2-3. A 1000-VA 230/115-V transformer has been tested to determine its equivalent circuit. The results of the tests are shown below.

Open-circuit test	Short-circuit test
$V_{OC} = 230$ V	$V_{SC} = 19.1$ V
$I_{OC} = 0.45$ A	$I_{SC} = 8.7$ A
$P_{OC} = 30$ W	$P_{SC} = 42.3$ W

All data given were taken from the primary side of the transformer.

(a) Find the equivalent circuit of this transformer referred to the low-voltage side of the transformer.

(b) Find the transformer's voltage regulation at rated conditions and (1) 0.8 PF lagging, (2) 1.0 PF, (3) 0.8 PF leading.

(c) Determine the transformer's efficiency at rated conditions and 0.8 PF lagging.

SOLUTION

(a) OPEN CIRCUIT TEST:

$$\left|Y_{\text{EX}}\right| = \left|G_C - jB_M\right| = \frac{0.45\text{ A}}{230\text{ V}} = 0.001957$$

$$\theta = \cos^{-1}\frac{P_{\text{OC}}}{V_{\text{OC}}I_{\text{OC}}} = \cos^{-1}\frac{30\text{ W}}{(230\text{ V})(0.45\text{ A})} = 73.15°$$

$$Y_{\text{EX}} = G_C - jB_M = 0.001957\angle -73.15°\text{ mho} = 0.000567 - j0.001873\text{ mho}$$

$$R_C = \frac{1}{G_C} = 1763\,\Omega$$

$$X_M = \frac{1}{B_M} = 534\,\Omega$$

SHORT CIRCUIT TEST:

$$\left|Z_{EQ}\right| = \left|R_{EQ} + jX_{EQ}\right| = \frac{19.1 \text{ V}}{8.7 \text{ A}} = 2.2\ \Omega$$

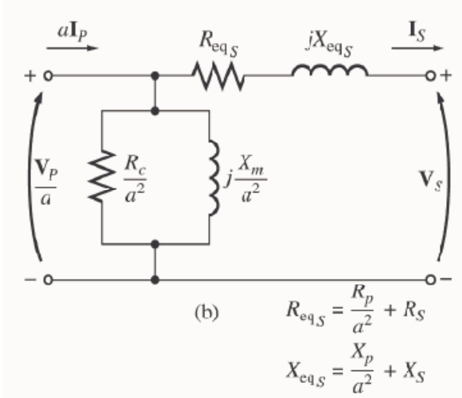
$$\theta = \cos^{-1}\frac{P_{SC}}{V_{SC}I_{SC}} = \cos^{-1}\frac{42.3 \text{ W}}{(19.1 \text{ V})(8.7 \text{ A})} = 75.3°$$

$$Z_{EQ} = R_{EQ} + jX_{EQ} = 2.20\angle 75.3°\ \Omega = 0.558 + j2.128\,\Omega$$

$$R_{EQ} = 0.558\,\Omega$$

$$X_{EQ} = j2.128\,\Omega$$

To convert the equivalent circuit to the secondary side, divide each impedance by the square of the turns ratio (a = 230/115 = 2). The resulting equivalent circuit is shown below:



$R_{EQ,s} = 0.140\,\Omega$ $\quad$ $X_{EQ,s} = j0.532\,\Omega$

$R_{C,s} = 441\Omega$ $\quad$ $X_{M,s} = 134\,\Omega$

(b) To find the required voltage regulation, we will use the equivalent circuit of the transformer referred to the secondary side. The rated secondary current is

$$I_S = \frac{1000\text{ VA}}{115\text{V}} = 8.70\text{ A}$$

We will now calculate the primary voltage referred to the secondary side and use the voltage regulation equation for each power factor.

(1) **<u>0.8 PF Lagging:</u>**

$$\mathbf{V}_P' = \mathbf{V}_S + Z_{EQ}\mathbf{I}_S = 115\angle 0°\text{ V} + (0.140 + j0.532\,\Omega)(8.7\angle -36.87°\text{ A})$$

$$\mathbf{V}_P' = 118.8\angle 1.4°\text{ V}$$

$$\text{VR} = \frac{118.8\text{-}115}{115}\times 100\% = 3.3\%$$

(2) **<u>1.0 PF:</u>**

$$\mathbf{V}_P' = \mathbf{V}_S + Z_{EQ}\mathbf{I}_S = 115\angle 0°\text{ V} + (0.140 + j0.532\,\Omega)(8.7\angle 0°\text{ A})$$

$$\mathbf{V}_P' = 116.3\angle 2.28°\text{ V}$$

$$\text{VR} = \frac{116.3\text{-}115}{115} \times 100\% = 1.1\%$$

(3) **<u>0.8 PF Leading:</u>**

$$\mathbf{V}_P' = \mathbf{V}_S + Z_{\text{EQ}}\mathbf{I}_S = 115\angle 0° \text{ V} + (0.140 + j0.532\ \Omega)(8.7\angle 36.87° \text{ A})$$

$$\mathbf{V}_P' = 113.3\angle 2.24° \text{ V}$$

$$\text{VR} = \frac{113.3\text{-}115}{115} \times 100\% = -1.5\%$$

(c) At rated conditions and 0.8 PF lagging, the output power of this transformer is

$$P_{\text{OUT}} = V_S I_S \cos\theta = (115 \text{ V})(8.7 \text{ A})(0.8) = 800 \text{ W}$$

The copper and core losses of this transformer are

$$P_{\text{CU}} = I_S^2 R_{\text{EQ},S} = (8.7 \text{ A})^2 (0.140\ \Omega) = 10.6 \text{ W}$$

$$P_{\text{core}} = \frac{\left(V_P'\right)^2}{R_C} = \frac{(118.8 \text{ V})^2}{441\ \Omega} = 32.0 \text{ W}$$

Therefore the efficiency of this transformer at these conditions is

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{OUT}} + P_{\text{CU}} + P_{\text{core}}} \times 100\% = \frac{800 \text{ W}}{800 \text{ W} + 10.6 \text{ W} + 32.0 \text{ W}} = 94.9\%$$

2-6. A 15 kVA 8000/230 V distribution transformer has an impedance referred to the primary of 80 + j300 Ω. The components of the excitation branch referred to the primary side are $R_C = 350\text{ k}\Omega$ and $X_M = 70\text{ k}\Omega$.

