Chapter 2 – Problems

2.1

The minimum number of pins required by dual-op-amp is 8. Each op-amp has 2 input terminals (4 pins) and one output terminal (2 pins). Another 2 pins are required for power.
Similarly, the minimum number of pins required by quad-op-amp is 14:
$4 \times 2 + 4 \times 1 + 2 = 14$

2.2

Refer to Fig. P2.2. $v_+ = v_I \frac{1K\Omega}{1M\Omega + 1K\Omega} = \frac{4}{1001} V$

$v_o = A v_+ \Rightarrow A = \frac{4}{4/1001} = 1001$

2.3

The voltage at the positive input has to be $-3.020V$.

$v_+ = -3.020\,V$, $A = \frac{v_o}{(v_+ - v_-)} = \frac{-2}{-3.020-(-3)} = 100$

2.4

#	v_1	v_2	$v_d = v_2 - v_1$	v_o	v_o/v_d
1	0.00	0.00	0.00	0.00	–
2	1.00	1.00	0.00	0.00	–
3	(a)	1.00	(b)	1.00	
4	1.00	1.10	0.10	10.1	101
5	2.01	2.00	-0.01	-0.99	99
6	1.99	2.00	0.01	1.00	100
7	5.10	(c)	(d)	-5.10	

experiments 4, 5, 6 show that the gain is approximately 100 V/V. The missing entry for experiment #3 can be predicted as follows:

(b) $v_d = \frac{v_o}{A} = \frac{1.00}{100} = 0.01\,V$.

(a) $v_1 = v_2 - v_d = 1.00 - 0.01 = 0.99\,V$

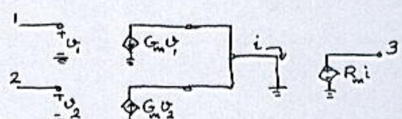
The missing entries for experiment #7:

(d) $v_d = \frac{-5.10}{100} = -0.051\,V$

(c) $v_2 = v_1 + v_d = 5.10 - 0.051 = 5.049\,V$

All the results seem to be reasonable.

2.5



$A = G_m R_m = 100 \times 10^{-3} \times 10^{6} = 100{,}000\ V/V$

2.6

$v_{CM} = 1V \sin(2\pi 60)t \quad = \frac{1}{2}(v_1 + v_2)$

$v_d = 0.01 \sin(2\pi 1000)t = v_1 - v_2$

$v_1 = v_{CM} - v_d/2 = \sin(120\pi)t - 0.005 \sin 2000\pi t$

$v_2 = v_{CM} + v_d/2 = \sin 120\pi t + 0.005 \sin 2000\pi t$

2.7

$v_d = R(G_{m2} v_2 - G_{m1} v_1)$ Refer to Fig. 2.4.

$v_o = v_3 = \mu v_d = \mu R(G_{m2} v_2 - G_{m1} v_1)$

$v_o = \mu R(G_m v_2 + \frac{1}{2}\Delta G_m v_2 - G_m v_1 + \frac{1}{2}\Delta G_m v_1)$

$v_o = \mu R G_m \underbrace{(v_2 - v_1)}_{v_{Id}} + \frac{1}{2}\mu R \Delta G_m \underbrace{(v_1 + v_2)}_{2v_{ICM}}$

we have $v_o = A_d v_{Id} + A_{CM} v_{ICM}$

$\Rightarrow A_d = \mu R G_m \quad , \quad A_{CM} = \mu R \Delta G_m$

$CMRR = 20 \log \left|\frac{A_d}{A_{CM}}\right| = 20 \log \frac{G_m}{\Delta G_m}$

cont.

$20 \log_{10} A_d = 80\,dB \Rightarrow A_d = 10^4$

$\frac{A_{cM}}{A_d} = \frac{\Delta G_m}{G_m} \Rightarrow A_{cm} = 10^4 \times \frac{0.1}{100} = 10$

$CMRR = 20 \log \frac{G_m}{\Delta G_m} = 20 \log \frac{1}{0.1/100} = 60$

2.8

circuit	v_o/v_i (V/V)	Rin (kΩ)	
a	$\frac{-100}{10} = -10$	10	
b	−10	10	
c	−10	10	virtual ground
d	−10	10	no current in 10kΩ

2.9

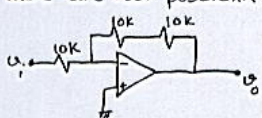
closed loop gain = −1 V/V . for $v_i = 5V \Rightarrow v_o = -5\,V$

