5-1. At a location in Europe, it is necessary to supply 300 kW of 60-Hz power. The only power sources available operate at 50 Hz. It is decided to generate the power by means of a motor-generator set consisting of a synchronous motor driving a synchronous generator. How many poles should each of the two machines have in order to convert 50-Hz power to 60-Hz power?

SOLUTION The speed of a synchronous machine is related to its frequency by the equation

$$n_m = \frac{120 f_e}{P}$$

To make a 50 Hz and a 60 Hz machine have the same mechanical speed so that they can be coupled together, we see that

$$n_{sync} = \frac{120\left(50 \text{ Hz}\right)}{P_1} = \frac{120\left(60 \text{ Hz}\right)}{P_2}$$

$$\frac{P_2}{P_1} = \frac{6}{5} = \frac{12}{10}$$

Therefore, a 10-pole synchronous motor must be coupled to a 12-pole synchronous generator to accomplish this frequency conversion.

5-2. A 2300-V 1000-kVA 0.8-PF-lagging 60-Hz two-pole Y-connected synchronous generator has a synchronous reactance of 1.1 Ω and an armature resistance of 0.15 Ω. At 60 Hz, its friction and windage losses are 24 kW, and its core losses are 18 kW. The field circuit has a dc voltage of 200 V, and the maximum I_F is 10 A. The resistance of the field circuit is adjustable over the range from 20 to 200 Ω. The OCC of this generator is shown in Figure P5-1.

(a) How much field current is required to make V_T equal to 2300 V when the generator is running at no load?

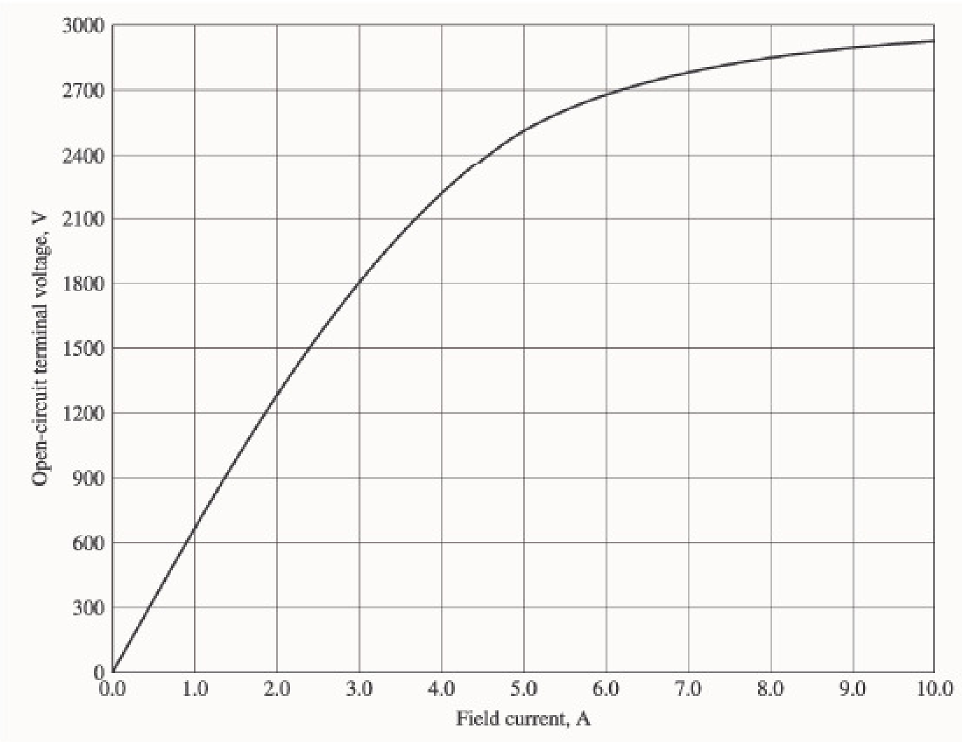
(b) What is the internal generated voltage of this machine at rated conditions?

(c) How much field current is required to make V_T equal to 2300 V when the generator is running at rated conditions?

(d) How much power and torque must the generator's prime mover be capable of supplying?

(e) Construct a capability curve for this generator.

Note: An electronic version of this open circuit characteristic can be found in file `p51_occ.dat`, which can be used with MATLAB programs. Column 1 contains field current in amps, and column 2 contains open-circuit terminal voltage in volts.



SOLUTION

(a) If the no-load terminal voltage is 2300 V, the required field current can be read directly from the open-circuit characteristic. It is 4.25 A.

(b) This generator is Y-connected, so $I_L = I_A$. At rated conditions, the line and phase current in this generator is

$$I_A = I_L = \frac{P}{\sqrt{3}\, V_L} = \frac{1000 \text{ kVA}}{\sqrt{3}(2300 \text{ V})} = 251 \text{ A} \text{ at an angle of } -36.87°$$

The phase voltage of this machine is $V_\phi = V_T / \sqrt{3} = 1328 \text{ V}$. The internal generated voltage of the machine is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A \mathbf{I}_A + jX_S \mathbf{I}_A$$
$$\mathbf{E}_A = 1328\angle 0° + (0.15\ \Omega)(251\angle -36.87° \text{ A}) + j(1.1\ \Omega)(251\angle -36.87° \text{ A})$$
$$\mathbf{E}_A = 1537\angle 7.4° \text{ V}$$

(c) The equivalent open-circuit terminal voltage corresponding to an E_A of 1537 volts is

$$V_{T,oc} = \sqrt{3}(1527 \text{ V}) = 2662 \text{ V}$$

From the OCC, the required field current is 5.9 A.

(d) The input power to this generator is equal to the output power plus losses. The rated output power is

$$P_{\text{OUT}} = (1000 \text{ kVA})(0.8) = 800 \text{ kW}$$

$$P_{\text{CU}} = 3 I_A^{\,2} R_A = 3(251 \text{ A})^2 (0.15\ \Omega) = 28.4 \text{ kW}$$

$$P_{\text{F\&W}} = 24 \text{ kW}$$

$$P_{\text{core}} = 18 \text{ kW}$$

$$P_{\text{stray}} = (\text{assumed } 0)$$

$$P_{\text{IN}} = P_{\text{OUT}} + P_{\text{CU}} + P_{\text{F\&W}} + P_{\text{core}} + P_{\text{stray}} = 870.4 \text{ kW}$$

Therefore the prime mover must be capable of supplying 175 kW. Since the generator is a two-pole 60 Hz. machine, to must be turning at 3600 r/min. The required torque is

$$\tau_{\text{APP}} = \frac{P_{\text{IN}}}{\omega_m} = \frac{175.2 \text{ kW}}{(3600 \text{ r/min})\left(\frac{1 \text{ min}}{60 \text{s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)} = 465 \text{ N} \cdot \text{m}$$

(e) The rotor current limit of the capability curve would be drawn from an origin of

$$Q = -\frac{3V_\phi^2}{X_S} = -\frac{3(1328 \text{ V})^2}{1.1 \, \Omega} = -4810 \text{ kVAR}$$

