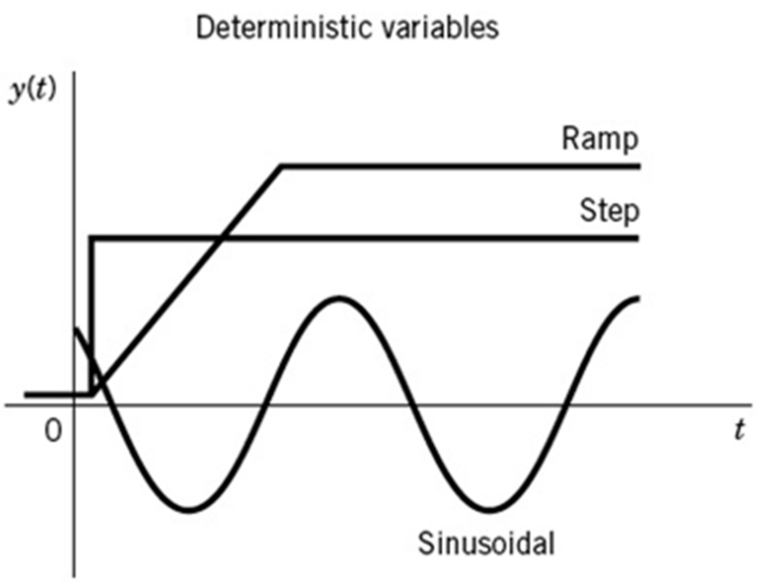
Instrumentation & Measurements MECH 430

Chapter 2
Static and Dynamic Characteristics of Signals

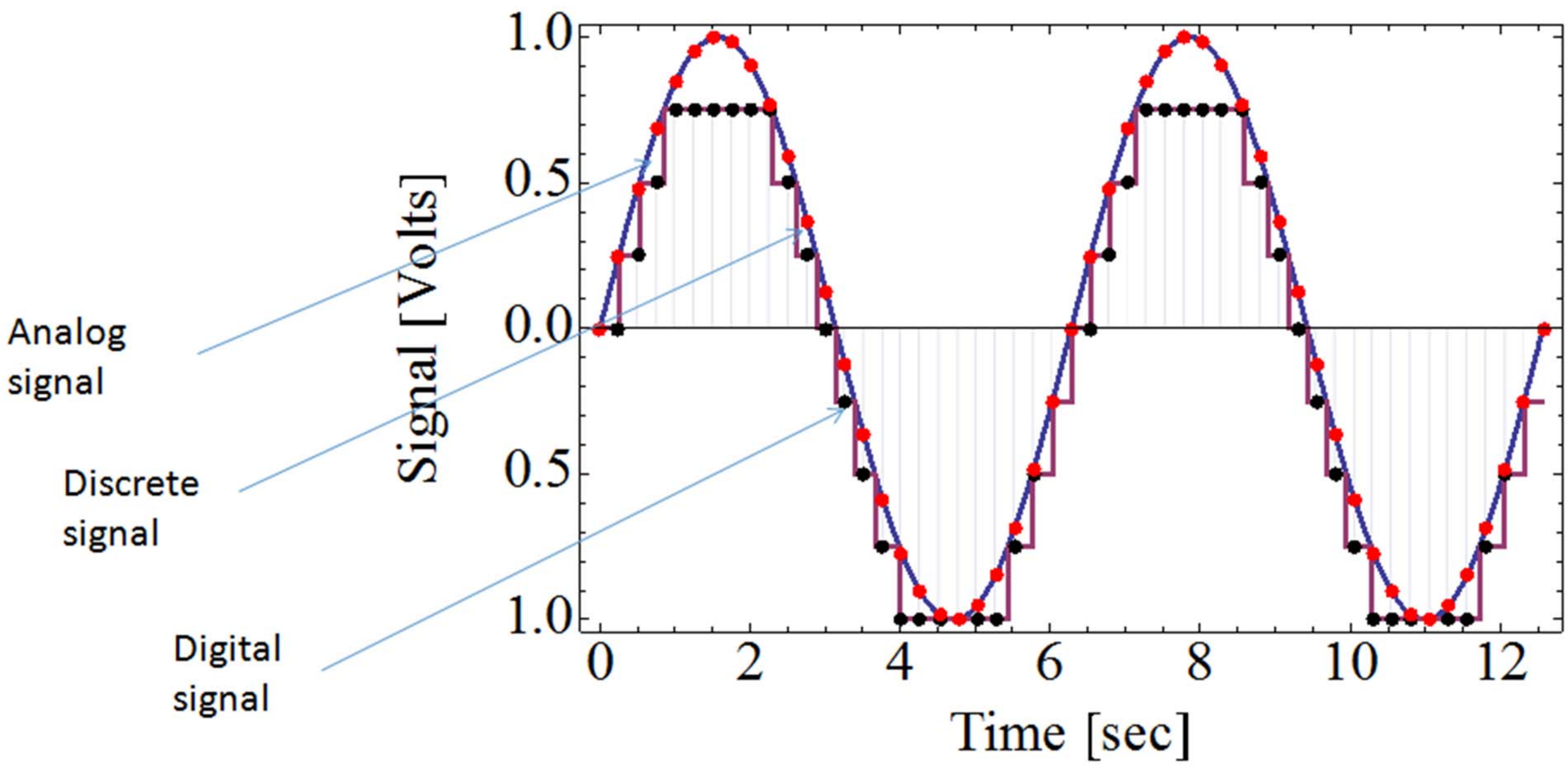


2.2 Signals and Waveforms

- A *signal* is used to transmit information
- In the context of measurement systems, both the input and the output have a *waveform*, which can be represented as a *signal*.
- Signals may be classified as:
  - Analog signals
  - Discrete signals
  - Digital signals
- A *Discrete Signal* results from the *sampling* of an analog signal at repeated finite time intervals. We estimate the value of the signal in between discrete values by assuming it is equal to the last available value (Sample and Hold).
- The magnitude of a *Digital Signal* are also discrete by the effect of *Quantization*.

2.2 Signals and Waveforms

- Three types of signals:



2.2 Signals and Waveforms

- Signals may also be characterized as *static* or *dynamic*.