(a) If the primary voltage is 7967 V and the load impedance is Z_L = 3.2 + j1.5 Ω, what is the secondary voltage of the transformer? What is the voltage regulation of the transformer?

(b) If the load is disconnected and a capacitor of –j3.5 Ω is connected in its place, what is the secondary voltage of the transformer? What is its voltage regulation under these conditions?

SOLUTION

(a) The easiest way to solve this problem is to refer all components to the *primary* side of the transformer. The turns ratio is a = 8000/230 = 34.78. Thus the load impedance referred to the primary side is

$$Z_L' = (34.78)^2(3.2 + j1.5\ \Omega) = 3871 + j1815\ \Omega$$

The referred secondary current is

$$\mathbf{I}_S' = \frac{7967\angle 0°\text{ V}}{(80 + j300\ \Omega) + (3871 + j1815\ \Omega)} = \frac{7967\angle 0°\text{ V}}{4481\angle 28.2°\ \Omega} = 1.78\angle -28.2°\text{ A}$$

and the referred secondary voltage is

$$\mathbf{V}_S' = \mathbf{I}_S' Z_L' = (1.78\angle -28.2°\text{ A})(3871 + j1815\ \Omega) = 7610\angle -3.1°\text{ V}$$

The actual secondary voltage is thus

$$\mathbf{V}_S = \frac{\mathbf{V}_S'}{a} = \frac{7610\angle -3.1°\text{ V}}{34.78} = 218.8\angle -3.1°\text{ V}$$

The voltage regulation is

$$\text{VR} = \frac{7967\text{-}7610}{7610} \times 100\% = 4.7\%$$

(b) The easiest way to solve this problem is to refer all components to the *primary* side of the transformer. The turns ratio is again a = 34.78. Thus the load impedance referred to the primary side is

$$Z_L' = (34.78)^2(-j3.5\ \Omega) = -j4234\ \Omega$$

The referred secondary current is

$$\mathbf{I}_S' = \frac{7967\angle 0°\text{ V}}{(80 + j300\ \Omega) + (-j4234\ \Omega)} = \frac{7967\angle 0°\text{ V}}{3935\angle -88.8°\ \Omega} = 2.025\angle 88.8°\text{ A}$$

and the referred secondary voltage is

$$\mathbf{V}_S' = \mathbf{I}_S' Z_L' = (2.25\angle 88.8°\text{ A})(-j4234\ \Omega) = 8573\angle -1.2°\text{ V}$$

The actual secondary voltage is thus

$$\mathbf{V}_S = \frac{\mathbf{V}_S'}{a} = \frac{8573\angle -1.2°\text{ V}}{34.78} = 246.5\angle -1.2°\text{ V}$$

The voltage regulation is

$$\text{VR} = \frac{7967 - 8573}{8573} \times 100\% = -7.07\%$$

2-8. A 200-MVA 15/200-kV single-phase power transformer has a per-unit resistance of 1.2 percent and a per-unit reactance of 5 percent (data taken from the transformer's nameplate). The magnetizing impedance is $j80$ per unit.

(a) Find the equivalent circuit referred to the low voltage side of this transformer.

(b) Calculate the voltage regulation of this transformer for a full-load current at power factor of 0.8 lagging.

(c) Assume that the primary voltage of this transformer is a constant 15 kV, and plot the secondary voltage as a function of load current for currents from no-load to full-load. Repeat this process for power factors of 0.8 lagging, 1.0, and 0.8 leading.

SOLUTION

(a) The base impedance of this transformer referred to the primary (low-voltage) side is

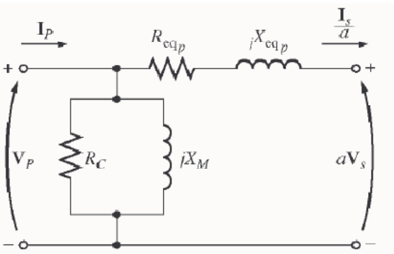
$$Z_{\text{base}} = \frac{V_{\text{base}}^{\ 2}}{S_{\text{base}}} = \frac{(15\text{ kV})^2}{200\text{ MVA}} = 1.125\ \Omega$$

so $R_{\text{EQ}} = (0.012)(1.125\ \Omega) = 0.0135\ \Omega$

$X_{\text{EQ}} = (0.05)(1.125\ \Omega) = 0.0563\ \Omega$

$X_M = (100)(1.125\ \Omega) = 112.5\ \Omega$

The equivalent circuit is



$R_{\text{EQ},P} = 0.0135\ \Omega$ $\quad$ $X_{\text{EQ},P} = j0.0563\ \Omega$

$R_C =$ not specified $\quad$ $X_M = 112.5\ \Omega$

(b) If the load on the *secondary* side of the transformer is 200 MVA at 0.8 PF lagging, and the referred secondary voltage is 15 kV, then the referred secondary current is

$$I_S' = \frac{P_{\text{LOAD}}}{V_S\ \text{PF}} = \frac{200\text{ MVA}}{(15\text{ kV})(0.8)} = 16{,}667\text{ A}$$

$$\mathbf{I}_S' = 16{,}667\angle -36.87°\text{ A}$$

The voltage on the primary side of the transformer is

$$\mathbf{V}_P = \mathbf{V}_S' + \mathbf{I}_S' Z_{\text{EQ},P}$$

$$\mathbf{V}_P = 15{,}000\angle 0°\text{ V} + (16{,}667\angle -36.87°\text{ A})(0.0135 + j0.0563\ \Omega) = 15{,}755\angle 2.24°\text{ V}$$