The radius of the rotor current limit is

$$D_E = \frac{3V_\phi E_A}{X_S} = \frac{3(1328 \text{ V})(1537 \text{ V})}{1.1 \, \Omega} = 5567 \text{ kVA}$$

The stator current limit is a circle at the origin of radius

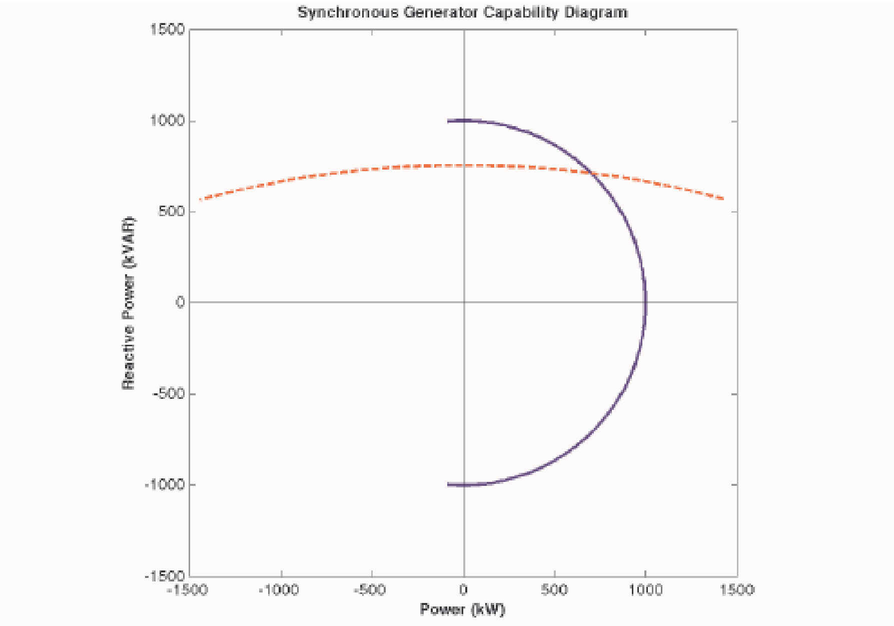
$$S = 3V_\phi I_A = 3(1328 \text{ V})(251 \text{ A}) = 1000 \text{ kVA}$$

A MATLAB program that plots this capability diagram is shown below:

```
% M-file: prob5_2.m
% M-file to display a capability curve for a
% synchronous generator.

% Calculate the waveforms for times from 0 to 1/30 s
Q = -4810;
DE = 5567;
S = 1000;

% Get points for stator current limit
theta = -95:1:95;            % Angle in degrees
rad = theta * pi / 180;      % Angle in radians
s_curve = S .* ( cos(rad) + j*sin(rad) );

% Get points for rotor current limit
orig = j*Q;
theta = 75:1:105;            % Angle in degrees
rad = theta * pi / 180;      % Angle in radians
r_curve = orig + DE .* ( cos(rad) + j*sin(rad) );

% Plot the capability diagram
figure(1);
plot(real(s_curve),imag(s_curve),'b','LineWidth',2.0);
hold on;
plot(real(r_curve),imag(r_curve),'r--','LineWidth',2.0);

% Add x and y axes
```

```
plot( [-1500 1500],[0 0],'k');
plot( [0,0],[-1500 1500],'k');

% Set titles and axes
title ('\bfSynchronous Generator Capability Diagram');
xlabel('\bfPower (kW)');
ylabel('\bfReactive Power (kVAR)');
axis( [ -1500 1500 -1500 1500] );
axis square;
hold off;
```

The resulting capability diagram is shown below:



5-3. Assume that the field current of the generator in Problem 5-2 has been adjusted to a value of 4.5 A.

(a) What will the terminal voltage of this generator be if it is connected to a Δ-connected load with an impedance of $20\angle 30° \ \Omega$?

(b) Sketch the phasor diagram of this generator.

(c) What is the efficiency of the generator at these conditions?

(d) Now assume that another identical Δ-connected load is to be paralleled with the first one. What happens to the phasor diagram for the generator?

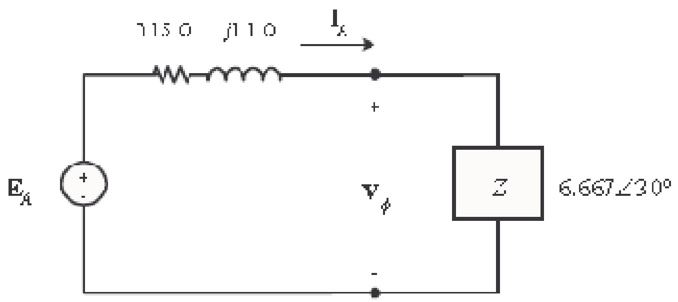
(e) What is the new terminal voltage after the load has been added?

(f) What must be done to restore the terminal voltage to its original value?

SOLUTION

(a) If the field current is 4.5 A, the open-circuit terminal voltage will be about 2385 V, and the phase voltage in the generator will be $2385/\sqrt{3} = 1377$ V.

The load is Δ-connected with three impedances of $20\angle 30° \ \Omega$. From the Y-Δ transform, this load is equivalent to a Y-connected load with three impedances of $6.667\angle 30° \ \Omega$. The resulting per-phase equivalent circuit is shown below:



The magnitude of the phase current flowing in this generator is

$$I_A = \frac{E_A}{|R_A + jX_S + Z|} = \frac{1377\text{ V}}{|0.15 + j1.1 + 6.667\angle 30°|} = \frac{1377\text{ V}}{1.829\ \Omega} = 186\text{ A}$$

Therefore, the magnitude of the phase voltage is

$$V_\phi = I_A Z = (186\text{ A})(6.667\ \Omega) = 1240\text{ V}$$

and the terminal voltage is

$$V_T = \sqrt{3}\,V_\phi = \sqrt{3}\,(1240\text{ V}) = 2148\text{ V}$$

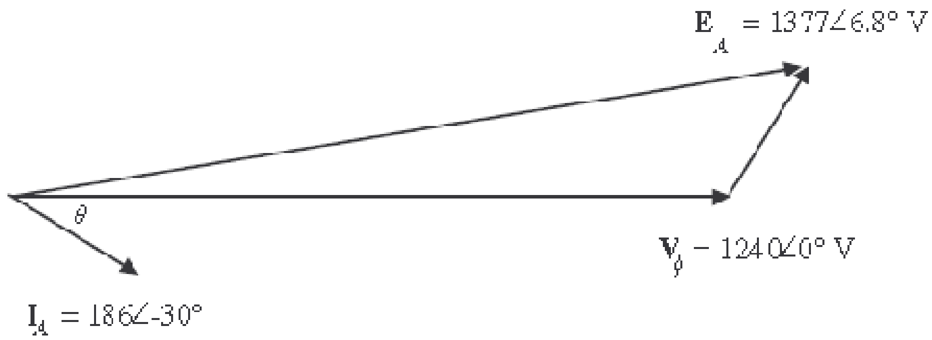
(b) Armature current is $\mathbf{I}_A = 186\angle -30°\text{ A}$, and the phase voltage is $\mathbf{V}_\phi = 1240\angle 0°\text{ V}$. Therefore, the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$

$$\mathbf{E}_A = 1240\angle 0° + (0.15\ \Omega)(186\angle -30°\text{ A}) + j(1.1\ \Omega)(186\angle -30°\text{ A})$$

$$\mathbf{E}_A = 1377\angle 6.8°\text{ V}$$

The resulting phasor diagram is shown below (not to scale):



