7-4. A three-phase, 60-Hz induction motor runs at 890 r/min at no load and at 840 r/min at full load.

(a) How many poles does this motor have?

(b) What is the slip at rated load?

(c) What is the speed at one-quarter of the rated load?

(d) What is the rotor's electrical frequency at one-quarter of the rated load?

SOLUTION

(a) This machine has 8 poles, which produces a synchronous speed of

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(60\text{ Hz})}{8} = 900\text{ r/min}$$

(b) The slip at rated load is

$$s = \frac{n_{\text{sync}} - n_m}{n_{\text{sync}}} \times 100\% = \frac{900 - 840}{900} \times 100\% = 6.67\%$$

(c) The motor is operating in the linear region of its torque-speed curve, so the slip at ¼ load will be

$$s = 0.25(0.0667) = 0.0167$$

The resulting speed is

$$n_m = (1 - s)\ n_{\text{sync}} = (1 - 0.0167)(900\text{ r/min}) = 885\text{ r/min}$$

(d) The electrical frequency at ¼ load is

$$f_r = s f_e = (0.0167)(60\text{ Hz}) = 1.00\text{ Hz}$$

7-5. A 50-kW, 440-V, 50-Hz, six-pole induction motor has a slip of 6 percent when operating at full-load conditions. At full-load conditions, the friction and windage losses are 300 W, and the core losses are 600 W. Find the following values for full-load conditions:

(a) The shaft speed n_m

(b) The output power in watts

(c) The load torque τ_{load} in newton-meters

(d) The induced torque τ_{ind} in newton-meters

(e) The rotor frequency in hertz

SOLUTION

(a) The synchronous speed of this machine is

$$n_{\text{sync}} = \frac{120 f_e}{P} = \frac{120(50\text{ Hz})}{6} = 1000 \text{ r/min}$$

Therefore, the shaft speed is

$$n_m = (1-s)\ n_{\text{sync}} = (1-0.06)(1000 \text{ r/min}) = 940 \text{ r/min}$$

(b) The output power in watts is 50 kW (stated in the problem).

(c) The load torque is

$$\tau_{\text{load}} = \frac{P_{\text{OUT}}}{\omega_m} = \frac{50 \text{ kW}}{(940 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60\ s}\right)} = 508 \text{ N}\cdot\text{m}$$

(d) The induced torque can be found as follows:

$$P_{\text{conv}} = P_{\text{OUT}} + P_{\text{F\&W}} + P_{\text{core}} + P_{\text{misc}} = 50 \text{ kW} + 300 \text{ W} + 600 \text{ W} + 0 \text{ W} = 50.9 \text{ kW}$$

$$\tau_{\text{ind}} = \frac{P_{\text{conv}}}{\omega_m} = \frac{50.9 \text{ kW}}{(940 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60\ s}\right)} = 517 \text{ N}\cdot\text{m}$$

(e) The rotor frequency is

$$f_r = s f_e = (0.06)(50 \text{ Hz}) = 3.00 \text{ Hz}$$

7-7. A 208-V, two-pole, 60-Hz Y-connected wound-rotor induction motor is rated at 15 hp. Its equivalent circuit components are