Therefore the voltage regulation of the transformer is

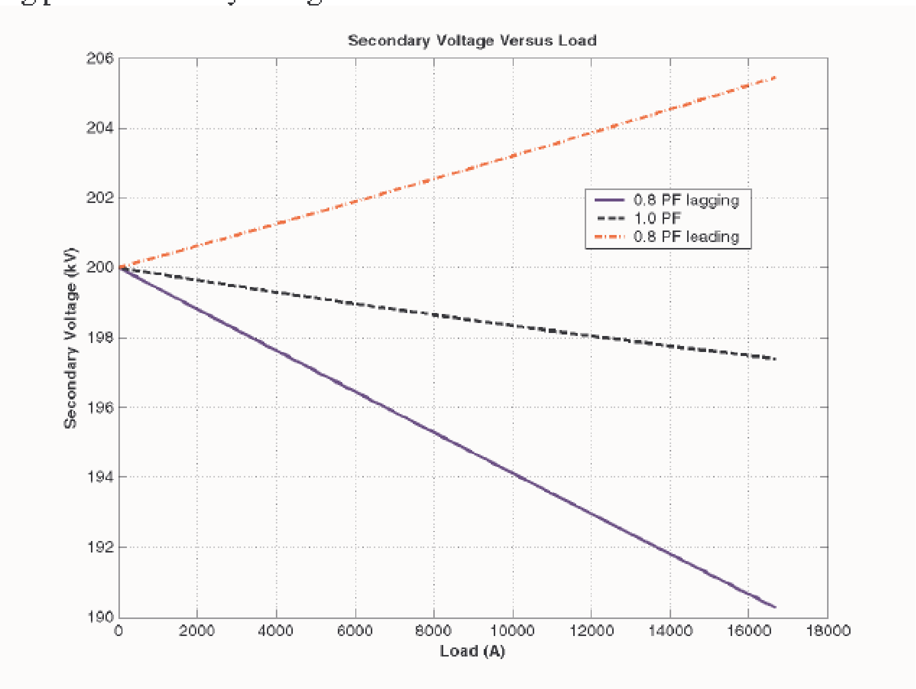
$$\text{VR} = \frac{15{,}755\text{-}15{,}000}{15{,}000} \times 100\% = 5.03\%$$

(c) This problem is repetitive in nature, and is ideally suited for MATLAB. A program to calculate the secondary voltage of the transformer as a function of load is shown below:

```
% M-file: prob2_8.m
% M-file to calculate and plot the secondary voltage
% of a transformer as a function of load for power
% factors of 0.8 lagging, 1.0, and 0.8 leading.
% These calculations are done using an equivalent
% circuit referred to the primary side.

% Define values for this transformer
VP = 15000;                  % Primary voltage (V)
amps = 0:166.67:16667;       % Current values (A)
Req = 0.0135;                % Equivalent R (ohms)
Xeq = 0.0563;                % Equivalent X (ohms)

% Calculate the current values for the three
% power factors.  The first row of I contains
% the lagging currents, the second row contains
% the unity currents, and the third row contains
```

```
% the leading currents.
I(1,:) = amps .* ( 0.8 - j*0.6);  % Lagging
I(2,:) = amps .* ( 1.0        );  % Unity
I(3,:) = amps .* ( 0.8 + j*0.6);  % Leading

% Calculate VS referred to the primary side
% for each current and power factor.
aVS = VP - (Req.*I + j.*Xeq.*I);

% Refer the secondary voltages back to the
% secondary side using the turns ratio.
VS = aVS * (200/15);

% Plot the secondary voltage (in kV!) versus load
plot(amps,abs(VS(1,:)/1000),'b-','LineWidth',2.0);
hold on;
plot(amps,abs(VS(2,:)/1000),'k--','LineWidth',2.0);
plot(amps,abs(VS(3,:)/1000),'r-.','LineWidth',2.0);
title ('\bfSecondary Voltage Versus Load');
xlabel ('\bfLoad (A)');
ylabel ('\bfSecondary Voltage (kV)');
legend('0.8 PF lagging','1.0 PF','0.8 PF leading');
grid on;
hold off;
```

The resulting plot of secondary voltage versus load is shown below:



2-10. A 13,800/480 V three-phase Y-Δ-connected transformer bank consists of three identical 100-kVA 7967/480-V transformers. It is supplied with power directly from a large constant-voltage bus. In the short-circuit test, the recorded values on the high-voltage side for one of these transformers are

$$V_{SC} = 560 \text{ V} \qquad I_{SC} = 12.6 \text{ A} \qquad P_{SC} = 3300 \text{ W}$$

(a) If this bank delivers a rated load at 0.85 PF lagging and rated voltage, what is the line-to-line voltage on the primary of the transformer bank?

(b) What is the voltage regulation under these conditions?

(c) Assume that the primary voltage of this transformer bank is a constant 13.8 kV, and plot the secondary voltage as a function of load current for currents from no-load to full-load. Repeat this process for power factors of 0.85 lagging, 1.0, and 0.85 leading.

(d) Plot the voltage regulation of this transformer as a function of load current for currents from no-load to full load. Repeat this process for power factors of 0.85 lagging, 1.0, and 0.85 leading.

SOLUTION From the short-circuit information, it is possible to determine the per-phase impedance of the transformer bank referred to the high-voltage side. The primary side of this transformer is Y-connected, so the short-circuit phase voltage is

$$V_{\phi SC} = \frac{V_{SC}}{\sqrt{3}} = \frac{560 \text{ V}}{\sqrt{3}} = 323.3 \text{ V}$$

the short-circuit phase current is

$$I_{\phi,SC} = I_{SC} = 12.6 \text{ A}$$

and the power per phase is

$$P_{\phi,SC} = \frac{P_{SC}}{3} = 1100 \text{ W}$$

Thus the per phase impedance is

$$\left|Z_{EQ}\right| = \left|R_{EQ} + jX_{EQ}\right| = \frac{323.3 \text{ V}}{12.6 \text{ A}} = 25.66\ \Omega$$

$$\theta = \cos^{-1}\frac{P_{SC}}{V_{SC} I_{SC}} = \cos^{-1}\frac{1100 \text{ W}}{(323.3 \text{ V})(12.6 \text{ A})} = 74.3°$$

$$Z_{EQ} = R_{EQ} + jX_{EQ} = 25.66\angle 74.3°\ \Omega = 6.94 + j24.7\ \Omega$$

$$R_{EQ} = 6.94\ \Omega$$
$$X_{EQ} = j24.7\ \Omega$$

(a) If this Y-Δ transformer bank delivers rated kVA (300 kVA) at 0.85 power factor lagging while the secondary voltage is at rated value, then each transformer delivers 100 kVA at a voltage of 480 V and 0.85 PF lagging. Referred to the *primary side* of one of the transformers, the load on each transformer is equivalent to 100 kVA at 7967 V and 0.85 PF lagging. The equivalent current flowing in the secondary of one transformer referred to the primary side is

$$I_{\phi,S}' = \frac{100 \text{ kVA}}{7967 \text{ V}} = 12.55 \text{ A}$$

$$\mathbf{I}_{\phi,S}' = 12.55\angle -31.79° \text{ A}$$

The voltage on the primary side of a single transformer is thus

$$\mathbf{V}_{\phi,P} = \mathbf{V}_{\phi,S}' + \mathbf{I}_{\phi,S}' Z_{EQ,P}$$

$$\mathbf{V}_{\phi,P} = 7967\angle 0° \text{ V} + (12.55\angle -31.79° \text{ A})(6.94 + j24.7\ \Omega) = 8207\angle 1.52° \text{ V}$$