Gain would be in the range of $\frac{-0.95}{1.05}$ to $\frac{-1.05}{0.95}$: $-0.9 < G < -1.1$

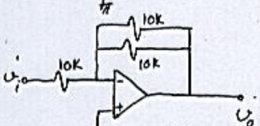
For $v_i = 5 \Rightarrow -4.5 < v_o < -5.5V$

2.10

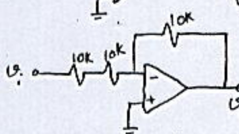
There are four possibilities :



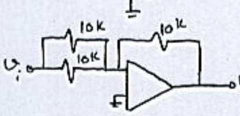
$\frac{v_o}{v_i} = -2\,V/V \quad R_{in} = 10k\Omega$



$\frac{v_o}{v_i} = -0.5\,V/V \quad R_{in} = 10\,k\Omega$



$\frac{v_o}{v_i} = -0.5\,V/V \quad R_{in} = 20\,k\Omega$



$\frac{v_o}{v_i} = -2\,V/V \quad R_{in} = 5\,k\Omega$

2.11

a. G = −1 V/V

b. G = −10 V/V

c. G = −0.1 V/V

d. G = −100 V/V

e. G = −10 V/V

2.12

a. $G = -1\,V/V = -\frac{R_2}{R_1} \Rightarrow R_1 = R_2 = 10k\Omega$

b. $G = -2\,V/V = -\frac{R_2}{R_1} \Rightarrow R_1 = 10k\Omega,\ R_2 = 20k\Omega$

c. $G = -0.5\,V/V = -\frac{R_2}{R_1} \Rightarrow R_1 = 20k\Omega,\ R_2 = 10k\Omega$

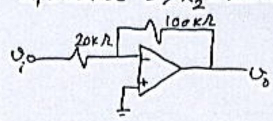
d. $G = -100\,V/V = -\frac{R_2}{R_1} \Rightarrow R_1 = 10k\Omega,\ R_2 = 1M\Omega$

2.13

$\frac{v_o}{v_i} = -5 = -\frac{R_2}{R_1} \Rightarrow R_2 = 5R_1$

$R_1 + R_2 = 120k\Omega \Rightarrow 5R_1 + R_1 = 120k\Omega \Rightarrow$

$R_1 = 20k\Omega \Rightarrow R_2 = 100\,k\Omega$



2.14

$20 \log|G| = 26\,dB \Rightarrow G = -19.95\,V/V = \frac{v_o}{v_i} = -\frac{R_2}{R_1}$

$\Rightarrow R_2 = 19.95 R_1 \leq 10M\Omega$

For largest possible input resistance, select

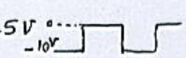
$R_2 = 10M\Omega \Rightarrow R_1 \simeq 500k\Omega$

$R_{in} = 500\,k\Omega$

2.15

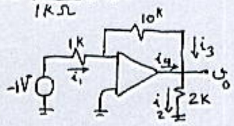
$G = \frac{v_o}{v_i} = -\frac{R_2}{R_1} = -\frac{100}{10} = -10$

$v_{low} = -10V,\ v_{high} = 0,\ v_{avg} = -5V$



2.16

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1} \Rightarrow v_o = -1 \times -\frac{10\,k\Omega}{1\,k\Omega} = 10\,V$$



$$i_2 = \frac{v_o}{2\,k\Omega} = 5\,mA$$

$$i_1 = i_3 = \frac{v_o}{10\,k\Omega} = 1\,mA$$

$$i_4 = i_2 - i_3 = 4\,mA$$

This additional current comes from the output of the op-amp.

2.17

$$|\text{Gain}| = \frac{R_2}{R_1} = \frac{R_2\,(1+x/100)}{R_1(1+x/100)} \simeq \frac{R_2}{R_1}\left(1 \pm \frac{2x}{100}\right)$$

for small x

$\Rightarrow 2x\%$ is the tolerance on the closed loop gain (G).