$R_1 = 0.200\ \Omega$ $\quad R_2 = 0.120\ \Omega$ $\quad X_M = 15.0\ \Omega$

$X_1 = 0.410\ \Omega$ $\quad X_2 = 0.410\ \Omega$

$P_{mech} = 250$ W $\quad P_{misc} \approx 0$ $\quad P_{core} = 180$ W

For a slip of 0.05, find

(a) The line current I_L

(b) The stator copper losses P_{SCL}

(c) The air-gap power P_{AG}

(d) The power converted from electrical to mechanical form P_{conv}

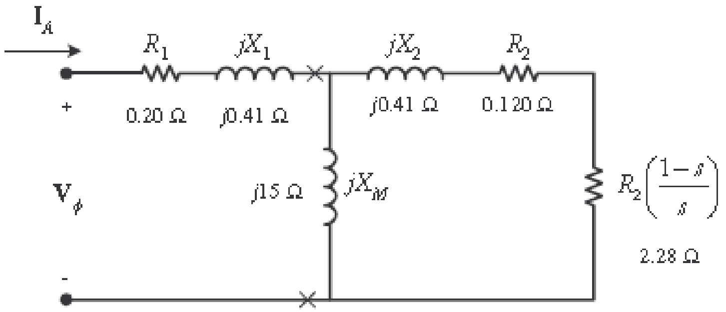
(e) The induced torque τ_{ind}

(f) The load torque τ_{load}

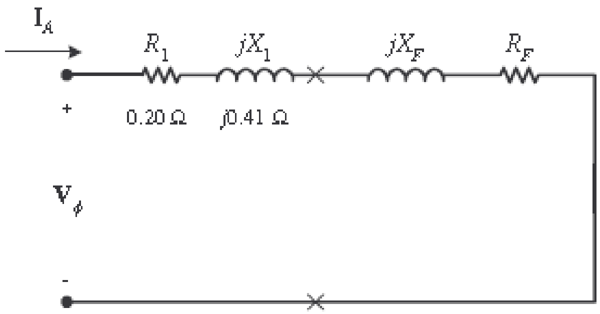
(g) The overall machine efficiency

(h) The motor speed in revolutions per minute and radians per second

SOLUTION The equivalent circuit of this induction motor is shown below:



(a) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\dfrac{1}{jX_M} + \dfrac{1}{Z_2}} = \frac{1}{\dfrac{1}{j15\ \Omega} + \dfrac{1}{2.40 + j0.41}} = 2.220 + j0.745 = 2.34\angle 18.5°\ \Omega$$

The phase voltage is $208/\sqrt{3} = 120$ V, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{120\angle 0^\circ \text{ V}}{0.20\ \Omega\ + j0.41\ \Omega + \ 2.22\ \Omega + j0.745\ \Omega}$$

$$I_L = I_A = 44.8\angle -25.5^\circ \text{ A}$$

(b) The stator copper losses are

$$P_{SCL} = 3I_A{}^2R_1 = 3(44.8 \text{ A})^2(0.20\ \Omega) = 1205 \text{ W}$$

(c) The air gap power is $P_{AG} = 3I_2{}^2\frac{R_2}{s} = 3I_A{}^2R_F$

(Note that $3I_A{}^2R_F$ is equal to $3I_2{}^2\frac{R_2}{s}$, since the only resistance in the original rotor circuit was R_2/s, and the resistance in the Thevenin equivalent circuit is R_F. The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{AG} = 3I_2{}^2\frac{R_2}{s} = 3I_A{}^2R_F = 3(44.8 \text{ A})^2(2.220\ \Omega) = 13.4 \text{ kW}$$

(d) The power converted from electrical to mechanical form is

$$P_{conv} = (1-s)P_{AG} = (1-0.05)(13.4 \text{ kW}) = 12.73 \text{ kW}$$

(e) The induced torque in the motor is

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{13.4 \text{ kW}}{(3600 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 35.5 \text{ N}\cdot\text{m}$$

(f) The output power of this motor is

$$P_{OUT} = P_{conv} - P_{mech} - P_{core} - P_{misc} = 12.73 \text{ kW} - 250 \text{ W} - 180 \text{ W} - 0 \text{ W} = 12.3 \text{ kW}$$

The output speed is

$$n_m = (1-s)\ n_{sync} = (1-0.05)(3600 \text{ r/min}) = 3420 \text{ r/min}$$

Therefore the load torque is

$$\tau_{load} = \frac{P_{OUT}}{\omega_m} = \frac{12.3 \text{ kW}}{(3420 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)} = 34.3 \text{ N}\cdot\text{m}$$

(g) The overall efficiency is

$$\eta = \frac{P_{OUT}}{P_{IN}} \times 100\% = \frac{P_{OUT}}{3V_\phi I_A \cos\theta} \times 100\%$$

$$\eta = \frac{12.3 \text{ kW}}{3(120 \text{ V})(44.8 \text{ A})\cos 25.5^\circ} \times 100\% = 84.5\%$$

(h) The motor speed in revolutions per minute is 3420 r/min. The motor speed in radians per second is

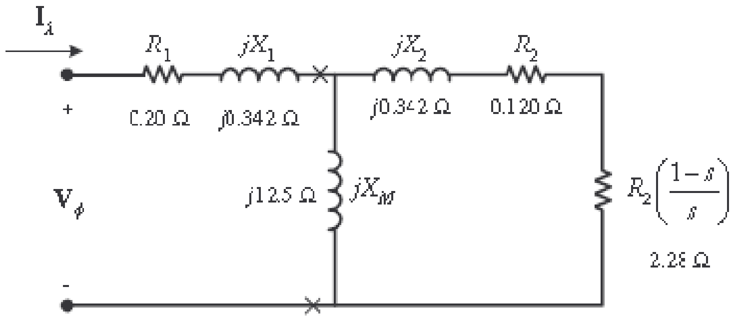
$$\omega_m = (3420 \text{ r/min})\left(\frac{2\pi \text{ rad}}{1 \text{ r}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 358 \text{ rad/s}$$

7-11. If the motor in Problem 7-7 is to be operated on a 50-Hz power system, what must be done to its supply voltage? Why? What will the equivalent circuit component values be at 50 Hz? Answer the questions in Problem 7-7 for operation at 50 Hz with a slip of 0.05 and the proper voltage for this machine.