(c) The efficiency of the generator under these conditions 3can be found as follows:

$$P_{OUT} = 3\,V_\phi\,I_A\cos\theta = 3(1240\text{ V})(186\text{ A})(0.8) = 554\text{ kW}$$

$$P_{CU} = 3I_A^{\,2}R_A = 3(186\text{ A})^2(0.15\ \Omega) = 15.6\text{ kW}$$

$$P_{F\&W} = 24\text{ kW}$$

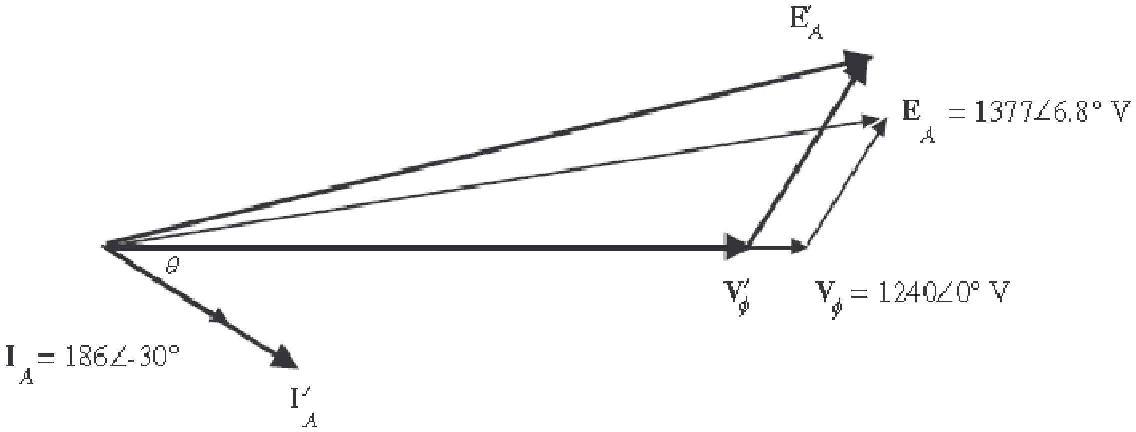
$$P_{core} = 18\text{ kW}$$

$$P_{stray} = (\text{assumed } 0)$$

$$P_{IN} = P_{OUT} + P_{CU} + P_{F\&W} + P_{core} + P_{stray} = 612\text{ kW}$$

$$\eta = \frac{P_{OUT}}{P_{IN}} \times 100\% = \frac{554 \text{ kW}}{612 \text{ kW}} \times 100\% = 90.5\%$$

(d) When the new load is added, the total current flow increases at the same phase angle. Therefore, $jX_S\mathbf{I}_S$ increases in length at the same angle, while the magnitude of E_A must remain constant. Therefore, $\mathbf{E}_A$ "swings" out along the arc of constant magnitude until the new $jX_S\mathbf{I}_S$ fits exactly between $\mathbf{V}_\phi$ and $\mathbf{E}_A$.



(e) The new impedance per phase will be half of the old value, so $Z = 3.333\angle 30°\ \Omega$. The magnitude of the phase current flowing in this generator is

$$I_A = \frac{E_A}{\left|R_A + jX_S + Z\right|} = \frac{1377 \text{ V}}{\left|0.15 + j1.1 + 3.333\angle 30°\right|} = \frac{1377 \text{ V}}{1.829\ \Omega} = 335 \text{ A}$$

Therefore, the magnitude of the phase voltage is

$$V_\phi = I_A Z = (335 \text{ A})(3.333\ \Omega) = 1117 \text{ V}$$

and the terminal voltage is

$$V_T = \sqrt{3}\ V_\phi = \sqrt{3}\ (1117 \text{ V}) = 1934 \text{ V}$$

(f) To restore the terminal voltage to its original value, increase the field current I_F.

5-4. Assume that the field current of the generator in Problem 5-2 is adjusted to achieve rated voltage (2300 V) at full load conditions in each of the questions below.

(a) What is the efficiency of the generator at rated load?

(b) What is the voltage regulation of the generator if it is loaded to rated kilovoltamperes with 0.8-PF-lagging loads?

(c) What is the voltage regulation of the generator if it is loaded to rated kilovoltamperes with 0.8-PF-leading loads?

(d) What is the voltage regulation of the generator if it is loaded to rated kilovoltamperes with unity-power-factor loads?

(e) Use MATLAB to plot the terminal voltage of the generator as a function of load for all three power factors.

SOLUTION

(a) This generator is Y-connected, so $I_L = I_A$. At rated conditions, the line and phase current in this generator is

$$I_A = I_L = \frac{P}{\sqrt{3}\,V_L} = \frac{1000\text{ kVA}}{\sqrt{3}(2300\text{ V})} = 251\text{ A} \text{ at an angle of } -36.87°$$

The phase voltage of this machine is $V_\phi = V_T/\sqrt{3} = 1328\text{ V}$. The internal generated voltage of the machine is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = 1328\angle 0° + (0.15\ \Omega)(251\angle -36.87°\text{ A}) + j(1.1\ \Omega)(251\angle -36.87°\text{ A})$$
$$\mathbf{E}_A = 1537\angle 7.4°\text{ V}$$

The input power to this generator is equal to the output power plus losses. The rated output power is

$$P_{\text{OUT}} = (1000\text{ kVA})(0.8) = 800\text{ kW}$$

$$P_{\text{CU}} = 3I_A^{\,2}R_A = 3(251\text{ A})^2(0.15\ \Omega) = 28.4\text{ kW}$$

$$P_{\text{F\&W}} = 24\text{ kW}$$

$$P_{\text{core}} = 18\text{ kW}$$

$$P_{\text{stray}} = (\text{assumed } 0)$$

$$P_{\text{IN}} = P_{\text{OUT}} + P_{\text{CU}} + P_{\text{F\&W}} + P_{\text{core}} + P_{\text{stray}} = 870.4\text{ kW}$$

$$\eta = \frac{P_{\text{OUT}}}{P_{\text{IN}}} \times 100\% = \frac{800\text{ kW}}{870.4\text{ kW}} \times 100\% = 91.9\%$$

