.7 \text{ A}$$

(b) Since this load is Δ-connected, the phase voltage is

$$V_{\phi 2} = 480 \text{ V}$$

The phase current can be derived from the equation $S = 3V_\phi I_\phi$ as follows:

$$I_{\phi 2} = \frac{S}{3V_\phi} = \frac{80 \text{ kVA}}{3(480 \text{ V})} = 55.56 \text{ A}$$

(c) The real and reactive power supplied by the generator when the switch is open is just the sum of the real and reactive powers of Loads 1 and 2.

$$P_1 = 100 \text{ kW}$$

$$Q_1 = P\tan\theta = P\tan\left(\cos^{-1}\text{PF}\right) = (100 \text{ kW})(\tan 25.84°) = 48.4 \text{ kvar}$$

$$P_2 = S\cos\theta = (80 \text{ kVA})(0.8) = 64 \text{ kW}$$

$$Q_2 = S\sin\theta = (80 \text{ kVA})(0.6) = 48 \text{ kvar}$$

$$P_G = P_1 + P_2 = 100 \text{ kW} + 64 \text{ kW} = 164 \text{ kW}$$

$$Q_G = Q_1 + Q_2 = 48.4 \text{ kvar} + 48 \text{ kvar} = 96.4 \text{ kvar}$$

(d) The line current when the switch is open is given by $I_L = \dfrac{P}{\sqrt{3}\, V_L \cos\theta}$, where $\theta = \tan^{-1}\dfrac{Q_G}{P_G}$.

$$\theta = \tan^{-1}\frac{Q_G}{P_G} = \tan^{-1}\frac{96.4 \text{ kvar}}{164 \text{ kW}} = 30.45°$$

$$I_L = \frac{P}{\sqrt{3}\, V_L \cos\theta} = \frac{164 \text{ kW}}{\sqrt{3}(480 \text{ V})\cos(30.45°)} = 228.8 \text{ A}$$

(e) The real and reactive power supplied by the generator when the switch is closed is just the sum of the real and reactive powers of Loads 1, 2, and 3. The powers of Loads 1 and 2 have already been calculated. The real and reactive power of Load 3 are:

$$P_3 = 80 \text{ kW}$$

$$Q_3 - P\tan\theta - P\tan\left(\cos^{-1}\text{PF}\right) - (80 \text{ kW}) \tan(\ \ 31.79\ ) \qquad 49.6 \text{ kvar}$$

$$P_G = P_1 + P_2 + P_3 = 100 \text{ kW} + 64 \text{ kW} + 80 \text{ kW} = 244 \text{ kW}$$

$$Q_G = Q_1 + Q_2 + Q_3 = 48.4 \text{ kvar} + 48 \text{ kvar} - 49.6 \text{ kvar} = 46.8 \text{ kvar}$$

(f) The line current when the switch is closed is given by $I_L = \dfrac{P}{\sqrt{3}\ V_L \cos\theta}$, where $\theta = \tan^{-1}\dfrac{Q_G}{P_G}$.

$$\theta = \tan^{-1}\frac{Q_G}{P_G} = \tan^{-1}\frac{46.8 \text{ kvar}}{244 \text{ kW}} = 10.86°$$

$$I_L = \frac{P}{\sqrt{3}\ V_L \cos\theta} = \frac{244 \text{ kW}}{\sqrt{3}(480 \text{ V})\cos(10.86°)} = 298.8 \text{ A}$$

(g) The total line current from the generator is 298.8 A. The line currents to each individual load are:

$$I_{L1} = \frac{P_1}{\sqrt{3}\ V_L \cos\theta_1} = \frac{100 \text{ kW}}{\sqrt{3}(480 \text{ V})(0.9)} = 133.6 \text{ A}$$

$$I_{L2} = \frac{S_2}{\sqrt{3}\ V_L} = \frac{80 \text{ kVA}}{\sqrt{3}(480 \text{ V})} = 96.2 \text{ A}$$

$$I_{L3} = \frac{P_3}{\sqrt{3}\ V_L \cos\theta_3} = \frac{80 \text{ kW}}{\sqrt{3}(480 \text{ V})(0.85)} = 113.2 \text{ A}$$

The sum of the three individual line currents is 343 A, while the current supplied by the generator is 298.8 A. These values are *not* the same, because the three loads have different impedance angles. Essentially, Load 3 is supplying some of the reactive power being consumed by Loads 1 and 2, so that it does not have to come from the generator.

A-4. Prove that the line voltage of a Y-connected generator with an *acb* phase sequence lags the corresponding phase voltage by 30°. Draw a phasor diagram showing the phase and line voltages for this generator.

SOLUTION If the generator has an *acb* phase sequence, then the three phase voltages will be

$$\mathbf{V}_{an} = V_\phi \angle 0°$$

$$\mathbf{V}_{bn} = V_\phi \angle -240°$$

$$\mathbf{V}_{cn} = V_\phi \angle -120°$$

The relationship between line voltage and phase voltage is derived below. By Kirchhoff's voltage law, the line-to-line voltage $\mathbf{V}_{ab}$ is given by

$$\mathbf{V}_{ab} = \mathbf{V}_a - \mathbf{V}_b$$

$$\mathbf{V}_{ab} = V_\phi \angle 0° - V_\phi \angle -240°$$

$$\mathbf{V}_{ab} = V_\phi - \left(-\frac{1}{2}V_\phi + j\frac{\sqrt{3}}{2}V_\phi\right) = \frac{3}{2}V_\phi - j\frac{\sqrt{3}}{2}V_\phi$$

$$\mathbf{V}_{ab} = \sqrt{3}V_\phi \left(\frac{\sqrt{3}}{2} - j\frac{1}{2}\right)$$

$$\mathbf{V}_{ab} = \sqrt{3}V_\phi \angle -30°$$

Thus the line voltage *lags* the corresponding phase voltage by 30°. The phasor diagram for this connection is shown below.