The line-to-line voltage on the primary of the transformer is

$$V_{LL,P} = \sqrt{3}\, V_{\phi,P} = \sqrt{3}(8207 \text{ V}) = 14.22 \text{ kV}$$

(b) The voltage regulation of the transformer is

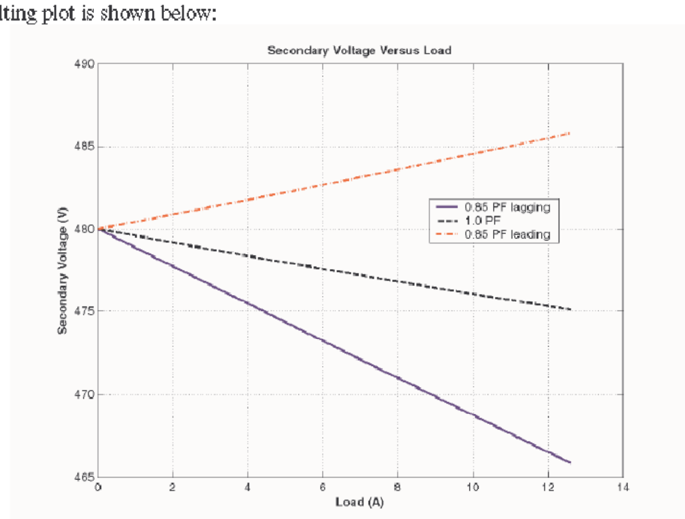
$$\text{VR} = \frac{8207\text{-}7967}{7967} \times 100\% = 3.01\%$$

Note: It is much easier to solve problems of this sort in the per-unit system, as we shall see in the next problem.

(c) This sort of repetitive operation is best performed with MATLAB. A suitable MATLAB program is shown below:

```
% M-file: prob2_10c.m
% M-file to calculate and plot the secondary voltage
% of a three-phase Y-delta transformer bank as a
% function of load for power factors of 0.85 lagging,
% 1.0, and 0.85 leading.  These calculations are done
% using an equivalent circuit referred to the primary side.

% Define values for this transformer
VL = 13800;                   % Primary line voltage (V)
VPP = VL / sqrt(3);           % Primary phase voltage (V)
amps = 0:0.0126:12.6;         % Phase current values (A)
Req = 6.94;                   % Equivalent R (ohms)
Xeq = 24.7;                   % Equivalent X (ohms)

% Calculate the current values for the three
% power factors.  The first row of I contains
% the lagging currents, the second row contains
% the unity currents, and the third row contains
% the leading currents.
re = 0.85;
im = sin(acos(re));
I(1,:) = amps .* ( re - j*im);  % Lagging
I(2,:) = amps .* ( 1.0      );  % Unity
I(3,:) = amps .* ( re + j*im);  % Leading

% Calculate secondary phase voltage referred
% to the primary side for each current and
% power factor.
aVSP = VPP - (Req.*I + j.*Xeq.*I);

% Refer the secondary phase voltages back to
% the secondary side using the turns ratio.
% Because this is a delta-connected secondary,
% this is also the line voltage.
VSP = aVSP * (480/7967);

% Plot the secondary voltage versus load
plot(amps,abs(VSP(1,:)),'b-','LineWidth',2.0);
hold on;
plot(amps,abs(VSP(2,:)),'k--','LineWidth',2.0);
plot(amps,abs(VSP(3,:)),'r-.','LineWidth',2.0);
title ('\bfSecondary Voltage Versus Load');
xlabel ('\bfLoad (A)');
ylabel ('\bfSecondary Voltage (V)');
legend('0.85 PF lagging','1.0 PF','0.85 PF leading');
grid on;
hold off;
```

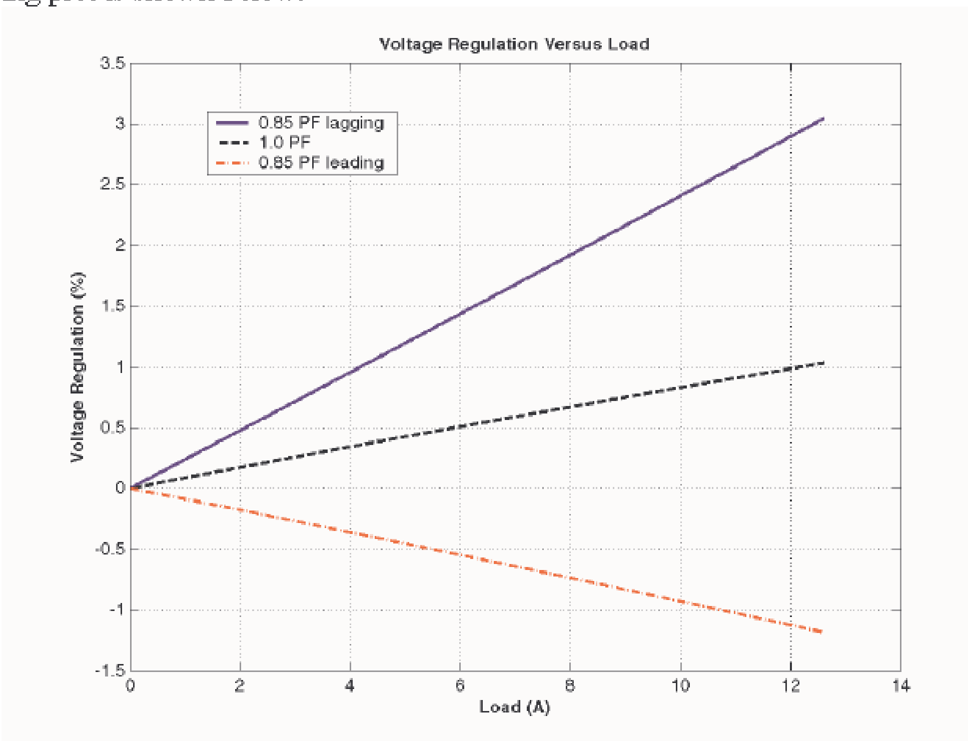
The resulting plot is shown below:



(d) This sort of repetitive operation is best performed with MATLAB. A suitable MATLAB program is shown below:

```
% M-file: prob2_10d.m

% M-file to calculate and plot the voltage regulation
% of a three-phase Y-delta transformer bank as a
% function of load for power factors of 0.85 lagging,
% 1.0, and 0.85 leading.  These calculations are done
% using an equivalent circuit referred to the primary side.

% Define values for this transformer
VL = 13800;                  % Primary line voltage (V)
VPP = VL / sqrt(3);          % Primary phase voltage (V)
amps = 0:0.0126:12.6;        % Phase current values (A)
Req = 6.94;                  % Equivalent R (ohms)
Xeq = 24.7;                  % Equivalent X (ohms)

% Calculate the current values for the three
% power factors.  The first row of I contains
% the lagging currents, the second row contains
% the unity currents, and the third row contains
% the leading currents.
re = 0.85;
im = sin(acos(re));
I(1,:) = amps .* ( re - j*im);  % Lagging
I(2,:) = amps .* ( 1.0      );  % Unity
I(3,:) = amps .* ( re + j*im);  % Leading

% Calculate secondary phase voltage referred
% to the primary side for each current and
% power factor.
aVSP = VPP - (Req.*I + j.*Xeq.*I);

% Calculate the voltage regulation.
VR = (VPP - abs(aVSP)) ./ abs(aVSP) .* 100;

% Plot the voltage regulation versus load
plot(amps,VR(1,:),'b-','LineWidth',2.0);
hold on;
plot(amps,VR(2,:),'k--','LineWidth',2.0);
plot(amps,VR(3,:),'r-.','LineWidth',2.0);
title ('\bfVoltage Regulation Versus Load');
xlabel ('\bfLoad (A)');
ylabel ('\bfVoltage Regulation (%)');
legend('0.85 PF lagging','1.0 PF','0.85 PF leading');
grid on;
hold off;
```

The resulting plot is shown below:



2-11. A 100,000-kVA 230/115-kV Δ-Δ three-phase power transformer has a per-unit resistance of 0.02 pu and a per-unit reactance of 0.055 pu. The excitation branch elements are $R_C = 110$ pu and $X_M = 20$ pu.

(a) If this transformer supplies a load of 80 MVA at 0.85 PF lagging, draw the phasor diagram of one phase of the transformer.

(b) What is the voltage regulation of the transformer bank under these conditions?

(c) Sketch the equivalent circuit referred to the low-voltage side of one phase of this transformer. Calculate all of the transformer impedances referred to the low-voltage side.

SOLUTION

(a) The transformer supplies a load of 80 MVA at 0.85 PF lagging. Therefore, the secondary line current of the transformer is

$$I_{LS} = \frac{S}{\sqrt{3}V_{LS}} = \frac{80{,}000{,}000 \text{ VA}}{\sqrt{3}(115{,}000 \text{ V})} = 402 \text{ A}$$

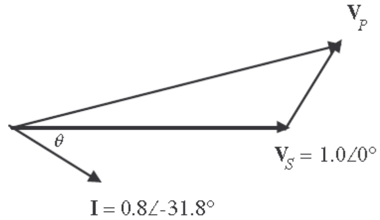
The base value of the secondary line current is

$$I_{LS,\text{base}} = \frac{S_{\text{base}}}{\sqrt{3}V_{LS,\text{base}}} = \frac{100{,}000{,}000 \text{ VA}}{\sqrt{3}(115{,}000 \text{ V})} = 502 \text{ A}$$

so the per-unit secondary current is

$$\mathbf{I}_{LS,\text{pu}} = \frac{I_{LS}}{I_{LS,\text{pu}}} = \frac{402 \text{ A}}{502 \text{ A}} \angle \cos^{-1}(0.85) = 0.8\angle -31.8°$$

The per-unit phasor diagram is shown below:



(b) The per-unit primary voltage of this transformer is

$$\mathbf{V}_P = \mathbf{V}_S + \mathbf{I}\, Z_{EQ} = 1.0\angle 0° + (0.8\angle -31.8°)(0.02 + j0.055) = 1.037\angle 1.6°$$

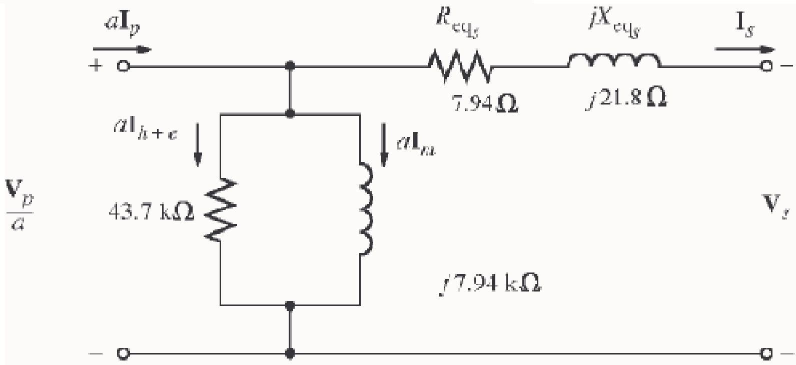
and the voltage regulation is

$$VR = \frac{1.037 - 1.0}{1.0} \times 100\% = 3.7\%$$

(c) The base impedance of the transformer referred to the low-voltage side is:

$$Z_{base} = \frac{3\, V_{\phi,base}{}^2}{S_{base}} = \frac{3(115\text{ kV})^2}{100\text{ MVA}} = 397\ \Omega$$

Each per-unit impedance is converted to actual ohms referred to the low-voltage side by multiplying it by this base impedance. The resulting equivalent circuit is shown below:



$$R_{EQ,S} = (0.02)(397\ \Omega) = 7.94\ \Omega$$
$$X_{EQ,S} = (0.055)(397\ \Omega) = 21.8\ \Omega$$
$$R_C = (110)(397\ \Omega) = 43.7\text{ k}\Omega$$
$$X_M = (20)(397\ \Omega) = 7.94\text{ k}\Omega$$

Note how easy it was to solve this problem in per-unit, compared with Problem 2-10 above.