(b) If the generator is loaded to rated kVA with lagging loads, the phase voltage is $\mathbf{V}_\phi = 1328\angle 0°\text{ V}$ and the internal generated voltage is $\mathbf{E}_A = 1537\angle 7.4°\text{ V}$. Therefore, the phase voltage at no-load would be $\mathbf{V}_\phi = 1537\angle 0°\text{ V}$. The voltage regulation would be:

$$\text{VR} = \frac{1537 - 1328}{1328} \times 100\% = 15.7\%$$

(c) If the generator is loaded to rated kVA with leading loads, the phase voltage is $\mathbf{V}_\phi = 1328\angle 0°\text{ V}$ and the internal generated voltage is

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$
$$\mathbf{E}_A = 1328\angle 0° + (0.15\ \Omega)(251\angle 36.87°\text{ A}) + j(1.1\ \Omega)(251\angle 36.87°\text{ A})$$
$$\mathbf{E}_A = 1217\angle 11.5°\text{ V}$$

The voltage regulation would be:

$$\text{VR} = \frac{1217 - 1328}{1328} \times 100\% = -8.4\%$$

(d) If the generator is loaded to rated kVA at unity power factor, the phase voltage is $\mathbf{V}_\phi = 1328\angle 0°\text{ V}$ and the internal generated voltage is

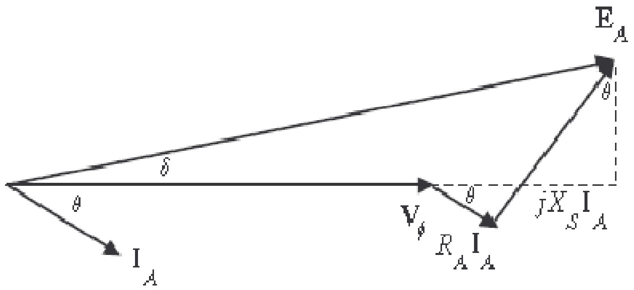
$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$

$$\mathbf{E}_A = 1328\angle 0° + (0.15\ \Omega)(251\angle 0°\ \text{A}) + j(1.1\ \Omega)(251\angle 0°\ \text{A})$$
$$\mathbf{E}_A = 1393\angle 11.4°\ \text{V}$$

The voltage regulation would be:

$$\text{VR} = \frac{1393-1328}{1328} \times 100\% = 4.9\%$$

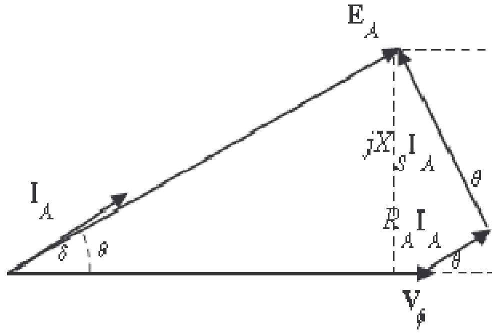
(e) For this problem, we will assume that the terminal voltage is adjusted to 2300 V at no load conditions, and see what happens to the voltage as load increases at 0.8 lagging, unity, and 0.8 leading power factors. Note that the maximum current will be 251 A in any case. A phasor diagram representing the situation at lagging power factor is shown below:



By the Pythagorean Theorem,

$$E_A^{\,2} = \left(V_\phi + R_A I_A \cos\theta + X_S I_A \sin\theta\right)^2 + \left(X_S I_A \cos\theta - R_A I_S \sin\theta\right)^2$$
$$V_\phi = \sqrt{E_A^{\,2} - \left(X_S I_A \cos\theta - R_A I_S \sin\theta\right)^2} - R_A I_A \cos\theta - X_S I_A \sin\theta$$

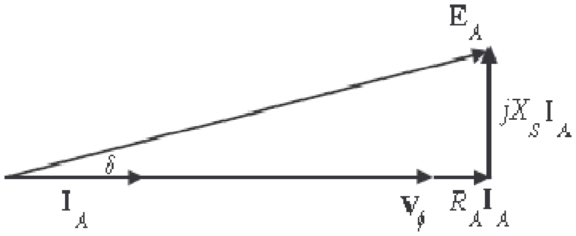
A phasor diagram representing the situation at leading power factor is shown below:



By the Pythagorean Theorem,

$$E_A^{\,2} = \left(V_\phi + R_A I_A \cos\theta - X_S I_A \sin\theta\right)^2 + \left(X_S I_A \cos\theta + R_A I_S \sin\theta\right)^2$$
$$V_\phi = \sqrt{E_A^{\,2} - \left(X_S I_A \cos\theta + R_A I_S \sin\theta\right)^2} - R_A I_A \cos\theta + X_S I_A \sin\theta$$

A phasor diagram representing the situation at unity power factor is shown below:



By the Pythagorean Theorem,

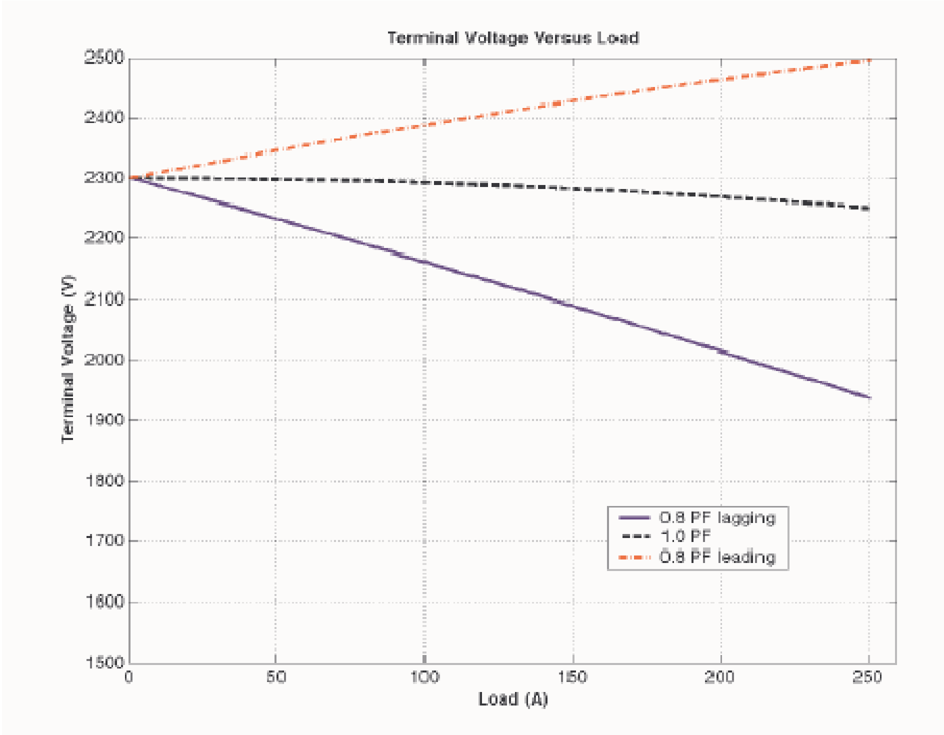
$$E_A{}^2 = V_\phi{}^2 + \left(X_S I_A\right)^2$$