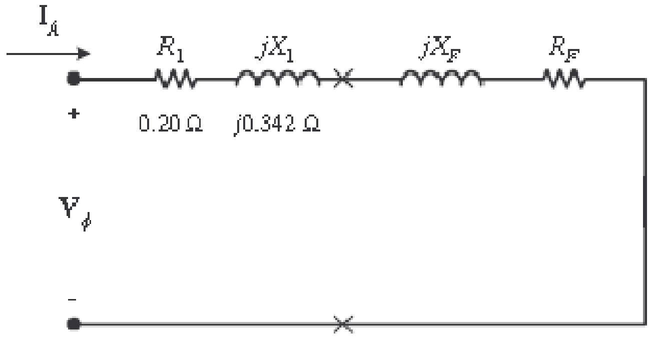
SOLUTION If the input frequency is decreased to 50 Hz, then the applied voltage must be decreased by 5/6 also. If this were not done, the flux in the motor would go into saturation, since

$$\phi = \frac{1}{N}\int_T v\, dt$$

and the period T would be increased. At 50 Hz, the resistances will be unchanged, but the reactances will be reduced to 5/6 of their previous values. The equivalent circuit of the induction motor at 50 Hz is shown below:



(a) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\frac{1}{jX_M} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{j12.5\ \Omega} + \frac{1}{2.40 + j0.342}} = 2.197 + j0.744 = 2.32\angle 18.7^\circ\ \Omega$$

The line voltage must be derated by 5/6, so the new line voltage is $V_T = 173.3$ V. The phase voltage is $173.3/\sqrt{3} = 100$ V, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{100\angle 0^\circ\ \text{V}}{0.20\ \Omega + j0.342\ \Omega + 2.197\ \Omega + j0.744\ \Omega}$$

$$I_L = I_A = 38.0\angle -24.4^\circ\ \text{A}$$

(b) The stator copper losses are

$$P_{SCL} = 3 I_A^{\,2} R_1 = 3(38\ \text{A})^2 (0.20\ \Omega) = 866\ \text{W}$$

(c) The air gap power is $P_{AG} = 3I_2^{\,2}\frac{R_2}{s} = 3I_A^{\,2}R_F$

(Note that $3I_A^{\,2}R_F$ is equal to $3I_2^{\,2}\frac{R_2}{s}$, since the only resistance in the original rotor circuit was R_2/s, and the resistance in the Thevenin equivalent circuit is R_F. The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{AG} = 3I_2^{\,2}\frac{R_2}{s} = 3I_A^{\,2}R_F = 3(38\text{ A})^2(2.197\ \Omega) = 9.52\text{ kW}$$

(d) The power converted from electrical to mechanical form is

$$P_{conv} = (1-s)P_{AG} = (1-0.05)(9.52\text{ kW}) = 9.04\text{ kW}$$

(e) The induced torque in the motor is

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{9.52\text{ kW}}{(3000\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)} = 30.3\text{ N}\cdot\text{m}$$

(f) In the absence of better information, we will treat the mechanical and core losses as constant despite the change in speed. This is not true, but we don't have reason for a better guess. Therefore, the output power of this motor is

$$P_{OUT} = P_{conv} - P_{mech} - P_{core} - P_{misc} = 9.04\text{ kW} - 250\text{ W} - 180\text{ W} - 0\text{ W} = 8.61\text{ kW}$$

The output speed is

$$n_m = (1-s)\,n_{sync} = (1-0.05)(3000\text{ r/min}) = 2850\text{ r/min}$$

Therefore the load torque is

$$\tau_{load} = \frac{P_{OUT}}{\omega_m} = \frac{8.61\text{ kW}}{(2850\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)} = 28.8\text{ N}\cdot\text{m}$$

(g) The overall efficiency is

$$\eta = \frac{P_{OUT}}{P_{IN}}\times 100\% = \frac{P_{OUT}}{3V_\phi I_A\cos\theta}\times 100\%$$

$$\eta = \frac{8.61\text{ kW}}{3(100\text{ V})(38.0\text{ A})\cos 24.4^\circ}\times 100\% = 82.9\%$$

(h) The motor speed in revolutions per minute is 2850 r/min. The motor speed in radians per second is

$$\omega_m = (2850\text{ r/min})\left(\frac{2\pi\text{ rad}}{1\text{ r}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 298.5\text{ rad/s}$$

7-14. A 440-V 50-Hz two-pole Y-connected induction motor is rated at 75 kW. The equivalent circuit parameters are