2-12. An autotransformer is used to connect a 13.2-kV distribution line to a 13.8-kV distribution line. It must be capable of handling 2000 kVA. There are three phases, connected Y-Y with their neutrals solidly grounded.

(a) What must the N_C / N_{SE} turns ratio be to accomplish this connection?

(b) How much apparent power must the windings of each autotransformer handle?

(c) If one of the autotransformers were reconnected as an ordinary transformer, what would its ratings be?

SOLUTION

(a) The transformer is connected Y-Y, so the primary and secondary phase voltages are the line voltages divided by $\sqrt{3}$. The turns ratio of each autotransformer is given by

$$\frac{V_H}{V_L} = \frac{N_C + N_{SE}}{N_C} = \frac{13.8 \text{ kV}/\sqrt{3}}{13.2 \text{ kV}/\sqrt{3}}$$

$$13.2\, N_C + 13.2\, N_{SE} = 13.8\, N_C$$

$$13.2\, N_{SE} = 0.6\, N_C$$

Therefore, $N_C / N_{SE} = 22$.

(b) The power advantage of this autotransformer is

$$\frac{S_{IO}}{S_W} = \frac{N_C + N_{SE}}{N_C} = \frac{N_C + 22 N_C}{N_C} = 23$$

so 1/22 of the power in each transformer goes through the windings. Since 1/3 of the total power is associated with each phase, the windings in each autotransformer must handle

$$S_W = \frac{2000 \text{ kVA}}{(3)(22)} = 30.3 \text{ kVA}$$

(c) The voltages across each phase of the autotransformer are $13.8/\sqrt{3}$ = 7967 V and $13.2/\sqrt{3}$ = 7621 V. The voltage across the common winding (N_C) is 7621 kV, and the voltage across the series winding (N_{SE}) is 7967 kV – 7621 kV = 346 V. Therefore, a single phase of the autotransformer connected as an ordinary transformer would be rated at 7621/346 V and 30.3 kVA.

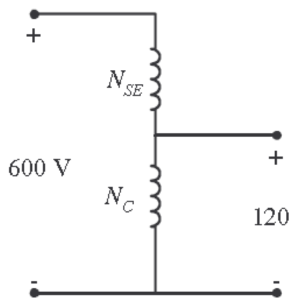
2-15. A 5000-VA 480/120-V conventional transformer is to be used to supply power from a 600-V source to a 120-V load. Consider the transformer to be ideal, and assume that all insulation can handle 600 V.

(a) Sketch the transformer connection that will do the required job.

(b) Find the kilovoltampere rating of the transformer in the configuration.

(c) Find the maximum primary and secondary currents under these conditions.

SOLUTION *(a)* For this configuration, the common winding must be the *smaller* of the two windings, and $N_{SE} = 4N_C$. The transformer connection is shown below:



(b) The kVA rating of the autotransformer can be found from the equation

$$S_{IO} = \frac{N_{SE} + N_C}{N_{SE}} S_W = \frac{4N_C + N_C}{4N_C}(5000 \text{ VA}) = 6250 \text{ VA}$$

(c) The maximum primary current for this configuration will be

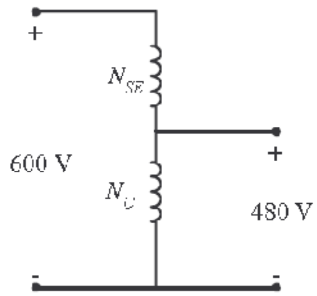
$$I_P = \frac{S}{V_P} = \frac{6250 \text{ VA}}{600 \text{ V}} = 10.4 \text{ A}$$

and the maximum secondary current is

$$I_S = \frac{S}{V_S} = \frac{6250 \text{ VA}}{120 \text{ V}} = 52.1 \text{ A}$$

2-16. A 5000-VA 480/120-V conventional transformer is to be used to supply power from a 600-V source to a 480-V load. Consider the transformer to be ideal, and assume that all insulation can handle 600 V. Answer the questions of Problem 2-15 for this transformer.

SOLUTION *(a)* For this configuration, the common winding must be the *larger* of the two windings, and $N_C = 4N_{SE}$. The transformer connection is shown below:



(b) The kVA rating of the autotransformer can be found from the equation

$$S_{IO} = \frac{N_{SE} + N_C}{N_{SE}} S_W = \frac{N_{SE} + 4N_{SE}}{N_{SE}} (5000 \text{ VA}) = 25{,}000 \text{ VA}$$

(c) The maximum primary current for this configuration will be

$$I_P = \frac{S}{V_P} = \frac{25{,}000 \text{ VA}}{600 \text{ V}} = 41.67 \text{ A}$$

and the maximum secondary current is

$$I_S = \frac{S}{V_S} = \frac{25{,}000 \text{ VA}}{480 \text{ V}} = 52.1 \text{ A}$$

Note that the apparent power handling capability of the autotransformer is *much* higher when there is only a small difference between primary and secondary voltages. Autotransformers are normally only used when there is a small difference between the two voltage levels.

2-19. A 20-kVA 20,000/480-V 60-Hz distribution transformer is tested with the following results:

Open-circuit test (measured from secondary side)	Short-circuit test (measured from primary side)
$V_{OC} = 480$ V	$V_{SC} = 1130$ V
$I_{OC} = 1.60$ A	$I_{SC} = 1.00$ A
$V_{OC} = 305$ W	$P_{SC} = 260$ W

(a) Find the per-unit equivalent circuit for this transformer at 60 Hz.

(b) What would the rating of this transformer be if it were operated on a 50-Hz power system?

(c) Sketch the equivalent circuit of this transformer referred to the primary side *if it is operating at 50 Hz*.