$$V_\phi = \sqrt{E_A{}^2 - \left(X_S I_A\right)^2}$$

The MATLAB program is shown below takes advantage of this fact.

```
% M-file: prob5_4e.m
% M-file to calculate and plot the terminal voltage
% of a synchronous generator as a function of load
% for power factors of 0.8 lagging, 1.0, and 0.8 leading.

% Define values for this generator
EA = 1328;                      % Internal gen voltage
I = 0:2.51:251;                 % Current values (A)
R = 0.15;                       % R (ohms)
X = 1.10;                       % XS (ohms)

% Calculate the voltage for the lagging PF case
VP_lag  = sqrt( EA^2 - (X.*I.*0.8 - R.*I.*0.6).^2 ) ...
        - R.*I.*0.8 - X.*I.*0.6;
VT_lag  = VP_lag .* sqrt(3);

% Calculate the voltage for the leading PF case
VP_lead = sqrt( EA^2 - (X.*I.*0.8 + R.*I.*0.6).^2 ) ...
        - R.*I.*0.8 + X.*I.*0.6;
VT_lead = VP_lead .* sqrt(3);

% Calculate the voltage for the unity PF case
VP_unity = sqrt( EA^2 - (X.*I).^2 );
VT_unity = VP_unity .* sqrt(3);

% Plot the terminal voltage versus load
plot(I,abs(VT_lag),'b-','LineWidth',2.0);
hold on;
plot(I,abs(VT_unity),'k--','LineWidth',2.0);
plot(I,abs(VT_lead),'r-.','LineWidth',2.0);
title ('\bfTerminal Voltage Versus Load');
xlabel ('\bfLoad (A)');
ylabel ('\bfTerminal Voltage (V)');
legend('0.8 PF lagging','1.0 PF','0.8 PF leading');
axis([0 260 1500 2500]);
grid on;
hold off;
```

The resulting plot is shown below:



5-10. A paper mill has installed three steam generators (boilers) to provide process steam and also to use some its waste products as an energy source. Since there is extra capacity, the mill has installed three 5-MW turbine generators to take advantage of the situation. Each generator is a 4160-V 6250-kVA 0.85-PF-lagging two-pole Y-connected synchronous generator with a synchronous reactance of 0.75 Ω and an armature resistance of 0.04 Ω. Generators 1 and 2 have a characteristic power-frequency slope s_P of 2.5 MW/Hz, and generators 2 and 3 have a slope of 3 MW/Hz.

(a) If the no-load frequency of each of the three generators is adjusted to 61 Hz, how much power will the three machines be supplying when actual system frequency is 60 Hz?

(b) What is the maximum power the three generators can supply in this condition without the ratings of one of them being exceeded? At what frequency does this limit occur? How much power does each generator supply at that point?

(c) What would have to be done to get all three generators to supply their rated real and reactive powers at an overall operating frequency of 60 Hz?

(d) What would the internal generated voltages of the three generators be under this condition?

SOLUTION

(a) If the system frequency is 60 Hz and the no-load frequencies of the generators are 61 Hz, then the power supplied by the generators will be

$$P_1 = s_{P1}\left(f_{nl1} - f_{sys}\right) = \left(2.5\text{ MW/Hz}\right)\left(61\text{ Hz} - 60\text{ Hz}\right) = 2.5\text{ MW}$$

$$P_2 = s_{P2}\left(f_{nl2} - f_{sys}\right) = \left(2.5\text{ MW/Hz}\right)\left(61\text{ Hz} - 60\text{ Hz}\right) = 2.5\text{ MW}$$

$$P_3 = s_{P3}\left(f_{nl3} - f_{sys}\right) = \left(3.0\text{ MW/Hz}\right)\left(61\text{ Hz} - 60\text{ Hz}\right) = 3.0\text{ MW}$$

Therefore the total power supplied by the generators is 8 MW.

(b) The maximum power supplied by any one generator is (6250 kVA)(0.85) = 5.31 MW. Generator 3 will be the first machine to reach that limit. Generator 3 will supply this power at a frequency of

$$5.31\text{ MW} = \left(3.0\text{ MW/Hz}\right)\left(61\text{ Hz } - f_{sys}\right)$$

$$f_{sys} = 59.23\text{ Hz}$$

At this point the power supplied by Generators 1 and 2 is

$$P_1 = P_2 = s_{P1}\left(f_{nl1} - f_{sys}\right) = \left(2.5\text{ MW/Hz}\right)\left(61\text{ Hz} - 59.23\text{ Hz}\right) = 4.425\text{ MW}$$

The total power supplied by the generators at this condition is 14.16 MW.

(c) To get each of the generators to supply 5.31 MW at 60 Hz, the no-load frequencies of Generator 1 and Generator 2 would have to be adjusted to 62.12 Hz, and the no-load frequency of Generator 3 would have to be adjusted to 61.77 Hz. The field currents of the three generators must then be adjusted to get them supplying a power factor of 0.85 lagging. At that point, each generator will be supplying its rated real and reactive power.

(d) Under the conditions of part *(c)*, which are the rated conditions of the generators, the internal generated voltage would be given by

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$

The phase voltage of the generators is 4160 V / $\sqrt{3}$ = 2402 V, and since the generators are Y-connected, their rated current is

$$I_A = I_L = \frac{S}{\sqrt{3}\ V_T} = \frac{6250\text{ kVA}}{\sqrt{3}\left(4160\text{ V}\right)} = 867\text{ A}$$

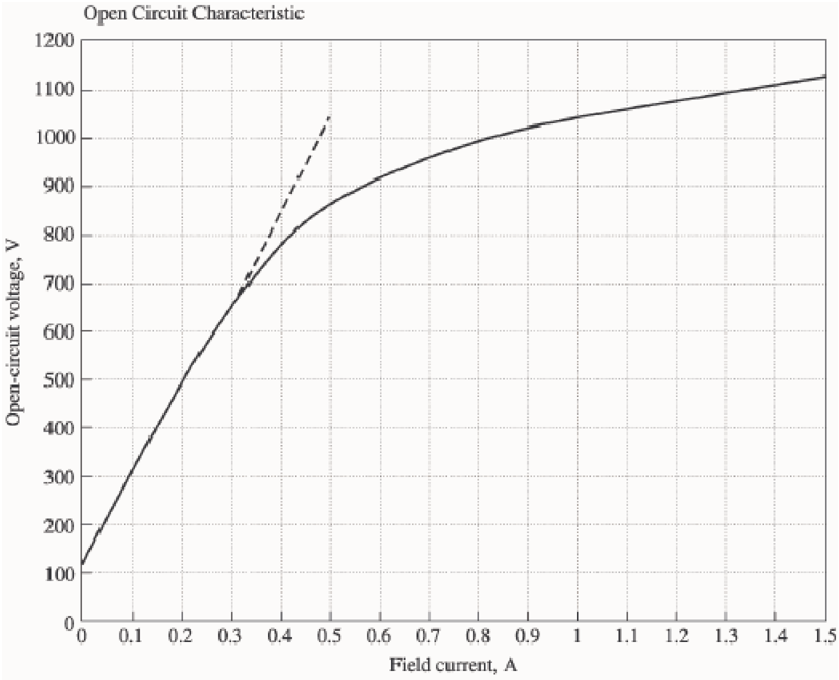
The power factor is 0.85 lagging, so $\mathbf{I}_A = 867\angle -31.8°\text{ A}$. Therefore,