$R_1 = 0.075\ \Omega$ $\quad R_2 = 0.065\ \Omega$ $\quad X_M = 7.2\ \Omega$

$X_1 = 0.17\ \Omega$ $\quad X_2 = 0.17\ \Omega$

$P_{F\&W} = 1.0\ \text{kW}$ $\quad P_{misc} = 150\ \text{W}$ $\quad P_{core} = 1.1\ \text{kW}$

For a slip of 0.04, find

(a) The line current I_L

(b) The stator power factor

(c) The rotor power factor

(d) The stator copper losses P_{SCL}

(e) The air-gap power P_{AG}

(f) The power converted from electrical to mechanical form P_{conv}

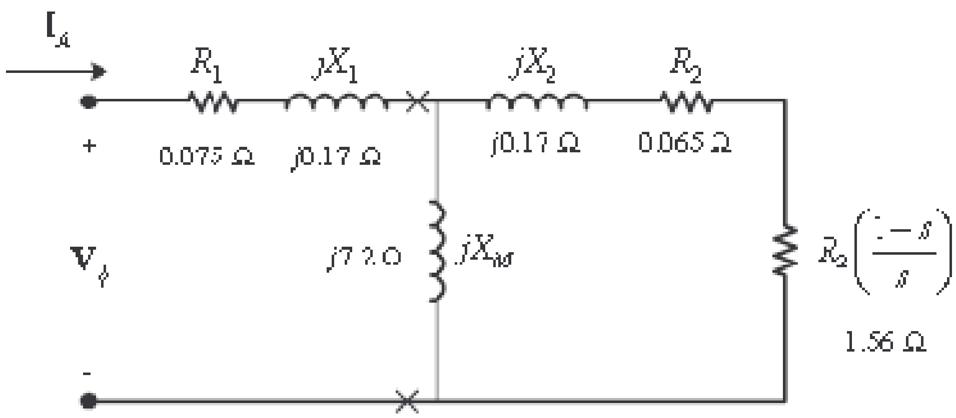
(g) The induced torque τ_{ind}

(h) The load torque τ_{load}

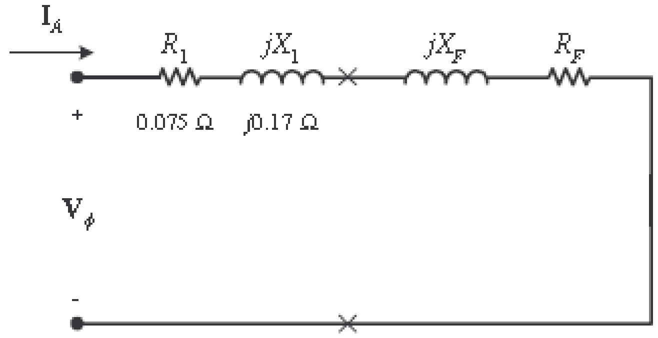
(i) The overall machine efficiency η

(j) The motor speed in revolutions per minute and radians per second

SOLUTION The equivalent circuit of this induction motor is shown below:



(*a*) The easiest way to find the line current (or armature current) is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M, and then calculate the current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M is:

$$Z_F = \frac{1}{\frac{1}{jX_M} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{j7.2\ \Omega} + \frac{1}{1.625 + j0.17}} = 1.539 + j0.364 = 1.58\angle 13.2°\ \Omega$$

The phase voltage is $440/\sqrt{3}$ = 254 V, so line current I_L is

$$I_L = I_A = \frac{V_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{254\angle 0°\ \text{V}}{0.075\,\Omega\ + j0.17\,\Omega + \ 1.539\,\Omega + j0.364\,\Omega}$$

$$I_L = I_A = 149.4\angle -18.3°\ \text{A}$$

(*b*) The stator power factor is

$$\text{PF} = \cos(18.3°) = 0.949 \text{ lagging}$$

(*c*) To find the rotor power factor, we must find the impedance angle of the rotor

$$\theta_R = \tan^{-1}\frac{X_2}{R_2 / s} = \tan^{-1}\frac{0.17}{1.625} = 5.97°$$

Therefore the rotor power factor is

$$\text{PF}_R = \cos 5.97° = 0.995 \text{ lagging}$$

(*d*) The stator copper losses are

$$P_{SCL} = 3I_A^{\ 2}R_1 = 3(149.4\ \text{A})^2(0.075\ \Omega) = 1675\ \text{W}$$

(*e*) The air gap power is $P_{AG} = 3I_2^{\ 2}\frac{R_2}{s} = 3I_A^{\ 2}R_F$

(Note that $3I_A{}^2R_F$ is equal to $3I_2{}^2\frac{R_2}{s}$, since the only resistance in the original rotor circuit was R_2/s, and the resistance in the Thevenin equivalent circuit is R_F. The power consumed by the Thevenin equivalent circuit must be the same as the power consumed by the original circuit.)

$$P_{AG} = 3I_2{}^2\frac{R_2}{s} = 3I_A{}^2R_F = 3(149.4\text{ A})^2(1.539\ \Omega) = 103\text{ kW}$$

(f) The power converted from electrical to mechanical form is

$$P_{conv} = (1-s)P_{AG} = (1-0.04)\ (103\text{ kW}) = 98.9\text{ kW}$$

(g) The synchronous speed of this motor is

$$n_{sync} = \frac{120 f_e}{P} = \frac{120(50\text{ Hz})}{2} = 3000\text{ r/min}$$

$$\omega_{sync} = (3000\text{ r/min})\ \frac{2\pi\text{ rad}}{1\text{ r}}\ \frac{1\text{ min}}{60\ s} = 314\text{ rad/s}$$

Therefore the induced torque in the motor is

$$\tau_{ind} = \frac{P_{AG}}{\omega_{sync}} = \frac{103\text{ kW}}{(3000\text{ r/min})\ \frac{2\pi\text{ rad}}{1\text{ r}}\ \frac{1\text{ min}}{60\ s}} = 327.9\text{ N}\cdot\text{m}$$

(h) The output power of this motor is

$$P_{OUT} = P_{conv} - P_{mech} - P_{core} - P_{misc} = 98.8\text{ kW} - 1.0\text{ kW} - 1.1\text{ kW} - 150\text{ W} = 96.6\text{ kW}$$

The output speed is

$$n_m = (1-s)\ n_{sync} = (1-0.04)\ (3000\text{ r/min}) = 2880\text{ r/min}$$

Therefore the load torque is

$$\tau_{load} = \frac{P_{OUT}}{\omega_m} = \frac{98.8\text{ kW}}{(2880\text{ r/min})\ \frac{2\pi\text{ rad}}{1\text{ r}}\ \frac{1\text{ min}}{60\ s}} = 327.6\text{ N}\cdot\text{m}$$

(i) The overall efficiency is

$$\eta = \frac{P_{OUT}}{P_{IN}}\times 100\% = \frac{P_{OUT}}{3V_\phi I_A\cos\theta}\times 100\%$$

$$\eta = \frac{96.6\text{ kW}}{3(254\text{ V})(149.4\text{ A})\cos(18.3°)}\times 100\% = 89.4\%$$

(j) The motor speed in revolutions per minute is 2880 r/min. The motor speed in radians per second is

$$\omega_m = (2880\text{ r/min})\ \frac{2\pi\text{ rad}}{1\text{ r}}\ \frac{1\text{ min}}{60\ s} = 301.6\text{ rad/s}$$

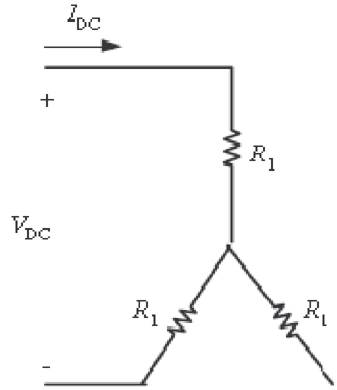
7-18. A 208 V, 60 Hz, six pole Y connected 25 hp design class B induction motor is tested in the laboratory, with the following results:

No load:	208 V, 22.0 A, 1200 W, 60 Hz
Locked rotor:	24.6 V, 64.5 A, 2200 W, 15 Hz
DC test:	13.5 V, 64 A

Find the equivalent circuit of this motor, and plot its torque-speed characteristic curve.

SOLUTION From the DC test,

$$2R_1 = \frac{13.5\text{ V}}{64\text{ A}} \qquad \Rightarrow \qquad R_1 = 0.105\ \Omega$$



In the no-load test, the line voltage is 208 V, so the phase voltage is 120 V. Therefore,

$$X_1 + X_M = \frac{V_\phi}{I_{A,nl}} = \frac{120\text{ V}}{22.0\text{ A}} = 5.455\ \Omega \quad @ \quad 60\text{ Hz}$$

In the locked-rotor test, the line voltage is 24.6 V, so the phase voltage is 14.2 V. From the locked-rotor test *at 15 Hz*,

$$\left|Z'_{LR}\right| = \left|R_{LR} + jX'_{LR}\right| = \frac{V_\phi}{I_{A,LR}} = \frac{14.2\text{ V}}{64.5\text{ A}} = 0.2202\ \Omega$$

$$\theta'_{LR} = \cos^{-1}\frac{P_{LR}}{S_{LR}} = \cos^{-1}\frac{2200\text{ W}}{\sqrt{3}(24.6\text{ V})(64.5\text{ A})} = 36.82°$$

Therefore,

$$R_{LR} = \left|Z'_{LR}\right| \cos\theta_{LR} = (0.2202\ \Omega)\cos(36.82°) = 0.176\ \Omega$$

$$\Rightarrow \qquad R_1 + R_2 = 0.176\ \Omega$$

$$\Rightarrow \qquad R_2 = 0.071\ \Omega$$

$$X'_{LR} = \left|Z'_{LR}\right| \sin\theta_{LR} = (0.2202\ \Omega)\sin(36.82°) = 0.132\ \Omega$$

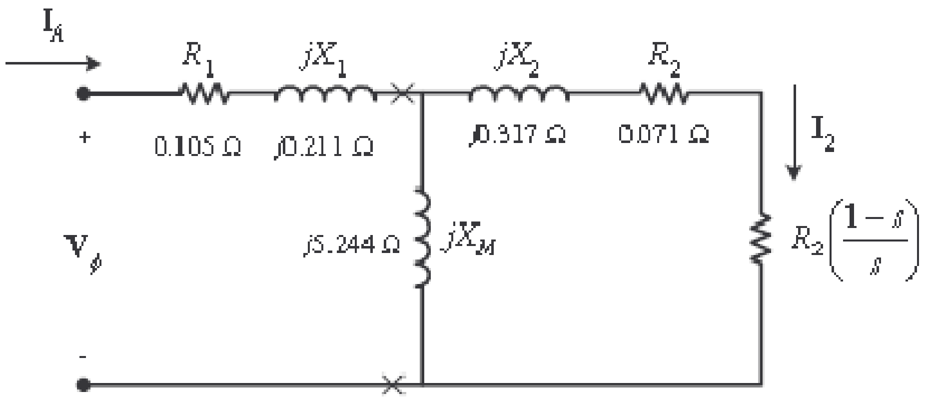
At a frequency of 60 Hz,

$$X_{LR} = \left(\frac{60\text{ Hz}}{15\text{ Hz}}\right) X'_{LR} = 0.528\ \Omega$$

For a Design Class B motor, the split is $X_1 = 0.211\ \Omega$ and $X_2 = 0.317\ \Omega$. Therefore,

$$X_M = 5.455\ \Omega - 0.211\ \Omega = 5.244\ \Omega$$

The resulting equivalent circuit is shown below:



7-19. A 460-V, four-pole, 50-hp, 60-Hz, Y-connected three-phase induction motor develops its full-load induced torque at 3.8 percent slip when operating at 60 Hz and 460 V. The per-phase circuit model impedances of the motor are

$R_1 = 0.33\ \Omega$ $\qquad X_M = 30\ \Omega$

$X_1 = 0.42\ \Omega$ $\qquad X_2 = 0.42\ \Omega$

Mechanical, core, and stray losses may be neglected in this problem.

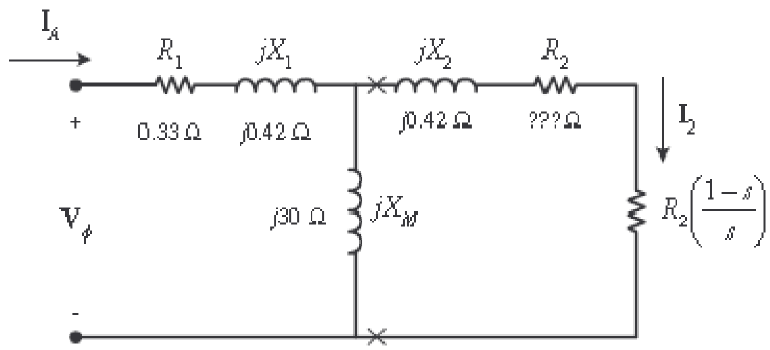
(a) Find the value of the rotor resistance R_2.

(b) Find τ_{max}, s_{max}, and the rotor speed at maximum torque for this motor.

(c) Find the starting torque of this motor.

(d) What code letter factor should be assigned to this motor?

SOLUTION The equivalent circuit for this motor is



The Thevenin equivalent of the input circuit is:

$$Z_{TH} = \frac{jX_M\left(R_1 + jX_1\right)}{R_1 + j\left(X_1 + X_M\right)} = \frac{(j30\ \Omega)(0.33\ \Omega\ + j0.42\ \Omega)}{0.33\ \Omega\ + j\left(0.42\ \Omega + 30\ \Omega\right)} = 0.321 + j0.418\ \Omega = 0.527\angle 52.5^\circ\ \Omega$$

$$\mathbf{V}_{TH} = \frac{jX_M}{R_1 + j(X_1 + X_M)}\mathbf{V}_\phi = \frac{(j30\ \Omega)}{0.33\ \Omega\ +\ j(0.42\ \Omega + 30\ \Omega)}(265.6\angle 0°\ \text{V}) = 262\angle 0.6°\ \text{V}$$

(a) If losses are neglected, the induced torque in a motor is equal to its load torque. At full load, the output power of this motor is 50 hp and its slip is 3.8%, so the induced torque is

$$n_m = (1 - 0.038)(1800\ \text{r/min}) = 1732\ \text{r/min}$$

$$\tau_{ind} = \tau_{load} = \frac{(50\ \text{hp})(746\ \text{W/hp})}{(1732\ \text{r/min})\left(\frac{2\pi\ \text{rad}}{1\ \text{r}}\right)\left(\frac{1\ \text{min}}{60\ s}\right)} = 205.7\ \text{N}\cdot\text{m}$$

The induced torque is given by the equation

$$\tau_{ind} = \frac{3V_{TH}^2 R_2 / s}{\omega_{sync}\left[(R_{TH} + R_2/s)^2 + (X_{TH} + X_2)^2\right]}$$

Substituting known values and solving for R_2/s yields

$$205.7\ \text{N}\cdot\text{m} = \frac{3(262\ \text{V})^2 R_2/s}{(188.5\ \text{rad/s})\left[(0.321 + R_2/s)^2 + (0.418 + 0.42)^2\right]}$$

$$38{,}774 = \frac{205{,}932\ R_2/s}{(0.321 + R_2/s)^2 + 0.702}$$

$$(0.321 + R_2/s)^2 + 0.702 = 5.311\ R_2/s$$

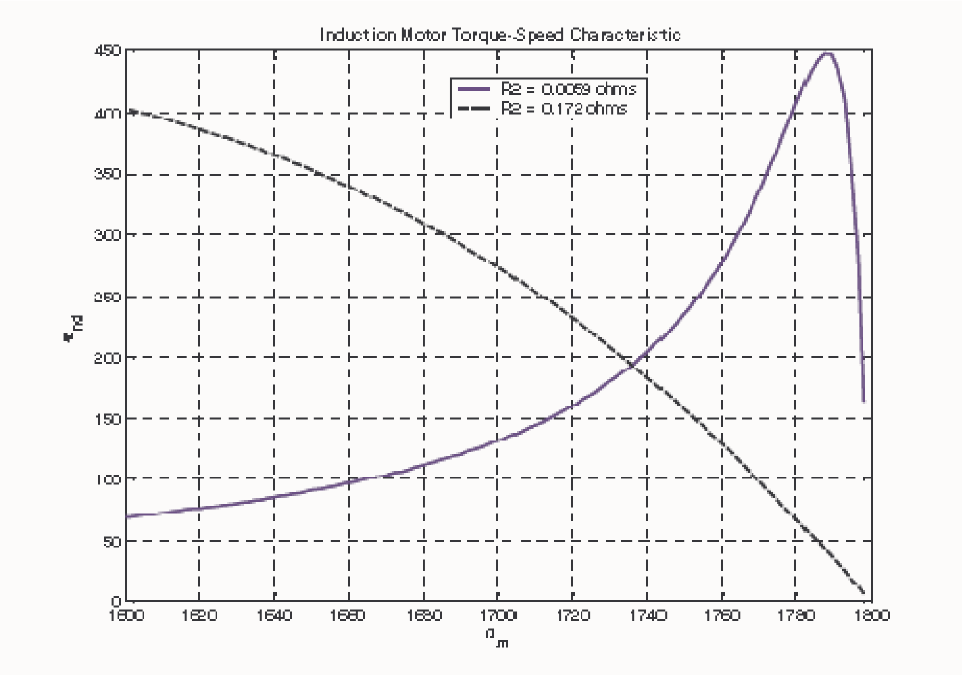
$$0.103 + 0.642 R_2/s + (R_2/s)^2 + 0.702 = 5.311\ R_2/s$$

$$\left(\frac{R_2}{s}\right)^2 - 4.669\left(\frac{R_2}{s}\right) + 0.702 = 0$$

$$\frac{R_2}{s} = 0.156,\quad 4.513$$

$$R_2 = 0.0059\ \Omega,\ 0.172\ \Omega$$

These two solutions represent two situations in which the torque-speed curve would go through this specific torque-speed point. The two curves are plotted below. As you can see, only the 0.172 Ω solution is realistic, since the 0.0059 Ω solution passes through this torque-speed point at an unstable location on the back side of the torque-speed curve.



(b) The slip at pullout torque can be found by calculating the Thevenin equivalent of the input circuit from the rotor back to the power supply, and then using that with the rotor circuit model. The Thevenin equivalent of the input circuit was calculate in part *(a)*. The slip at pullout torque is

$$s_{max} = \frac{R_2}{\sqrt{R_{TH}^2 + \left(X_{TH} + X_2\right)^2}}$$

$$s_{max} = \frac{0.172\ \Omega}{\sqrt{\left(0.321\ \Omega\right)^2 + \left(0.418\ \Omega\ + 0.420\ \Omega\right)^2}} = 0.192$$

The rotor speed a maximum torque is

$$n_{pullout} = (1 - s)\ n_{sync} = \left(1 - 0.192\right)\left(1800\ \text{r/min}\right) = 1454\ \text{r/min}$$

and the pullout torque of the motor is

$$\tau_{max} = \frac{3V_{TH}^2}{2\,\omega_{sync}\left[R_{TH} + \sqrt{R_{TH}^2 + \left(X_{TH} + X_2\right)^2}\right]}$$

$$\tau_{max} = \frac{3\left(262\ \text{V}\right)^2}{2\left(188.5\ \text{rad/s}\right)\left[0.321\ \Omega + \sqrt{\left(0.321\ \Omega\right)^2 + \left(0.418\ \Omega\ + 0.420\ \Omega\right)^2}\right]}$$

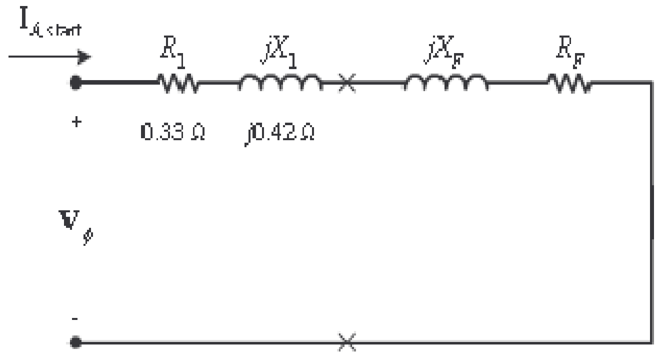
$$\tau_{max} = 448\ \text{N} \cdot \text{m}$$

(c) The starting torque of this motor is the torque at slip $s = 1$. It is

$$\tau_{ind} = \frac{3V_{TH}^2 R_2 / s}{\omega_{sync}\left[\left(R_{TH} + R_2 / s\right)^2 + \left(X_{TH} + X_2\right)^2\right]}$$

$$\tau_{ind} = \frac{3\left(262\ \text{V}\right)^2\left(0.172\ \Omega\right)}{\left(188.5\ \text{rad/s}\right)\left[\left(0.321 + 0.172\ \Omega\right)^2 + \left(0.418 + 0.420\right)^2\right]} = 199\ \text{N} \cdot \text{m}$$

(d) To determine the starting code letter, we must find the locked-rotor kVA per horsepower, which is equivalent to finding the starting kVA per horsepower. The easiest way to find the line current (or armature current) at starting is to get the equivalent impedance Z_F of the rotor circuit in parallel with jX_M at starting conditions, and then calculate the starting current as the phase voltage divided by the sum of the series impedances, as shown below.



The equivalent impedance of the rotor circuit in parallel with jX_M at starting conditions ($s = 1.0$) is:

$$Z_{F,start} = \frac{1}{\frac{1}{jX_M} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{j30\ \Omega} + \frac{1}{0.172 + j0.42}} = 0.167 + j0.415 = 0.448\angle 68.1^\circ\ \Omega$$

The phase voltage is $460/\sqrt{3} = 266$ V, so line current $\mathbf{I}_{L,start}$ is

$$\mathbf{I}_{L,start} = \mathbf{I}_A = \frac{\mathbf{V}_\phi}{R_1 + jX_1 + R_F + jX_F} = \frac{266\angle 0^\circ\ \text{V}}{0.33\ \Omega + j0.42\ \Omega + 0.167\ \Omega + j0.415\ \Omega}$$

$$\mathbf{I}_{L,start} = \mathbf{I}_A = 274\angle -59.2^\circ\ \text{A}$$

Therefore, the locked-rotor kVA of this motor is

$$S = \sqrt{3}\ V_T\ I_{L,rated} = \sqrt{3}\,(460\ \text{V})(274\ \text{A}) = 218\ \text{kVA}$$

and the kVA per horsepower is

$$\text{kVA/hp} = \frac{218\ \text{kVA}}{50\ \text{hp}} = 4.36\ \text{kVA/hp}$$

This motor would have **starting code letter D**, since letter D covers the range 4.00-4.50.