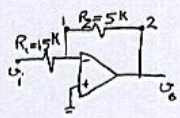
$G = -100\,V/V$, $x = 5 \Rightarrow -110 < G < -90$

or more precisely: $-100 \times \frac{105}{95} < G < -100\,\frac{95}{105}$

$-110.5 < G < -90.5$

2.18

$$G = \frac{v_o}{v_i} = -\frac{R_2}{R_1} \Rightarrow \frac{R_2}{R_1} = \frac{5}{15}$$



$v_1 = 0V \quad , v_2 = v_o = 5V$

For $\pm 1\%$ on R_1, R_2: $R_1 = 15 \pm 0.15\,K\Omega$

$R_2 = 5 \pm 0.05\,K\Omega$

$$v_o = v_i\,\frac{-R_2}{R_1} = 15\,\frac{R_2}{R_1} \Rightarrow 15 \times \frac{4.95}{15.15} \leq v_o \leq 15 \times \frac{5.05}{14.85}$$

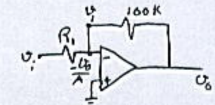
$\Rightarrow 4.9V \leq v_o \leq 5.1V$

For $v_i = -15 \pm 0.15\,V$ $\quad 14.85 \times \frac{4.95}{15.15} \leq v_o \leq 15.15 \times \frac{5.05}{14.85}$

$\Rightarrow 4.85V \leq v_o \leq 5.15V$

2.19

$$v_1 = \frac{-v_o}{A} = -\frac{v_o}{200}$$



$$\frac{v_o}{v_i} = -50\,V/V$$

$$\frac{v_i - \left(\frac{-v_o}{A}\right)}{R_1} = \frac{\left(\frac{-v_o}{A} - v_o\right)}{100\,k\Omega} \Rightarrow R_1 = 100K \times \frac{\frac{v_o}{200} - \frac{v_o}{50}}{\frac{-v_o}{200} - v_o}$$

$$\Rightarrow R_1 = 100K \times \frac{3}{201} = 1.49\,k\Omega$$

Shunt Resistor R_a: $R_a \parallel 2\,k\Omega = 1.49K$

$$\frac{R_a \times 2}{R_a + 2} = 1.49 \Rightarrow R_a = 5.84\,k\Omega$$

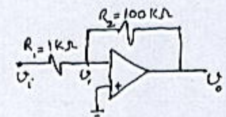
2.20

ⓐ

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1} \Rightarrow -100\,V/V = \frac{-R_2}{1\,K\Omega} \Rightarrow R_2 = 100\,K\Omega$$

ⓑ $A = 1000\,V/V$

$v_1 = \frac{-v_o}{A}$



$$\frac{v_i - v_1}{R_1} = \frac{v_1 - v_o}{R_2}$$

$$\frac{v_o}{v_i} = \frac{-R_2/R_1}{1 + (1 + \frac{R_2}{R_1})/A} = \frac{-100}{1 + \frac{101}{1000}} = -90.8\,V/V$$

$$\Rightarrow \frac{v_o}{v_i} = 90.8\,V/V$$

ⓒ Assume $R_1' = R_x \parallel R_1$ when $R_1 = 1\,K\Omega$

$$\frac{v_o}{v_i} = -100\,V/V$$

$$\frac{v_i - v_1}{R_1'} = \frac{v_1 - v_o}{R_2} \Rightarrow R_1' = R_2 \times \left(\frac{-v_o}{100} - \frac{-v_o}{1000}\right) \Big/ \left(\frac{-v_o}{1000} - v_o\right)$$

$$R_1' = \frac{1 - 0.1}{1.001} = 0.899\,K\Omega = \frac{R_1 R_x}{R_1 + R_x} = \frac{R_x}{1 + R_x}$$

$$\Rightarrow R_x = 8.9\,K\Omega \simeq 8.87\,K\Omega \pm 1\%$$

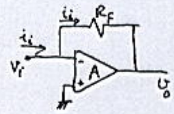
2.21

Voltage of the inverting input terminal

Cont.

will vary from $\frac{-10V}{1000}$ to $\frac{+10V}{1000}$. Thus the virtual ground will depart from the ideal voltage of zero by a maximum of $\pm 10\,mV$.

2.22



a) For $A = \infty$: $v_i = 0$

$v_o = -i_i R_F$

$R_m = \frac{v_o}{i_i} = -R_F$

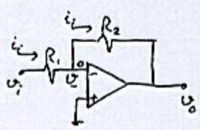
$R_{in} = \frac{v_i}{i_i} = 0$

b) For A = finite : $v_i = -\frac{v_o}{A}$, $v_o = v_i - i_i R_F$

$\Rightarrow v_o = \frac{-v_o}{A} - i_i R_F \Rightarrow R_m = \frac{v_o}{i_i} = -\frac{R_F}{1+\frac{1}{A}}$