$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$

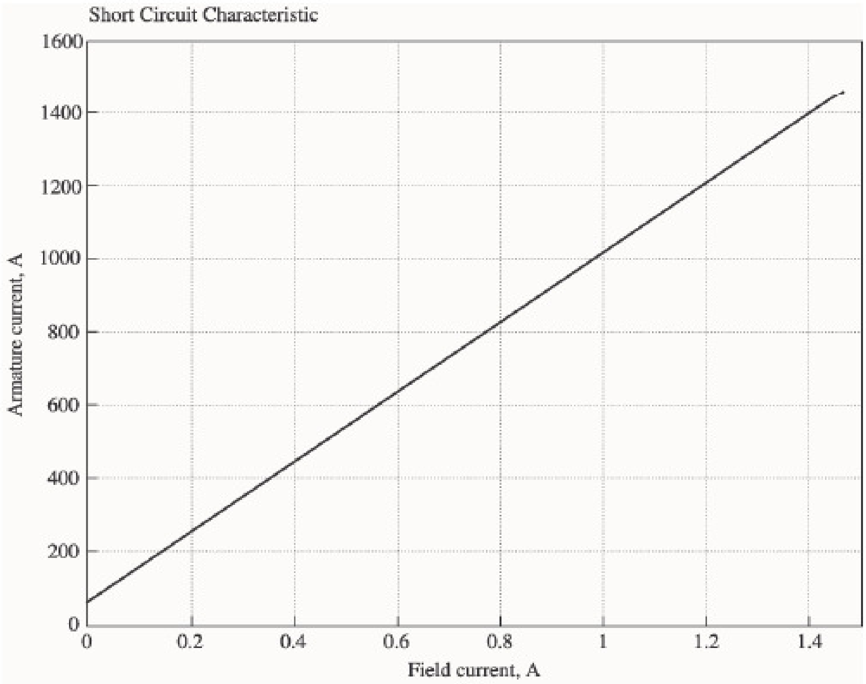
$$\mathbf{E}_A = 2402\angle 0° + \left(0.04\ \Omega\right)\left(867\angle -31.8°\text{ A}\right) + j\left(0.75\ \Omega\right)\left(867\angle -31.8°\text{ A}\right)$$

$\mathbf{E}_A = 2825\angle 10.9° \text{ V}$

Problems 5-11 to 5-21 refer to a two-pole Y-connected synchronous generator rated at 470 kVA, 480 V, 60 Hz, and 0.85 PF lagging. Its armature resistance R_A is 0.016 Ω. The core losses of this generator at rated conditions are 7 kW, and the friction and windage losses are 8 kW. The open-circuit and short-circuit characteristics are shown in Figure P5-2.



(a)



(b)

Note: An electronic version of the saturated open circuit characteristic can be found in file `p52 occ.dat`, and an electronic version of the air-gap characteristic can be found in file `p52_ag_occ.dat`. These files can be used with MATLAB programs. Column 1 contains field current in amps, and column 2 contains open-circuit terminal voltage in volts. An electronic version of the short circuit characteristic can be found in file `p52 scc.dat`. Column 1 contains field current in amps, and column 2 contains short-circuit terminal current in amps.

5-24. Two identical 600-kVA 480-V synchronous generators are connected in parallel to supply a load. The prime movers of the two generators happen to have different speed droop characteristics. When the field currents of the two generators are equal, one delivers 400 A at 0.9 PF lagging, while the other delivers 300 A at 0.72 PF lagging.

(a) What are the real power and the reactive power supplied by each generator to the load?

(b) What is the overall power factor of the load?

(c) In what direction must the field current on each generator be adjusted in order for them to operate at the same power factor?

SOLUTION

(a) The real and reactive powers are

$$P_1 = \sqrt{3}\, V_T\, I_L \cos\theta = \sqrt{3}(480\text{ V})(400\text{ A})(0.9) = 299\text{ kW}$$

$$Q_1 = \sqrt{3}\, V_T\, I_L \sin\theta = \sqrt{3}(480\text{ V})(400\text{ A})\sin\ \cos^{-1}(0.9) = 145\text{ kVAR}$$

$$P_2 = \sqrt{3}\, V_T\, I_L \cos\theta = \sqrt{3}(480\text{ V})(200\text{ A})(0.72) = 120\text{ kW}$$

$$Q_2 = \sqrt{3}\, V_T\, I_L \sin\theta = \sqrt{3}(480\text{ V})(200\text{ A})\sin\ \cos^{-1}(0.72) = 115\text{ kVAR}$$

(b) The overall power factor can be found from the total real and reactive power supplied to the load.

$$P_{\text{TOT}} = P_1 + P_2 = 299\text{ kW} + 120\text{ kW} = 419\text{ kW}$$

$$Q_{\text{TOT}} = Q_1 + Q_2 = 145\text{ kVAR} + 115\text{ kVAR} = 260\text{ kVAR}$$

The overall power factor is

$$\text{PF} = \cos\left(\tan^{-1}\frac{Q_{\text{TOT}}}{P_{\text{TOT}}}\right) = 0.850\text{ lagging}$$

(c) The field current of generator 1 should be increased, and the field current of generator 2 should be simultaneously decreased.

5-25. A generating station for a power system consists of four 120-MVA 15-kV 0.85-PF-lagging synchronous generators with identical speed droop characteristics operating in parallel. The governors on the generators' prime movers are adjusted to produce a 3-Hz drop from no load to full load. Three of these generators are each supplying a steady 75 MW at a frequency of 60 Hz, while the fourth generator (called the *swing generator*) handles all incremental load changes on the system while maintaining the system's frequency at 60 Hz.

(a) At a given instant, the total system loads are 260 MW at a frequency of 60 Hz. What are the no-load frequencies of each of the system's generators?

(b) If the system load rises to 290 MW and the generator's governor set points do not change, what will the new system frequency be?

(c) To what frequency must the no-load frequency of the swing generator be adjusted in order to restore the system frequency to 60 Hz?

(d) If the system is operating at the conditions described in part *(c)*, what would happen if the swing generator were tripped off the line (disconnected from the power line)?