- A static signal does not vary with time, Example, the diameter of a shaft. Voltage of a battery, outdoor temperature?
- A Dynamic signal is a time dependent signal. They can be *deterministic* or *non-deterministic*. They can be *simple periodic*, *complex periodic or aperiodic*.

2.2 Signals and Waveforms

Table 2.1 Classification of Waveforms

I. Static	$y(t) = A_0$
II. Dynamic	
Periodic waveforms	
Simple periodic waveform	$y(t) = A_0 + C\sin(\omega t + \phi)$
Complex periodic waveform	$y(t) = A_0 + \sum_{n=1}^{\infty} C_n \sin(n\omega t + \phi_n)$
Aperiodic waveforms	
Step[a]	$y(t) = A_0 U(t)$ $= A_0 \quad \text{for } t > 0$
Ramp	$y(t) = Kt \quad \text{for } 0 < t < t_f$
Pulse[b]	$y(t) = A_0 U(t) - A_0 U(t - t_1)$
III. Nondeterminisitic waveform	$y(t) \approx A_0 + \sum_{n=1}^{\infty} C_n \sin(\omega_n t + \phi_n)$

2.2 Signals and Waveforms

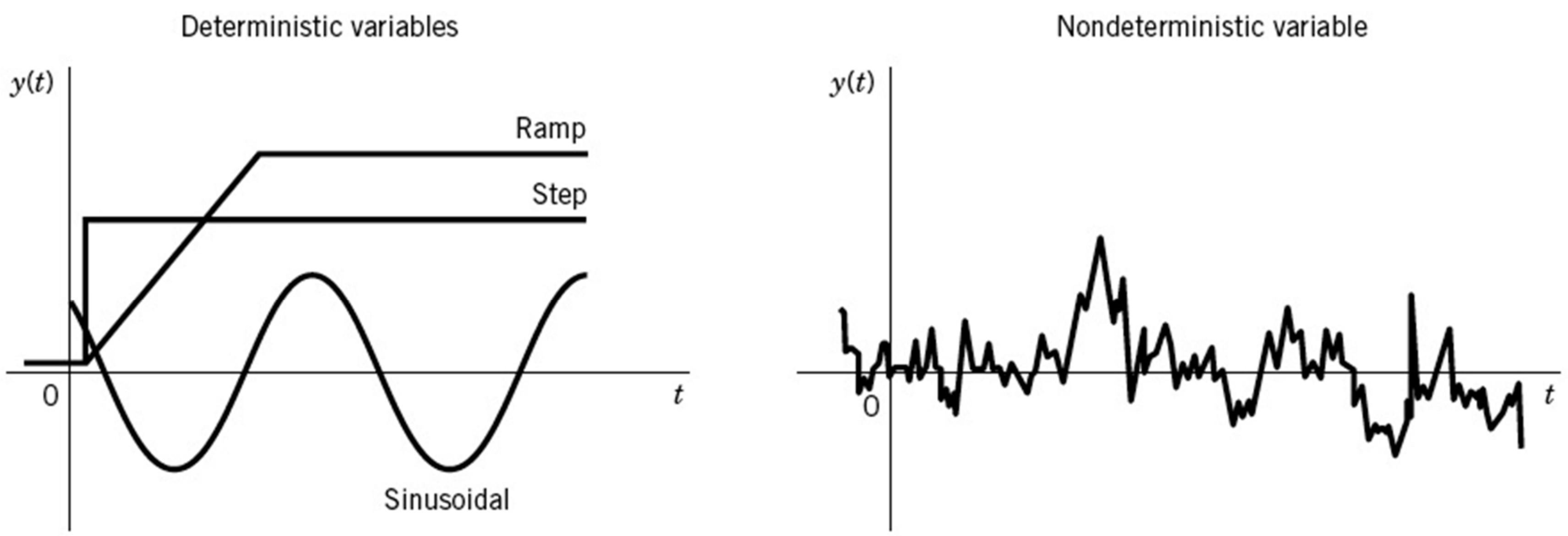


Figure 2.5 Examples of dynamic signals.