$R_i = \frac{v_i}{i_i} = \frac{R_F}{1+A}$

2.23



$v_o = -Av_- = v_- - i_i R_2$

$i_i R_2 = (1+A) v_-$

$v_- = \frac{i_i R_2}{1+A}$

Now: $v_i = i_i R_1 + v_- = i_i R_1 + i_i \frac{R_2}{1+A}$

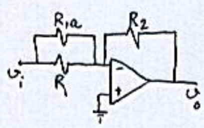
$R_{in} = \frac{v_i}{i_i} = R_1 + \frac{R_2}{1+A}$

2.24

$G = \frac{-R_2/R_1}{1 + \frac{1+R_2/R_1}{A}}$ Gain Error $\epsilon = \left(1+\frac{R_2}{R_1}\right)/A \times 100$

ϵ	0.1%	1%	10%
A	$1000(1+\frac{R_2}{R_1})$	$100(1+\frac{R_2}{R_1})$	$10(1+\frac{R_2}{R_1})$

Gain with R_{1a} :



$G \simeq \frac{\frac{R_2}{R_1}\left(1+\frac{R_1}{R_{1a}}\right)}{1+\frac{1+R_2/R_1}{A}}$

where we have neglected the effect of R_{1a} on the error on the denominator. To restore the gain to its nominal value of R_2/R_1 we use:

$\frac{R_1}{R_{1a}} = \frac{1+R_2/R_1}{A} = \frac{\epsilon}{100} \rightarrow R_{1a} = \frac{100R_1}{\epsilon}$

ϵ	0.1%	1%	10%
R_{1a}	$1000R_1$	$100R_1$	$10R_1$

2.25

$R_1' = R_1 \parallel R_c$ $G' = \frac{-R_2/R_1'}{1+\frac{1+R_2/R_1'}{A}}$

$G = -\frac{R_2}{R_1}$

In order for $G' = G$: $G = \frac{-R_2/R_1'}{1+\frac{1+R_2/R_1'}{A}} = \frac{-R_2}{R_1}$

$R_1' = \frac{R_1 R_c}{R_1+R_c}$

$\Rightarrow \frac{R_1+R_c}{R_1 R_c} = \frac{1}{R_1}\left(1+\frac{1+R_2\frac{(R_1+R_c)}{R_1 R_c}}{A}\right)$

$(R_1+R_c)A = AR_c + R_c + \frac{R_2}{R_1}(R_1+R_c)$

$R_1 A = R_c + GR_1 + GR_c$

$\frac{R_c}{R_1} = \frac{A-G}{1+G}$

2.26

$G = \frac{-R_2/R_1}{1+\frac{1+R_2/R_1}{A}}$ $G_{nominal} = \frac{-R_2}{R_1}$

$\epsilon = \left|\frac{G - G_{nominal}}{G_{nominal}}\right| = \left|\frac{G}{G_{nominal}} - 1\right|$

$\epsilon = \left|\frac{1}{1+\frac{1+R_2/R_1}{A}} - 1\right| = \left|\frac{-\frac{1+R_2/R_1}{A}}{1+\frac{1+R_2/R_1}{A}}\right| = \frac{1}{\frac{A}{1+\frac{R_2}{R_1}}+1}$

which can be rearranged to yield:

$\frac{A}{1+\frac{R_2}{R_1}} + 1 = \frac{1}{\epsilon} \Rightarrow A = \left(1+\frac{R_2}{R_1}\right)\left(\frac{1}{\epsilon}-1\right)$

or $A = (1 - G_{nominal})\left(\frac{1}{\epsilon}-1\right)$

For $G_{nominal} = -100\ V/V$ and $\epsilon = 10\% = 0.1$

$A = (1+100)\left(\frac{1}{0.1}-1\right) = 909\ V/V$

This is the minimum required value for A.