SOLUTION

(a) The full-load power of these generators is $(120\text{ MVA})(0.85) = 102\text{ MW}$ and the droop from no-load to full-load is 3 Hz. Therefore, the slope of the power-frequency curve for these four generators is

$$s_P = \frac{102\text{ MW}}{3\text{ Hz}} = 34\text{ MW/Hz}$$

If generators 1, 2, and 3 are supplying 75 MW each, then generator 4 must be supplying 35 MW. The no-load frequency of the first three generators is

$$P_1 = s_{P1}\left(f_{nl1} - f_{sys}\right)$$

$$75\text{ MW} = \left(34\text{ MW/Hz}\right)\left(f_{nl1} - 60\text{ Hz}\right)$$

$$f_{nl1} = 62.21\text{ Hz}$$

The no-load frequency of the fourth generator is

$$P_4 = s_{P4}\left(f_{nl4} - f_{sys}\right)$$

$$35\text{ MW} = \left(34\text{ MW/Hz}\right)\left(f_{nl1} - 60\text{ Hz}\right)$$

$$f_{nl1} = 61.03\text{ Hz}$$

(b) The setpoints of generators 1, 2, 3, and 4 do not change, so the new system frequency will be

$$P_{\text{LOAD}} = s_{P1}\left(f_{nl1} - f_{sys}\right) + s_{P2}\left(f_{nl2} - f_{sys}\right) + s_{P3}\left(f_{nl3} - f_{sys}\right) + s_{P4}\left(f_{nl4} - f_{sys}\right)$$

$$290\text{ MW} = (34)\left(62.21 - f_{sys}\right) + (34)\left(62.21 - f_{sys}\right) + (34)\left(62.21 - f_{sys}\right) + (34)\left(61.03 - f_{sys}\right)$$

$$8.529 = 247.66 - 4 f_{sys}$$

$$f_{sys} = 59.78\text{ Hz}$$

(c) The governor setpoints of the swing generator must be increased until the system frequency rises back to 60 Hz. At 60 Hz, the other three generators will be supplying 75 MW each, so the swing generator must supply 290 MW – 3(75 MW) = 65 MW at 60 Hz. Therefore, the swing generator's setpoints must be set to

$$P_4 = s_{P4}\left(f_{nl4} - f_{sys}\right)$$

$$65\text{ MW} = \left(34\text{ MW/Hz}\right)\left(f_{nl1} - 60\text{ Hz}\right)$$

$$f_{nl1} = 61.91\text{ Hz}$$

(d) If the swing generator trips off the line, the other three generators would have to supply all 290 MW of the load. Therefore, the system frequency will become

$$P_{\text{LOAD}} = s_{P1}\left(f_{nl1} - f_{sys}\right) + s_{P2}\left(f_{nl2} - f_{sys}\right) + s_{P3}\left(f_{nl3} - f_{sys}\right)$$

$$290\text{ MW} = (34)\left(62.21 - f_{sys}\right) + (34)\left(62.21 - f_{sys}\right) + (34)\left(62.21 - f_{sys}\right)$$

$$8.529 = 186.63 - 3 f_{sys}$$

$$f_{sys} = 59.37\text{ Hz}$$

Each generator will supply 96.7 MW to the loads.

5-27. A 25-MVA three-phase 13.8-kV two-pole 60-Hz synchronous generator was tested by the open-circuit test, and its air-gap voltage was extrapolated with the following results:

Open-circuit test

Field current, A	320	365	380	475	570
Line voltage, kV	13.0	13.8	14.1	15.2	16.0
Extrapolated air-gap voltage, kV	15.4	17.5	18.3	22.8	27.4

The short-circuit test was then performed with the following results:

Short-circuit test

Field current, A	320	365	380	475	570
Armature current, A	1040	1190	1240	1550	1885

The armature resistance is 0.24 Ω per phase.

(a) Find the unsaturated synchronous reactance of this generator in ohms per phase and in per-unit.

(b) Find the approximate saturated synchronous reactance X_S at a field current of 380 A. Express the answer both in ohms per phase and in per-unit.

(c) Find the approximate saturated synchronous reactance at a field current of 475 A. Express the answer both in ohms per phase and in per-unit.

(d) Find the short-circuit ratio for this generator.

SOLUTION

(a) The unsaturated synchronous reactance of this generator is the same at any field current, so we will look at it at a field current of 380 A. The extrapolated air-gap voltage at this point is 18.3 kV, and the short-circuit current is 1240 A. Since this generator is Y-connected, the phase voltage is $V_\phi = 18.3 \text{ kV}/\sqrt{3} = 10,566 \text{ V}$ and the armature current is $I_A = 1240 \text{ A}$. Therefore, the *unsaturated* synchronous reactance is

$$X_{Su} = \frac{10,566 \text{ V}}{1240 \text{ A}} = 8.52 \ \Omega$$

The base impedance of this generator is

$$Z_{base} = \frac{3 V_{\phi,base}^{\ 2}}{S_{base}} = \frac{3(7967 \text{ V})^2}{25,000,000 \text{ VA}} = 7.62 \ \Omega$$

Therefore, the per-unit unsaturated synchronous reactance is

$$X_{Su,pu} = \frac{8.52 \ \Omega}{7.62 \ \Omega} = 1.12$$

(b) The saturated synchronous reactance at a field current of 380 A can be found from the OCC and the SCC. The OCC voltage at $I_F = 380$ A is 14.1 kV, and the short circuit current is 1240 A. Since this generator is Y-connected, the corresponding phase voltage is $V_\phi = 14.1 \text{ kV}/\sqrt{3} = 8141 \text{ V}$ and the armature current is $I_A = 1240 \text{ A}$. Therefore, the *saturated* synchronous reactance is

$$X_{Su} = \frac{8141 \text{ V}}{1240 \text{ A}} = 6.57 \ \Omega$$

and the per-unit unsaturated synchronous reactance is

$$X_{Su,pu} = \frac{6.57 \ \Omega}{7.62 \ \Omega} = 0.862$$

(c) The saturated synchronous reactance at a field current of 475 A can be found from the OCC and the SCC. The OCC voltage at $I_F = 475$ A is 15.2 kV, and the short-circuit current is 1550 A. Since this generator is Y-connected, the corresponding phase voltage is $V_\phi = 15.2 \text{ kV}/\sqrt{3} = 8776 \text{ V}$ and the armature current is $I_A = 1550 \text{ A}$. Therefore, the *saturated* synchronous reactance is

$$X_{Su} = \frac{8776 \text{ V}}{1550 \text{ A}} = 5.66 \ \Omega$$

and the per-unit unsaturated synchronous reactance is

$$X_{Su,pu} = \frac{5.66 \ \Omega}{7.62 \ \Omega} = 0.743$$

(d) The rated voltage of this generator is 13.8 kV, which requires a field current of 365 A. The rated line and armature current of this generator is

$$I_L = \frac{25 \text{ MVA}}{\sqrt{3}(13.8 \text{ kV})} = 1046 \text{ A}$$

The field current required to produce a short-circuit current of 10465 A is about 320 A. Therefore, the short-circuit ratio of this generator is

$$\text{SCR} = \frac{365 \text{ A}}{320 \text{ A}} = 1.14$$

5-28. A 20-MVA 12.2-kV 0.8-PF-lagging Y-connected synchronous generator has a negligible armature resistance and a synchronous reactance of 1.1 per-unit. The generator is connected in parallel with a 60-Hz 12.2-kV infinite bus that is capable of supplying or consuming any amount of real or reactive power with no change in frequency or terminal voltage.

(a) What is the synchronous reactance of the generator in ohms?

(b) What is the internal generated voltage $\mathbf{E}_A$ of this generator under rated conditions?