2.3 Signal Analysis

- Signal Mean or DC-Component

Analog

$$\bar{y} = \frac{\int_{t_1}^{t_2} y(t)dt}{\int_{t_1}^{t_2} dt}$$

Discrete

$$\bar{y} = \frac{\sum_{i=1}^{N} y_i}{N}$$

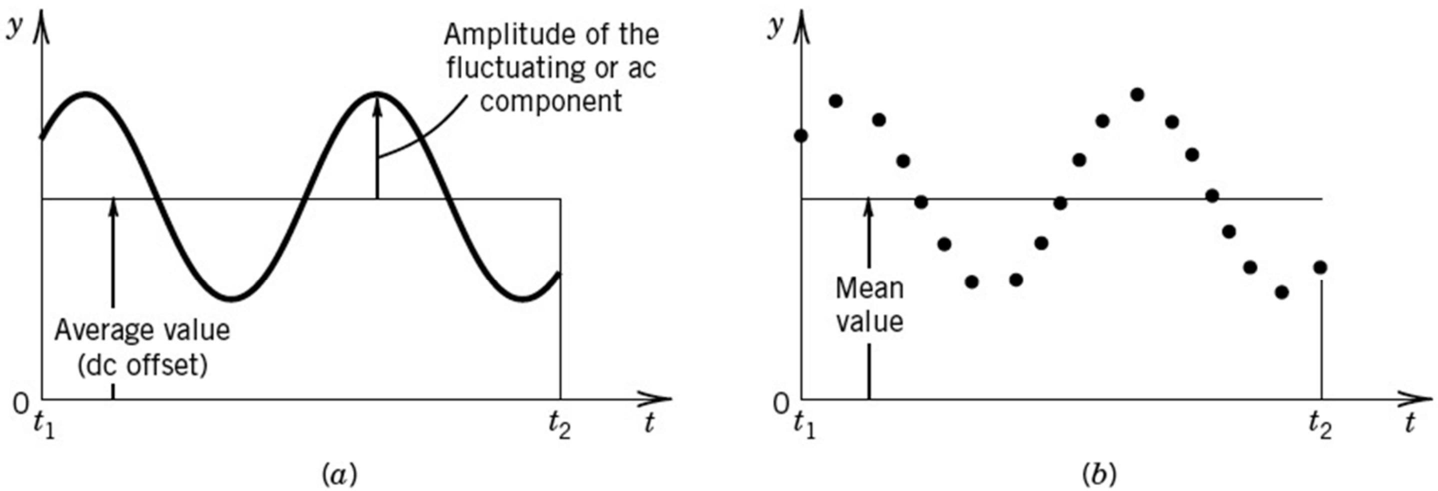


Figure 2.6 Analog and discrete representations of a dynamic signal.

2.3 Signal Analysis

- The root-mean-square (rms) is another measure that is related to the energy in an oscillating signal.
- If we have a sinusoidal current, the rms value is a constant current that would produce the same energy dissipation in a resistor.

Analog

$$y_{rms} = \sqrt{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} y^2 dt}$$

Discrete

$$y_{rms} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} {y_i}^2}$$

2.3 Signal Analysis

- Root Mean Square

$$P = I^2 R \text{ and } E = \int P dt$$

$$E_{DC} = E_{AC}$$

$$\int_{t_1}^{t_2} R {I_{DC}}^2 dt = \int_{t_1}^{t_2} R {I_{AC}}^2 dt$$

$$R {I_{DC}}^2 (t_2 - t_1) = R \int_{t_1}^{t_2} {I_{AC}}^2 dt$$

$$\rightarrow I_{DC} = \sqrt{\frac{1}{t_2 - t_1} \int_{t_1}^{t_2} {I_{AC}}^2 dt}$$

2.4 Signal Amplitude and Freq

- A function of time, $y(t)$, is said to be periodic with a period T (sec) if

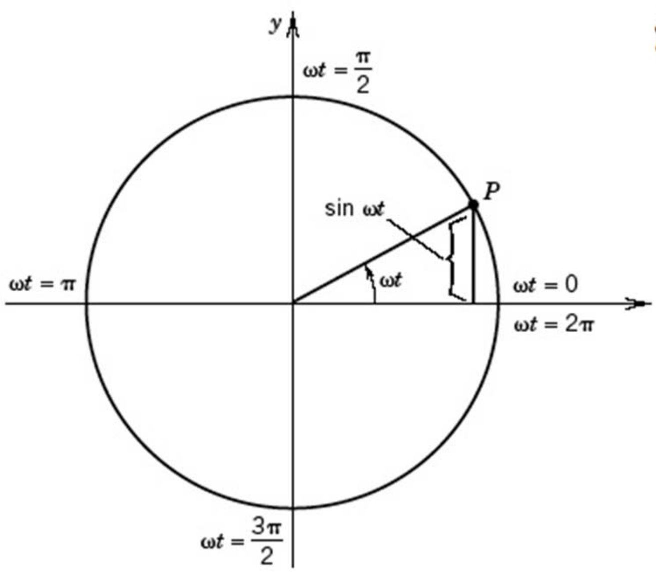
$$y(t+T) = y(t)$$

- Frequency is define as the inverse of period

$$f = \frac{1}{T}\left(\frac{1}{sec} = Hz\right)$$

- **Radial** or angular frequency is defined as

$$\pi f = \frac{2\pi}{T}\left(\frac{Radians}{sec}\right)$$



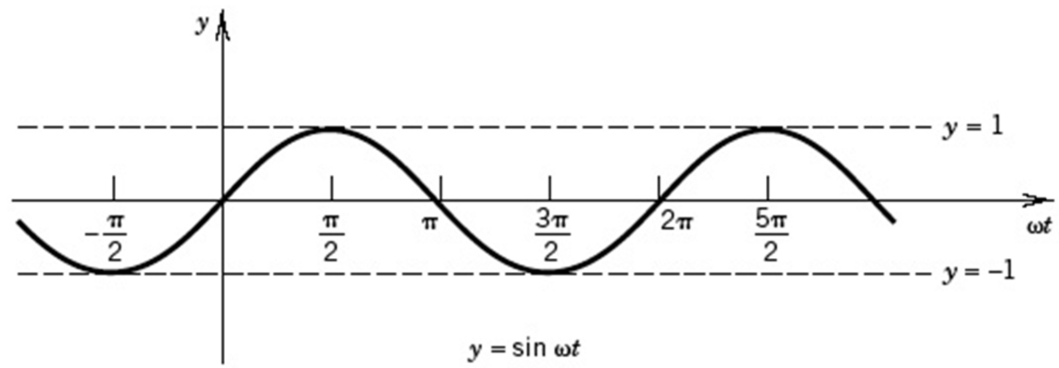


Figure 2.11 Relationship between cycles on the unit circle and circular frequency.

Fourier Series

- In many cases, signals do not have a clearly defined trigonometric form.
- We can represent such signal as a weighted sum of sinusoidal signals (sines and cosines).
- This decomposition into an infinite number of sines and cosines is called a *Fourier series*.

Fourier Series

- The Fourier series if given by

$$y(t) = A_0 + \sum_{i=1}^{\infty} (A_n \cos n\omega t + B_n \sin n\omega t)$$

$$\omega = \frac{2\pi}{T}$$

where

Period $T = 2\pi$	Arbitrary period $T = {}^{2\pi}/_{\omega}$
$A_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} y(t) dt$ $A_n = \frac{1}{\pi} \int_{-\pi}^{\pi} y(t) \cos nt \, dt$ $B_n = \frac{1}{\pi} \int_{-\pi}^{\pi} y(t) \sin nt \, dt$	$A_0 = \frac{1}{T} \int_{-T/2}^{T/2} y(t) dt$ $A_n = \frac{2}{T} \int_{-T/2}^{T/2} y(t) \cos n\omega t \, dt$ $B_n = \frac{2}{T} \int_{-T/2}^{T/2} y(t) \sin n\omega t \, dt$

Fourier Series

- For some special functions, a family of coefficient will be identically zero

$$y(t) = A_0 + \sum_{n=1}^{\infty}(A_n \cos n\omega t + B_n \sin n\omega t)$$

where

Odd functions $y(t) = -y(-t)$	Even functions $y(t) = y(-t)$
$A_0 = 0$ $A_n = 0$ $B_n = \frac{4}{T}\int_0^{T/2} y(t)\sin n\omega t\, dt$	$A_0 = \frac{2}{T}\int_0^{T/2} y(t)dt$ $A_n = \frac{4}{T}\int_0^{T/2} y(t)\cos n\omega t\, dt$ $B_n = 0$

Frequency content of white light

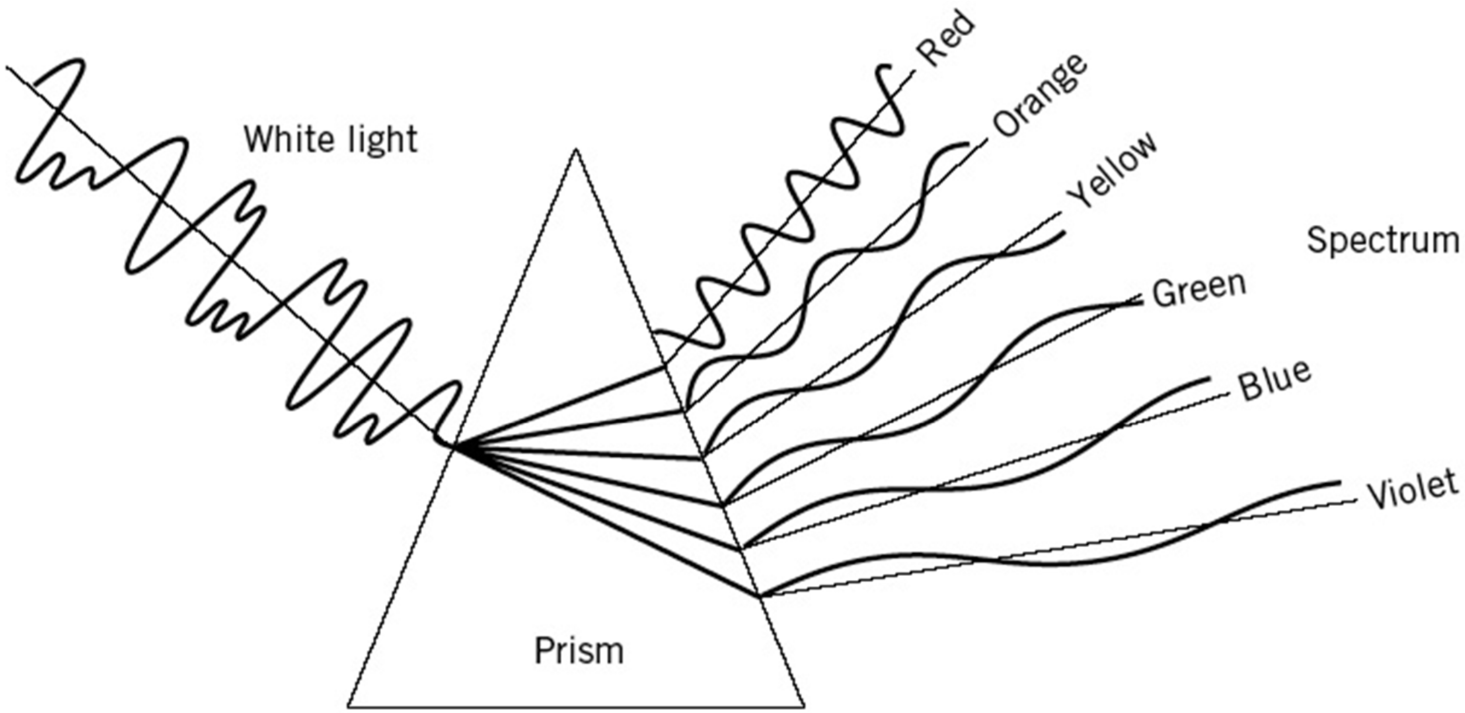


Figure 2.9 Separation of white light into its color spectrum. Color corresponds to a particular frequency or wavelength; light intensity corresponds to varying amplitudes.

Example

- Given the following function

$$y(t) = \begin{cases} -1\ for\ -5 < t \le 0 \\ 1\ for\ 0 < t \le 5 \end{cases}$$

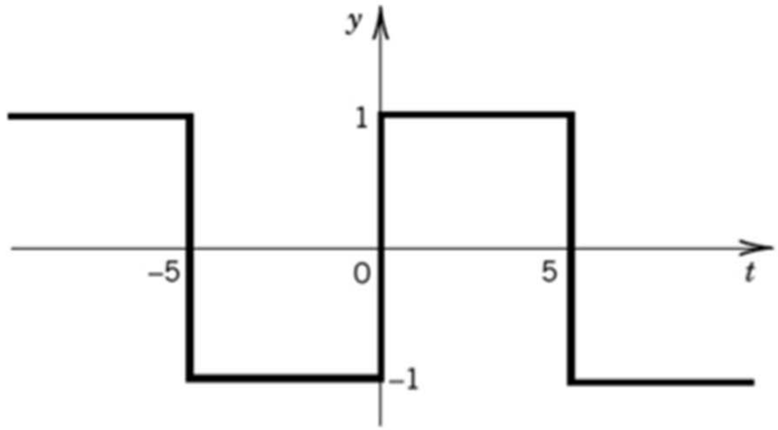


Figure 2.15 Function represented by a Fourier series in Example 2.3.2.

- So we know that the period is 10 (sec)

Example

- This is an odd function then the DC component is zero and the cosine coefficients are all zeros.
- The sine coefficients

$$B_n = \frac{4}{T}\int_0^{T/2} y(t)\sin n\omega t\,dt$$

$$= \frac{4}{10}\int_0^{5} y(t)\sin n\frac{2\pi}{10}t\,dt$$

$$= \frac{2}{5}\left(\int_0^{5} \sin n\frac{2\pi}{10}t\,dt\right)$$

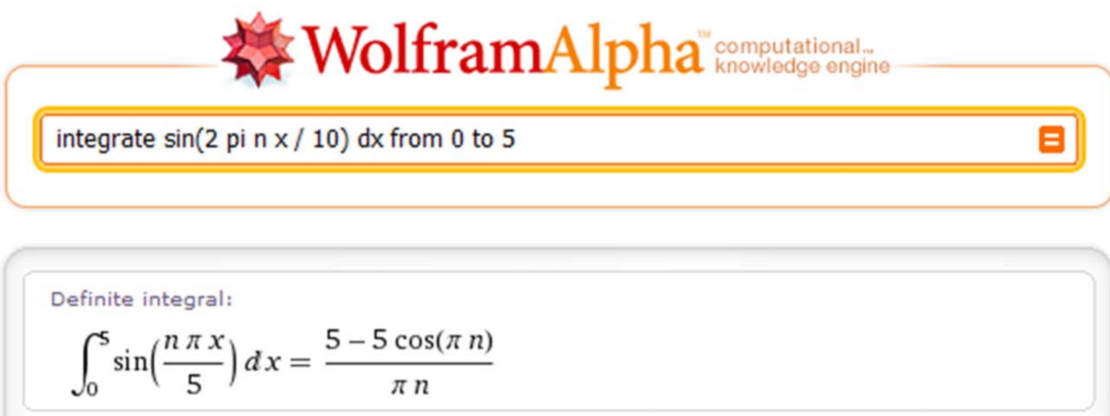
$$= 2\left(\frac{1-\cos n\pi}{n\pi}\right)$$

Example

- Use tools to compute integrals

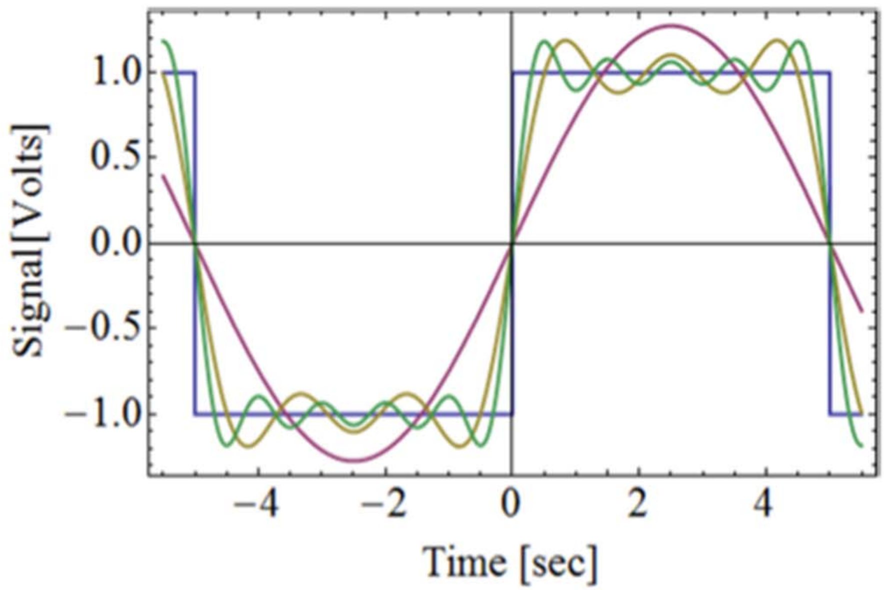
$$\int_0^5 \sin n \frac{2\pi}{10} t \, dt$$

- Take a look at http://www.wolframalpha.com



Example

- Given the following function
- $y(t) = \begin{cases} -1 \; for \; -5 < t \leq 0 \\ 1 \; for \; 0 < t \leq 5 \end{cases}$
- $y(t) \cong \frac{4}{\pi}\sin\frac{2\pi}{10}t + \frac{4}{3\pi}\sin\frac{6\pi}{10}t + \frac{4}{5\pi}\sin\frac{10\pi}{10}t + \frac{4}{7\pi}\sin\frac{14\pi}{10}t + \cdots$

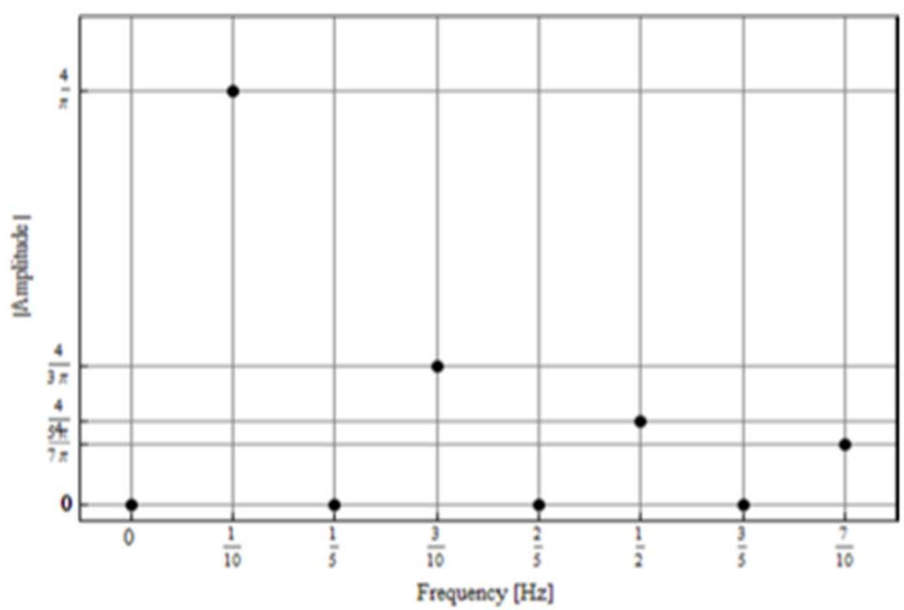


Note that the yellow line represents the sum of the first three terms

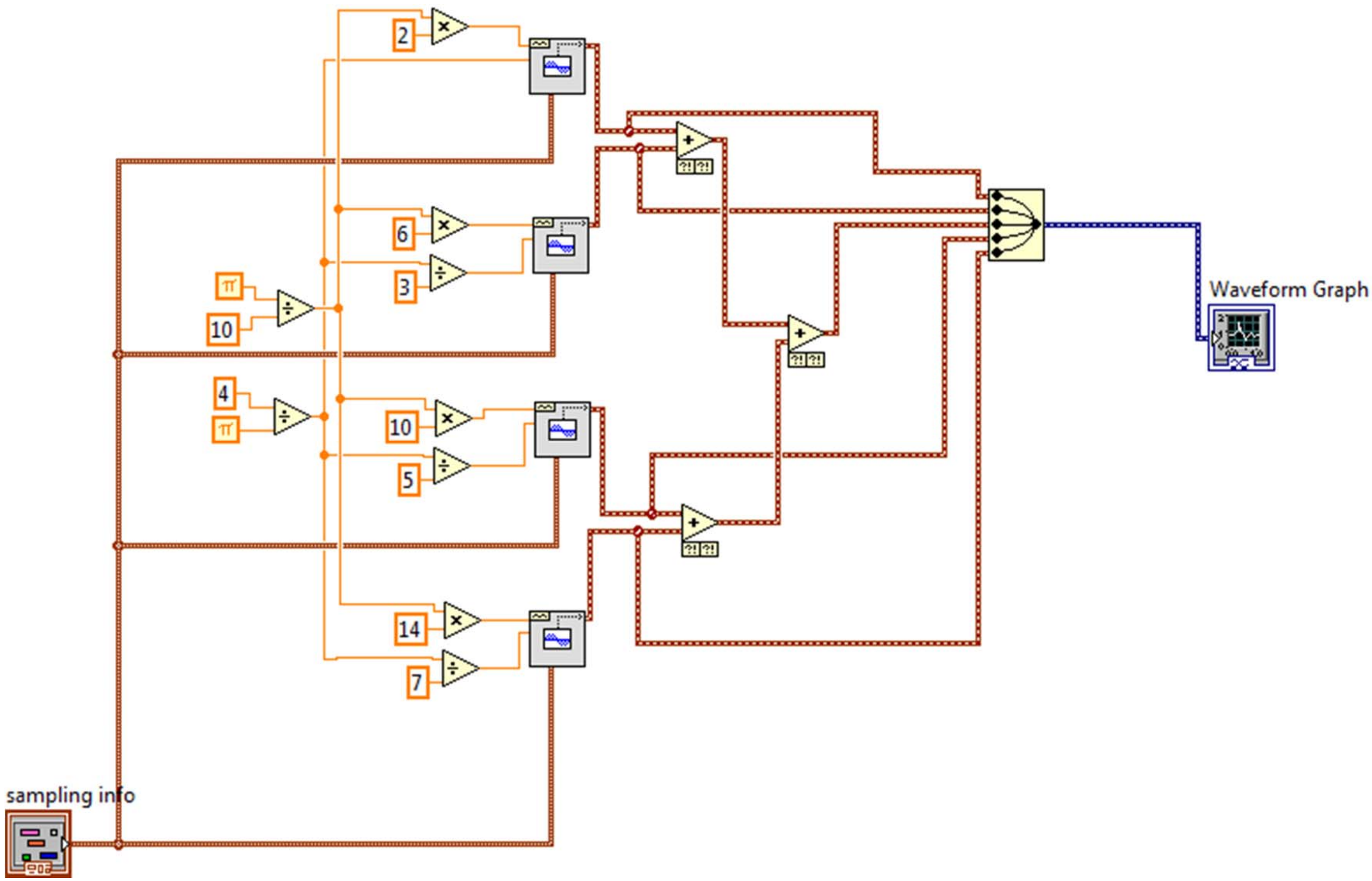
Example

- Plot the absolute value of the coefficient versus the corresponding frequency
- $y(t) \cong \frac{4}{\pi}\sin\frac{2\pi}{10}t + \frac{4}{3\pi}\sin\frac{6\pi}{10}t + \frac{4}{5\pi}\sin\frac{10\pi}{10}t + \frac{4}{7\pi}\sin\frac{14\pi}{10}t + \cdots$

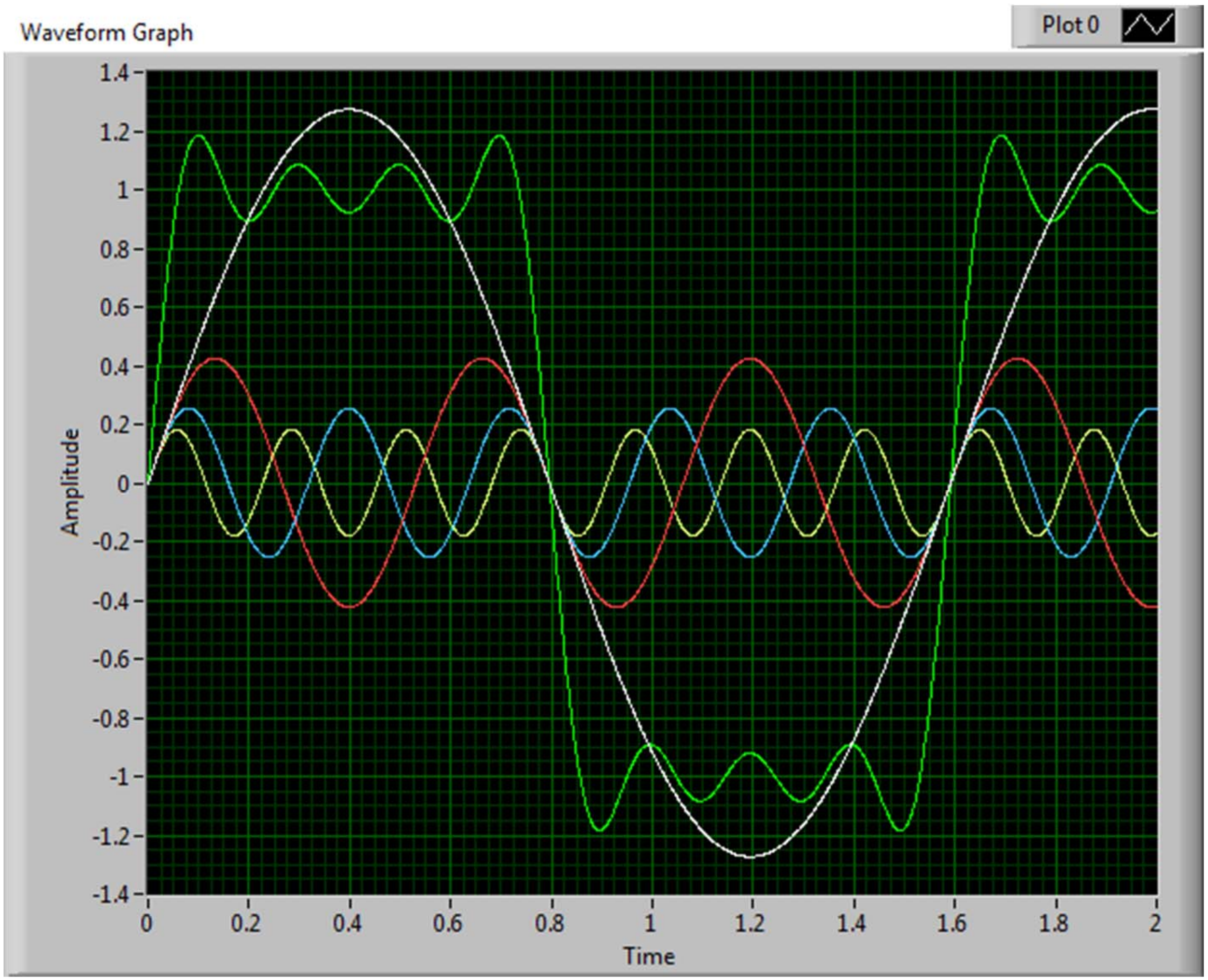
Frequency	Magnitude
0	0
1/10	$4/\pi$
2/10	0
3/10	$4/3\pi$
4/10	0
5/10	$4/5\pi$
6/10	0
7/10	$4/7\pi$



Example



Example



Waveform Graph
Plot 0
Amplitude
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-1.2
-1.4
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
Time