SOLUTION

(a) The base impedance of this transformer referred to the primary side is

$$Z_{\text{base},P} = \frac{(V_P)^2}{S} = \frac{(20{,}000\text{ V})^2}{20\text{ kVA}} = 20\text{ k}\Omega$$

The base impedance of this transformer referred to the secondary side is

$$Z_{\text{base},S} = \frac{(V_S)^2}{S} = \frac{(480\text{ V})^2}{20\text{ kVA}} = 11.52\ \Omega$$

The open circuit test yields the values for the excitation branch (referred to the *secondary* side):

$$|Y_{EX}| = \frac{I_{\phi,OC}}{V_{\phi,OC}} = \frac{1.60\text{ A}}{480\text{ V}} = 0.00333\ \text{S}$$

$$\theta = -\cos^{-1}\frac{P_{OC}}{V_{OC}\,I_{OC}} = -\cos^{-1}\frac{305\text{ W}}{(480\text{ V})(1.60\text{ A})} = -66.6^\circ$$

$$Y_{EX} = G_C - jB_M = 0.00333\angle -66.6^\circ = 0.00132 - j0.00306$$

$$R_C = 1/G_C = 757\ \Omega$$

$$X_M = 1/B_M = 327\ \Omega$$

The excitation branch elements can be expressed in per-unit as

$$R_C = \frac{757\ \Omega}{11.52\ \Omega} = 65.7\text{ pu} \qquad X_M = \frac{327\ \Omega}{11.52\ \Omega} = 28.4\text{ pu}$$

The short circuit test yields the values for the series impedances (referred to the *primary* side):

$$|Z_{EQ}| = \frac{V_{SC}}{I_{SC}} = \frac{1130\text{ V}}{1.00\text{ A}} = 1130\ \Omega$$

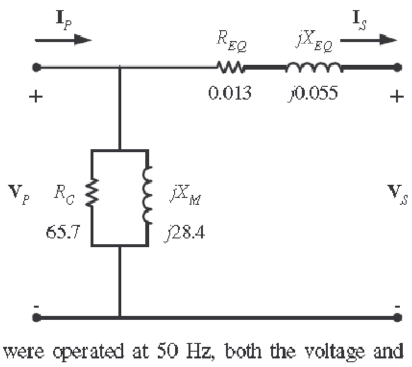
$$\theta = \cos^{-1}\frac{P_{SC}}{V_{SC}\,I_{SC}} = \cos^{-1}\frac{260\text{ W}}{(1130\text{ V})(1.00\text{ A})} = 76.7^\circ$$

$$Z_{EQ} = R_{EQ} + jX_{EQ} = 1130\angle 76.7^\circ = 260 + j1100\ \Omega$$

The resulting per-unit impedances are

$$R_{EQ} = \frac{260\ \Omega}{20{,}000\ \Omega} = 0.013\text{ pu} \qquad X_{EQ} = \frac{1100\ \Omega}{20{,}000\ \Omega} = 0.055\text{ pu}$$

The per-unit equivalent circuit is



(b) If this transformer were operated at 50 Hz, both the voltage and apparent power would have to be derated by a factor of 50/60, so its ratings would be 16.67 kVA, 16,667/400 V, and 50 Hz.

(*c*) The transformer parameters referred to the primary side *at 60 Hz* are:

$$R_C = Z_{\text{base}} R_{C,\text{pu}} = (20\ \text{k}\Omega)(65.7) = 1.31\ \text{M}\Omega$$

$$X_M = Z_{\text{base}} X_{M,\text{pu}} = (20\ \text{k}\Omega)(28.4) = 568\ \text{k}\Omega$$

$$R_{\text{EQ}} = Z_{\text{base}} R_{\text{EQ},\text{pu}} = (20\ \text{k}\Omega)(0.013) = 260\ \Omega$$

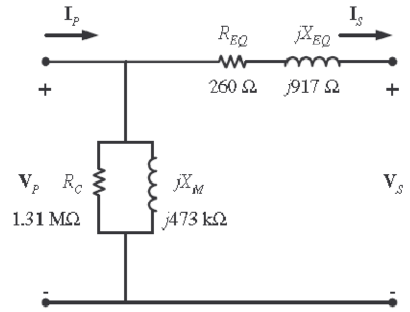
$$X_{\text{EQ}} = Z_{\text{base}} X_{\text{EQ},\text{pu}} = (20\ \text{k}\Omega)(0.055) = 1100\ \Omega$$

At 50 Hz, the resistance will be unaffected but the reactances are reduced in direct proportion to the decrease in frequency. At 50 Hz, the reactances are

$$X_M = \frac{50\ \text{Hz}}{60\ \text{Hz}}(568\ \text{k}\Omega) = 473\ \text{k}\Omega$$

$$X_{\text{EQ}} = \frac{50\ \text{Hz}}{60\ \text{Hz}}(1100\ \Omega) = 917\ \Omega$$

The resulting equivalent circuit referred to the primary at 50 Hz is shown below:



2-22. A single-phase 10-kVA 480/120-V transformer is to be used as an autotransformer tying a 600-V distribution line to a 480-V load. When it is tested as a conventional transformer, the following values are measured on the primary (480-V) side of the transformer:

Open-circuit test	Short-circuit test
$V_{OC} = 480$ V	$V_{SC} = 10.0$ V
$I_{OC} = 0.41$ A	$I_{SC} = 10.6$ A
$P_{OC} = 38$ W	$P_{SC} = 26$ W

(a) Find the per-unit equivalent circuit of this transformer when it is connected in the conventional manner. What is the efficiency of the transformer at rated conditions and unity power factor? What is the voltage regulation at those conditions?

(b) Sketch the transformer connections when it is used as a 600/480-V step-down autotransformer.

(c) What is the kilovoltampere rating of this transformer when it is used in the autotransformer connection?

(d) Answer the questions in *(a)* for the autotransformer connection.

SOLUTION

(a) The base impedance of this transformer referred to the primary side is

$$Z_{\text{base},P} = \frac{(V_P)^2}{S} = \frac{(480 \text{ V})^2}{10 \text{ kVA}} = 23.04\ \Omega$$

The open circuit test yields the values for the excitation branch (referred to the *primary* side):

$$|Y_{EX}| = \frac{I_{\phi,OC}}{V_{\phi,OC}} = \frac{0.41 \text{ A}}{480 \text{ V}} = 0.000854 \text{ S}$$

$$\theta = \cos^{-1}\frac{P_{OC}}{V_{OC} I_{OC}} = \cos^{-1}\frac{38 \text{ W}}{(480 \text{ V})(0.41 \text{ A})} = 78.87°$$

$$Y_{EX} = G_C - jB_M = 0.000854\angle -78.87° = 0.000165 - j0.000838$$

$$R_C = 1/G_C = 6063\ \Omega$$

$$X_M = 1/B_M = 1193\ \Omega$$

The excitation branch elements can be expressed in per-unit as

$$R_C = \frac{6063\ \Omega}{23.04\ \Omega} = 263 \text{ pu} \qquad X_M = \frac{1193\ \Omega}{23.04\ \Omega} = 51.8 \text{ pu}$$

The short circuit test yields the values for the series impedances (referred to the *primary* side):

$$|Z_{EQ}| = \frac{V_{SC}}{I_{SC}} = \frac{10.0 \text{ V}}{10.6 \text{ A}} = 0.943\ \Omega$$

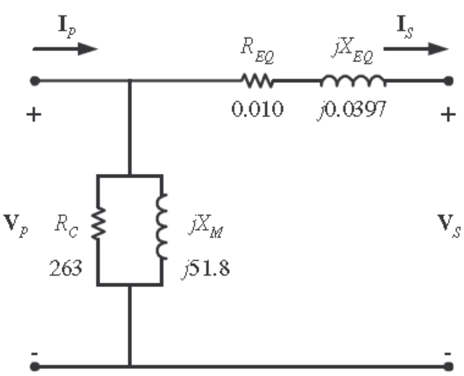
$$\theta = \cos^{-1}\frac{P_{SC}}{V_{SC} I_{SC}} = \cos^{-1}\frac{26 \text{ W}}{(10.0 \text{ V})(10.6 \text{ A})} = 75.8°$$

$$Z_{EQ} = R_{EQ} + jX_{EQ} = 0.943\angle 75.8° = 0.231 + j0.915\ \Omega$$

The resulting per-unit impedances are

$$R_{EQ} = \frac{0.231\ \Omega}{23.04\ \Omega} = 0.010 \text{ pu} \qquad X_{EQ} = \frac{0.915\ \Omega}{23.04\ \Omega} = 0.0397 \text{ pu}$$

The per-unit equivalent circuit is



At rated conditions and unity power factor, the input power to this transformer would be $P_{IN} = 1.0$ pu. The core losses (in resistor R_C) would be

$$P_{core} = \frac{V^2}{R_C} = \frac{(1.0)^2}{263} = 0.00380 \text{ pu}$$

The copper losses (in resistor R_{EQ}) would be

$$P_{CU} = I^2 R_{EQ} = (1.0)^2 (0.010) = 0.010 \text{ pu}$$

The output power of the transformer would be

$$P_{OUT} = P_{OUT} - P_{CU} - P_{core} = 1.0 - 0.010 - 0.0038 = 0.986$$

and the transformer efficiency would be

$$\eta = \frac{P_{OUT}}{P_{IN}} \times 100\% = \frac{0.986}{1.0} \times 100\% = 98.6\%$$

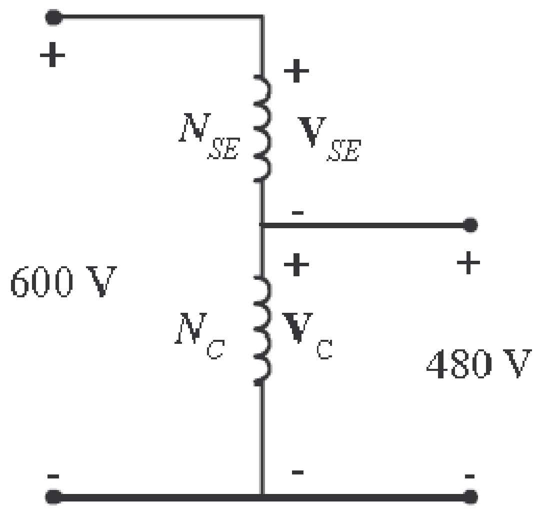
The output voltage of this transformer is

$$\mathbf{V}_{OUT} = \mathbf{V}_{IN} - \mathbf{I}Z_{EQ} = 1.0 - (1.0\angle 0°)(0.01 + j0.0397) = 0.991\angle -2.3°$$

The voltage regulation of the transformer is

$$\text{VR} = \frac{1.0 - 0.991}{0.991} \times 100\% = 0.9\%$$

(b) The autotransformer connection for 600/480 V stepdown operation is



(c) When used as an autotransformer, the kVA rating of this transformer becomes:

$$S_{IO} = \frac{N_C + N_{SE}}{N_{SE}} S_W = \frac{4+1}{1}(10 \text{ kVA}) = 50 \text{ kVA}$$

(d) As an autotransformer, the per-unit series impedance Z_{EQ} is decreased by the reciprocal of the power advantage, so the series impedance becomes

$$R_{EQ} = \frac{0.010}{5} = 0.002 \text{ pu}$$

$$X_{EQ} = \frac{0.0397}{5} = 0.00794 \text{ pu}$$

while the magnetization branch elements are basically unchanged. At rated conditions and unity power factor, the input power to this transformer would be P_{IN} = 1.0 pu. The core losses (in resistor R_C) would be

$$P_{core} = \frac{V^2}{R_C} = \frac{(1.0)^2}{263} = 0.00380 \text{ pu}$$

The copper losses (in resistor R_{EQ}) would be

$$P_{CU} = I^2 R_{EQ} = (1.0)^2 (0.002) = 0.002 \text{ pu}$$

The output power of the transformer would be

$$P_{OUT} = P_{OUT} - P_{CU} - P_{core} = 1.0 - 0.002 - 0.0038 = 0.994$$

and the transformer efficiency would be

$$\eta = \frac{P_{OUT}}{P_{IN}} \times 100\% = \frac{0.994}{1.0} \times 100\% = 99.4\%$$

The output voltage of this transformer is

$$\mathbf{V}_{OUT} = \mathbf{V}_{IN} - \mathbf{I}\mathbf{Z}_{EQ} = 1.0 - (1.0\angle 0°)(0.002 + j0.00794) = 0.998\angle -0.5°$$

The voltage regulation of the transformer is

$$\text{VR} = \frac{1.0 - 0.998}{0.998} \times 100\% = 0.2\%$$