(c) What is the armature current $\mathbf{I}_A$ in this machine at rated conditions?

(d) Suppose that the generator is initially operating at rated conditions. If the internal generated voltage $\mathbf{E}_A$ is decreased by 5 percent, what will the new armature current $\mathbf{I}_A$ be?

(e) Repeat part *(d)* for 10, 15, 20, and 25 percent reductions in $\mathbf{E}_A$.

(f) Plot the magnitude of the armature current I_A as a function of E_A. (You may wish to use MATLAB to create this plot.)

SOLUTION

(a) The rated phase voltage of this generator is 12.2 kV / $\sqrt{3}$ = 7044 V. The base impedance of this generator is

$$Z_{\text{base}} = \frac{3\,V_{\phi,\text{base}}^{\;2}}{S_{\text{base}}} = \frac{3(7044\text{ V})^2}{20{,}000{,}000\text{ VA}} = 7.44\ \Omega$$

Therefore,

$$R_A \approx 0\ \Omega \quad \text{(negligible)}$$

$$X_S = (1.1)(7.44\ \Omega) = 8.18\ \Omega$$

(b) The rated armature current is

$$I_A = I_L = \frac{S}{\sqrt{3}\,V_T} = \frac{20\text{ MVA}}{\sqrt{3}(12.2\text{ kV})} = 946\text{ A}$$

The power factor is 0.8 lagging, so $\mathbf{I}_A = 946\angle -36.87^\circ$ A. Therefore, the internal generated voltage is

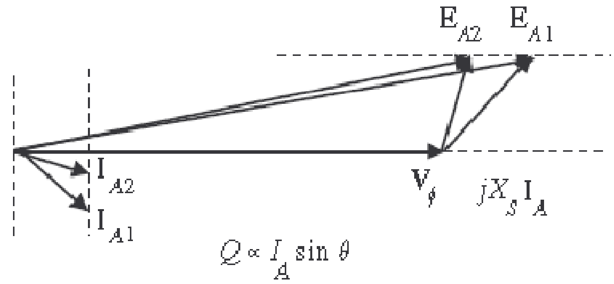
$$\mathbf{E}_A = \mathbf{V}_\phi + R_A\mathbf{I}_A + jX_S\mathbf{I}_A$$

$$\mathbf{E}_A = 7044\angle 0^\circ + j(8.18\ \Omega)(946\angle -36.87^\circ\text{ A})$$

$$\mathbf{E}_A = 13{,}230\angle 27.9^\circ\text{ V}$$

(c) From the above calculations, $\mathbf{I}_A = 946\angle -36.87^\circ$ A.

(d) If E_A is decreased by 5%, the armature current will change as shown below. Note that the infinite bus will keep V_ϕ and ω_m constant. Also, since the prime mover hasn't changed, the power supplied by the generator will be constant.



$$P = \frac{3V_\phi E_A}{X_S}\sin\delta = \text{constant}, \text{ so } E_{A1}\sin\delta_1 = E_{A2}\sin\delta_2$$

With a 5% decrease, $E_{A2} = 12,570 \text{ V}$, and

$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_2\right) = \sin^{-1}\left(\frac{13,230 \text{ V}}{12,570 \text{ V}}\sin 27.9°\right) = 29.5°$$

Therefore, the new armature current is

$$\mathbf{I}_A = \frac{\mathbf{E}_{A2} - \mathbf{V}_\phi}{jX_S} = \frac{12,570\angle 29.5° - 7044\angle 0°}{j8.18} = 894\angle -32.2° \text{ A}$$

(e) Repeating part *(d)*:

With a **10%** decrease, $E_{A2} = 11,907 \text{ V}$, and

$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_2\right) = \sin^{-1}\left(\frac{13,230 \text{ V}}{11,907 \text{ V}}\sin 27.9°\right) = 31.3°$$

Therefore, the new armature current is

$$\mathbf{I}_A = \frac{\mathbf{E}_{A2} - \mathbf{V}_\phi}{jX_S} = \frac{11,907\angle 31.3° - 7044\angle 0°}{j8.18} = 848\angle -26.8° \text{ A}$$

With a **15%** decrease, $E_{A2} = 11,246 \text{ V}$, and

$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_2\right) = \sin^{-1}\left(\frac{13,230 \text{ V}}{11,246 \text{ V}}\sin 27.9°\right) = 33.4°$$

Therefore, the new armature current is

$$\mathbf{I}_A = \frac{\mathbf{E}_{A2} - \mathbf{V}_\phi}{jX_S} = \frac{11,246\angle 33.4° - 7044\angle 0°}{j8.18} = 809\angle -20.7° \text{ A}$$

With a **20%** decrease, $E_{A2} = 10,584 \text{ V}$, and

$$\delta_2 = \sin^{-1}\left(\frac{E_{A1}}{E_{A2}}\sin\delta_2\right) = \sin^{-1}\left(\frac{13,230 \text{ V}}{10,584 \text{ V}}\sin 27.9°\right) = 35.8°$$

Therefore, the new armature current is

$$\mathbf{I}_A = \frac{\mathbf{E}_{A2} - \mathbf{V}_\phi}{jX_S} = \frac{10,584\angle 35.8° - 7044\angle 0°}{j8.18} = 780\angle -14.0°\text{ A}$$

With a **25%** decrease, $E_{A2} = 9,923\text{ V}$, and

$$\delta_2 = \sin^{-1} \frac{E_{A1}}{E_{A2}} \sin\delta_2 \quad = \sin^{-1} \frac{13,230\text{ V}}{9,923\text{ V}} \sin 27.9° \quad = 38.6°$$

Therefore, the new armature current is

$$\mathbf{I}_A = \frac{\mathbf{E}_{A2} - \mathbf{V}_\phi}{jX_S} = \frac{9,923\angle 38.6° - 7044\angle 0°}{j8.18} = 762\angle -6.6°\text{ A}$$

(f) A MATLAB program to plot the magnitude of the armature current I_A as a function of E_A is shown below.

```
% M-file: prob5_28f.m
% M-file to calculate and plot the armature current
% supplied to an infinite bus as Ea is varied.

% Define values for this generator
Ea = (0.65:0.01:1.00)*13230; % Ea
Vp = 7044;                    % Phase voltage
d1 = 27.9*pi/180;             % torque angle at full Ea
Xs = 8.18;                    % Xs (ohms)

% Calculate delta for each Ea
d = asin( 13230 ./ Ea .* sin(d1));

% Calculate Ia for each flux
Ea = Ea .* ( cos(d) + j.*sin(d) );
Ia = ( Ea - Vp ) ./ (j*Xs);

% Plot the armature current versus Ea
figure(1);
plot(abs(Ea)/1000,abs(Ia),'b-','LineWidth',2.0);
title ('\bfArmature current versus \itE_{A}\rm');
xlabel ('\bf\itE_{A}\rm\bf (kV)');
ylabel ('\bf\itI_{A}\rm\bf (A)');
grid on;
hold off;
```

The resulting plot is shown below: