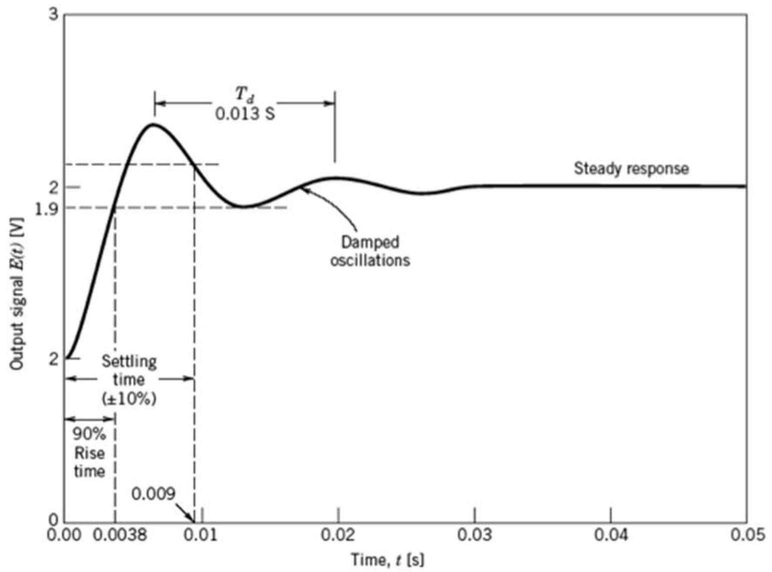
Instrumentation & Measurements
MECH 430

Chapter 3
Measurement System Behavior



3.1 Introduction

- This chapter introduces how to simulate measurement system behavior using mathematical modeling.
- Measurement systems respond differently to different types of input signals.
- Measurement system in this discussion refers to individual stages in the measurement setup or the complete setup.

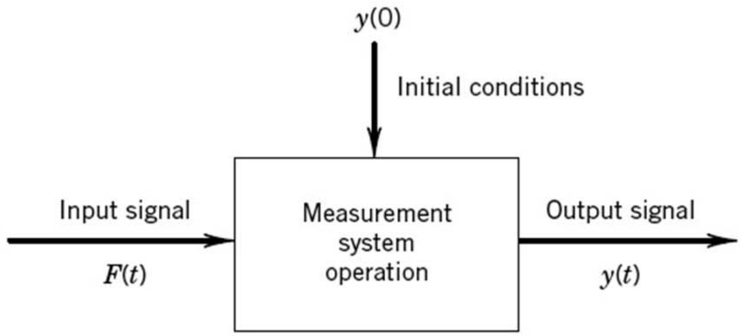


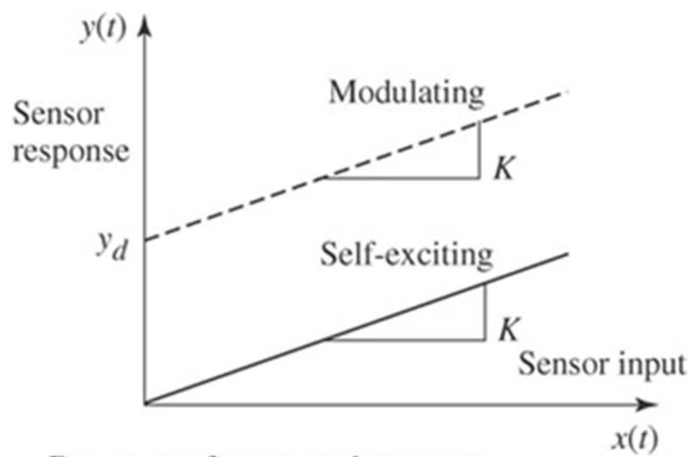
Figure 3.2 Measurement system operation on an input signal, $F(t)$, provides the output signal, $y(t)$.

3.3 Sensor Models

- Systems can be modeled using differential equations.
- The behavior of sensors can described as
  - Zero-order model
  - First-order model
  - Second-order model
- Higher order sensors are usually approximated by one of the models shown above

Zero Order Sensors

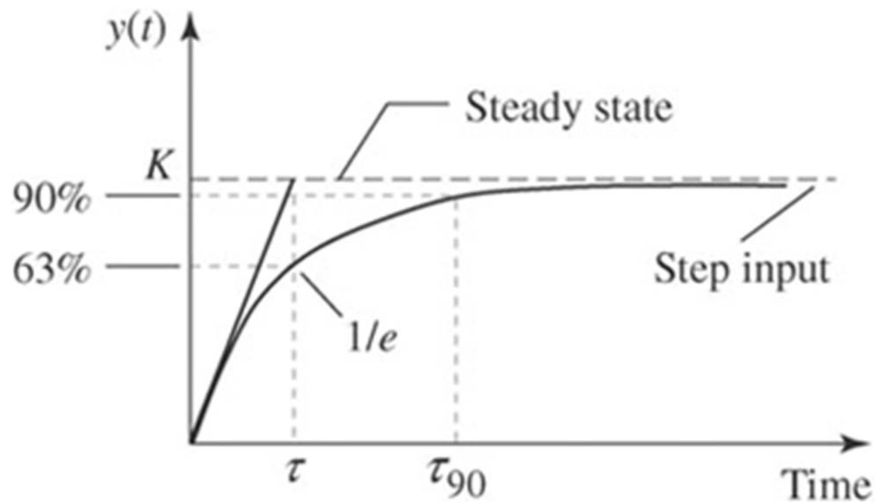
- Represent ideal or perfect dynamic performance
- Response governed by linear algebraic relation: y(t) = K x(t), K is sensor gain
- The system output respond to the input signal instantly.
- Example: potentiometers, tachometers & LVDTs



Response of a zero-order sensor.

First Order Sensors

- Behavior governed by: $$\tau \frac{dy(t)}{dt} + y(t) = Ku(t)$$
  where τ is time constant
- For a step input u(t) → $$y(t) = K + (y_0 - K)e^{(-t/\tau)}$$
- Transfer function: H(s)=Y(s)/U(s)=K/(τs+1)



Response of a first-order system to a step input.

First Order Sensors

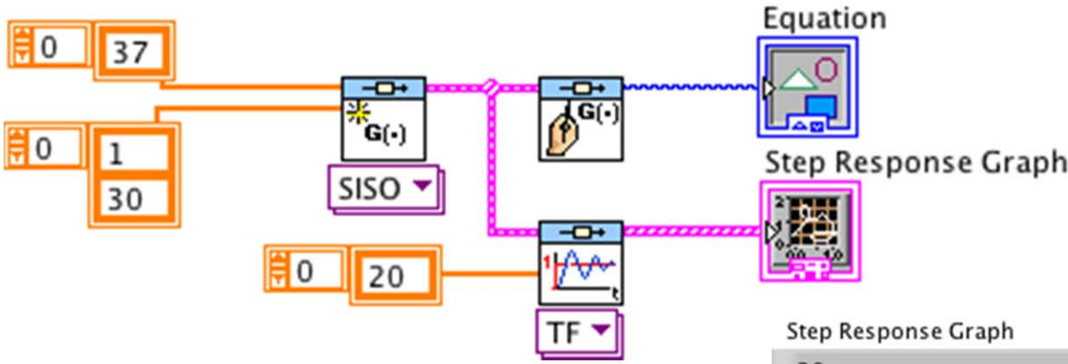
Table 3.1 First-Order System Response and Error Fraction

t/τ	% Response	Γ	% Error
0	0.0	1.0	100.0
1	63.2	0.368	36.8
2	86.5	0.135	13.5
2.3	90.0	0.100	10.0
3	95.0	0.050	5.0
5	99.3	0.007	0.7
∞	100.0	0.0	0.0

Example of first order sensor system

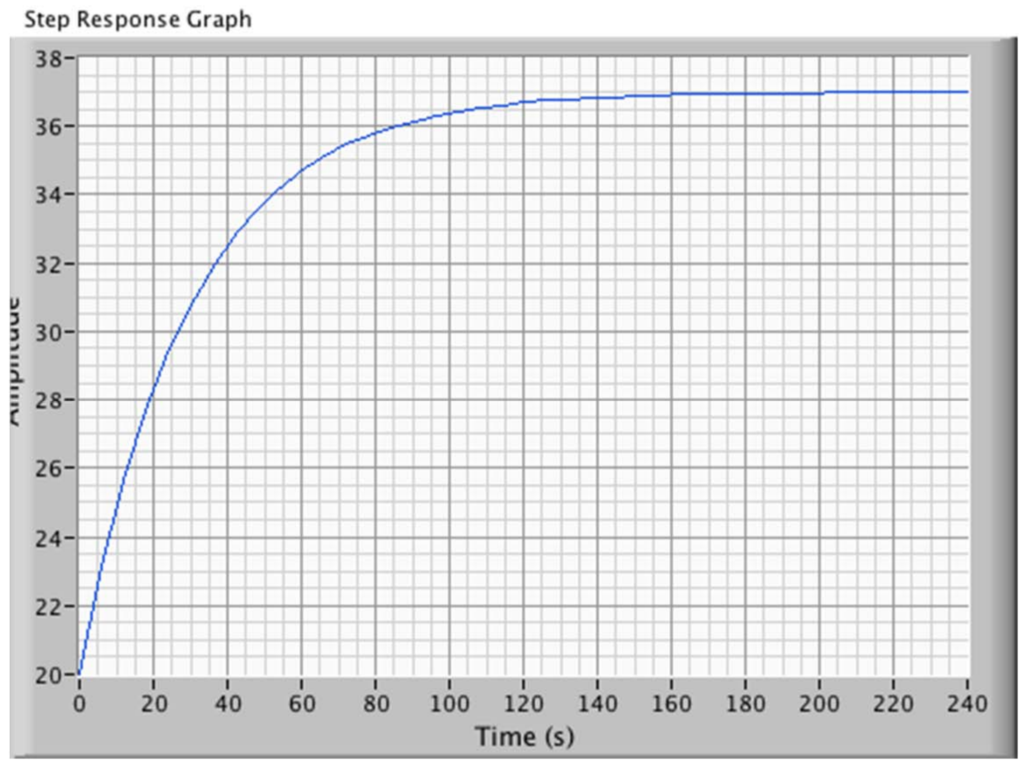
- Example 3.3: A thermometer with a time constant of 30 seconds is originally at room temperature, that is 20 degrees Celsius. It is suddenly exposed to a 37 degrees Celsius.
- Modeling with a first order differential equation:
- $30dT(t)/dt + T(t) = 37$
- The solution of which is:
- $T(t) = 37 + (20 – 37)\ e^{-t/30}$
- $T(t) = 37 – 17\ e^{-t/30}$ degrees Celsius

Example



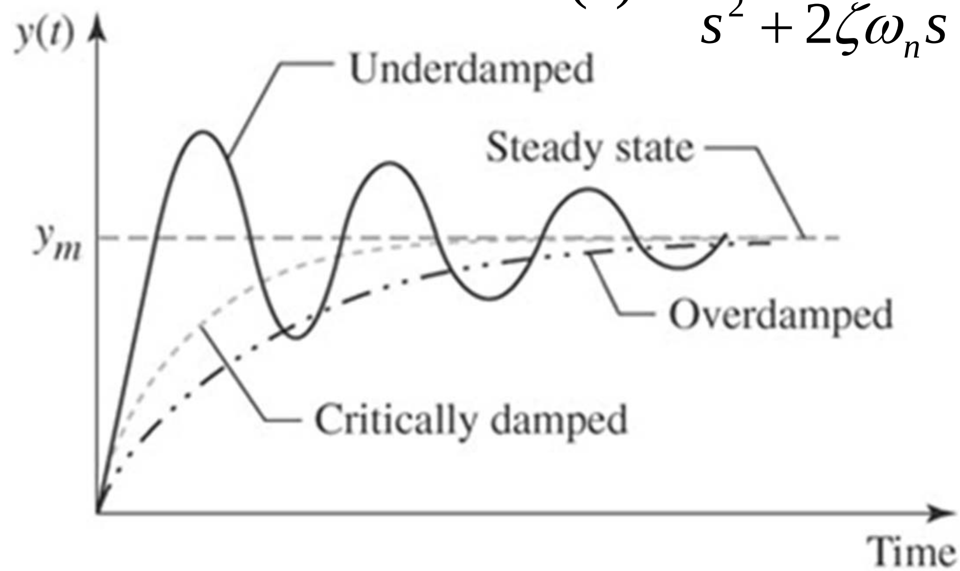
Equation

$$\frac{37}{30s + 1}$$



Second Order Sensors

- Behavior governed by: $\frac{d^2y(t)}{dt^2} + 2\zeta\omega_n \frac{dy(t)}{dt} + \omega_n^2 = Kx(t)$
- ζ is damping ratio and ω_n is natural frequency
- Transfer Function: $H(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2}$



Response of a second-order sensor to a step input.

Example:

Accelerometers

Unit-step response of a second order system

Depending on the value for ζ three forms of homogeneous solution are possible:

$0 \le \zeta < 1$ (underdamped system solution)

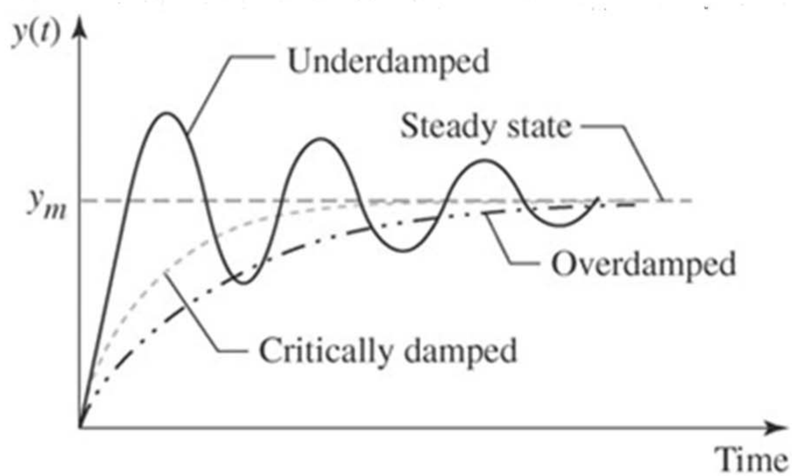
$$y_h(t) = Ce^{-\zeta\omega_n t}\sin\left(\omega_n\sqrt{1-\zeta^2}t + \Theta\right) \quad (3.14a)$$

$\zeta = 1$ (critically damped system solution)

$$y_h(t) = C_1 e^{\lambda_1 t} + C_2 t e^{\lambda_2 t} \quad (3.14b)$$

$\zeta > 1$ (overdamped system solution)

$$y_h(t) = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \quad (3.14c)$$



Response of a second-order sensor to a step input.

Unit-step response of a second order system