2.27

$$|G| = \frac{R_2/R_1}{1+\frac{1+\frac{R_2}{R_1}}{A}} \qquad A \longrightarrow A\left(1-\frac{x}{100}\right)$$
$$\qquad\qquad\qquad\qquad G \longrightarrow G'$$

$$|G'| = \frac{R_2/R_1}{1+\frac{1+R_2/R_1}{A\left(1-\frac{x}{100}\right)}}$$

For $|G'| = |G|\left(1-\frac{x}{100K}\right)$

$$\frac{R_2/R_1}{1+\frac{1+R_2/R_1}{A(1-x/100)}} = \frac{R_2/R_1}{1+\frac{1+R_2/R_1}{A}}\left(1-\frac{x}{100K}\right)$$

$$1+\frac{1+R_2/R_1}{A\left(1-\frac{x}{100}\right)} = \left(1+\frac{1+R_2/R_1}{A}\right)\Big/\left(1-\frac{x}{100K}\right)$$

$$1-\frac{x}{100K}+\frac{1+R_2/R_1}{A}\,\frac{1-x/100K}{1-x/100} = 1+\frac{1+R_2/R_1}{A}$$

$$\frac{1+R_2/R_1}{A}\,\frac{1-x/100K-1+x/100}{1-x/100} = \frac{x}{100K}$$

$$A = \frac{-1+K}{1-\frac{x}{100}}\left(1+R_2/R_1\right) = \left(\frac{K-1}{1-\frac{x}{100}}\right)\left(1+\frac{R_2}{R_1}\right)$$

For $\frac{R_2}{R_1} = 100$ $\quad x = 50 \quad K = 100:\ A = \frac{99}{0.5} \times 101 = 19998$

$A \simeq 2\times 10^4\ V/V$

Thus for $A = 2\times10^4\ V/V$, a reduction of 50% results in only 0.5% reduction of the closed loop gain whose nominal value is $\frac{R_2}{R_1}(100)$.

2.28

From the results of example 2.2, the gain of the circuit in fig. 2.8 is given by:

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1}\left(1+\frac{R_4}{R_2}+\frac{R_4}{R_3}\right)$$

For $R_1 = R_2 = R_4 = 1M\Omega \Rightarrow \frac{v_o}{v_i} = -\left(1+1+\frac{1}{R_3}\right)$

a) $\frac{v_o}{v_i} = -10\ V/V \Rightarrow 10 = 2+\frac{1}{R_3} \Rightarrow R_3 = \frac{1}{8}M\Omega = 125\,K\Omega$

b) $\frac{v_o}{v_i} = -100\ V/V \Rightarrow 100 = 2+\frac{1}{R_3} \Rightarrow R_3 = \frac{1}{98}M\Omega = 10.2\,K\Omega$

c) $\frac{v_o}{v_i} = -2\ V/V \Rightarrow 2 = 2+\frac{1}{R_3} \Rightarrow R_3 = \infty$: eliminate R_3.

2.29

$R_2/R_1 = 1000$, $R_2 = 100K\Omega \Rightarrow R_1 = 100\Omega$

a) $R_{in} = R_1 = 100\Omega$

b) $\frac{v_o}{v_i} = \frac{-R_2}{R_1}\left(1+\frac{R_4}{R_2}+\frac{R_4}{R_3}\right) = -1000$

IF $R_2 = R_1 = R_4 = 100K \Rightarrow R_3 = \frac{100K}{1000-2} \simeq 100\,\Omega$

$R_{in} = R_1 = 100K\Omega$

2.30

$$v_x = 0 - i_1 R_2\ ,\quad i_1 = \frac{v_I}{R_1} \Rightarrow v_x = -v_I\frac{R_2}{R_1}$$

$$\frac{v_x}{v_I} = -\frac{R_2}{R_1}$$

$$v_x = v_o\,\frac{R_2\|R_3}{R_2\|R_3+R_4} = v_o\,\frac{R_2R_3}{R_2R_3+R_4R_2+R_4R_3}$$

$$\frac{v_o}{v_x} = \frac{R_2R_3+R_2R_4+R_3R_4}{R_2R_3} = 1+\frac{R_4}{R_3}+\frac{R_4}{R_2}$$

$$\frac{v_o/v_x}{v_I/v_x} = \frac{v_o}{v_i} = \frac{(1+R_4/R_3+R_4/R_2)}{-R_1/R_2} \Rightarrow$$

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1}\left(1+\frac{R_4}{R_3}+\frac{R_4}{R_2}\right)$$

2.31

a) $R_1 = R$

$R_2 = R\|R + \frac{R}{2} = \frac{R}{2}+\frac{R}{2} = R$

$R_3 = R_2\|R + \frac{R}{2} = R\|R + \frac{R}{2} = R$

$R_4 = R_3\|R + \frac{R}{2} = R\|R + \frac{R}{2} = R$

b) $v_1 = RI = RI_1 \Rightarrow \underline{I_1 = I}$

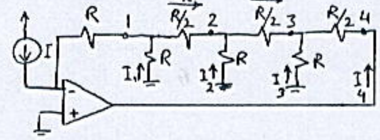
$I_{12} = I + I = 2I \Rightarrow v_1 + 2I\times\frac{R}{2} = RI_2$

$RI + RI = RI_2 \Rightarrow \underline{I_2 = 2I}$

$I_{13} = I_2 + I_{12} = 4I \Rightarrow v_2 + 4I\times\frac{R}{2} = RI_3$

$R\times 2I + 4I\times\frac{R}{2} = RI_3 \Rightarrow \underline{I_3 = 4I},\ I_4 = -(4I+4I)$

$\underline{I_4 = 8I}$



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c) $v_1 = I_1 R = IR$
$v_2 = I_2 R = 2IR$
$v_3 = -I_3 R = -4IR$
$v_4 = -I_3 R + I_4 \frac{R}{2} = -4IR - 8I\frac{R}{2} = -8IR$

2.32

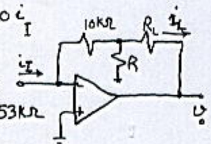
a) $I_1 = \frac{1V}{10k\Omega} = 0.1mA$
$I_2 = I_1 = 0.1\ mA \quad , I_2 \times 10k\Omega = I_3 \times 100\Omega \Rightarrow I_3 = 10^{mA}$
$v_x = 10mA \times 100\Omega = 1V$

b) $v_x = R_L I_L + v_0 \quad , I_L = I_2 + I_3 = 10.1mA$
$1V = R_L \times 10.1mA + v_0$
$R_L = \frac{1 - v_0}{10.1} \Rightarrow R_{L max} = \frac{1 - v_{0 min}}{10.1} = \frac{14}{10.1}$
$R_{L max} =$

c) $100\Omega \leq R_L \leq 1K\Omega$
I_L stays fixed at $10.1mA$
$v_0 = v_x - R_L I_L = 1 - R_L \times 10.1 \Rightarrow -9.1^V \leq v_0 \leq -0.01^V$

2.33

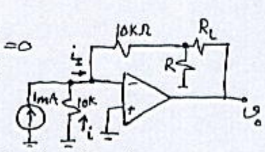
a) $\frac{i_L}{i_I} = 20 \Rightarrow i_L = 20 i_I$



$-10k\Omega \times i_I = R(i_I - i_L)$
$R = \frac{10k\Omega \times i_I}{20 i_I - i_I} = 0.53K\Omega$

b) $R_L = 1K\Omega \quad -12^V \leq v_0 \leq 12^V$
$v_0 = R_L i_L + 10K\Omega \times i_I = i_I (1K\Omega \times \frac{i_L}{i_I} + 10^{K\Omega})$
$v_0 = i_I (1 \times 20 + 10) = 30 i_I$
$i_I = \frac{v_0}{30} \Rightarrow \frac{-12}{30} \leq i_I \leq \frac{12}{30} \Rightarrow -0.4_{mA} \leq i_I \leq 0.4_{mA}$

c) $R_I = \frac{v_I}{i_I} = \frac{0}{i_I} = 0$



$v_- = 0 \Rightarrow i = 0$
$\Rightarrow i_I = 1mA$
From part a: $i_L = 20 \times i_I = 20\ mA$

2.34

$R_2 \gg R_3$, if we ignore the current across R_2 : $v_A = \frac{v_0 R_3}{R_3 + R_4}$

$\frac{v_I}{R_1} = \frac{0 - v_A}{R_2} \Rightarrow v_A = -\frac{R_2 v_I}{R_1}$

$v_0 \frac{R_3}{R_3 + R_4} = -\frac{R_2}{R_1} \times v_I \Rightarrow \frac{v_0}{v_I} = \frac{-R_2}{R_1}(1 + \frac{R_4}{R_3})$

Now if we recalculate v_A considering that There is a voltage divider between R_4 and $R_3 \| R_2$:

$$v_A = v_0 \frac{R_3 \| R_2}{R_4 + R_3 \| R_2} = v_0 \frac{R_3 R_2}{R_4(R_3 + R_2) + R_2 R_3}$$

$$v_A = v_0 \frac{R_2 R_3}{R_3 R_4 + R_2 R_4 + R_2 R_3}$$

$$v_A = v_0 \frac{1}{R_4/R_2 + \frac{R_4}{R_3} + 1}$$

$v_A = -\frac{R_2}{R_1} . v_I \Rightarrow \frac{v_0}{v_I} = \frac{-R_2}{R_1}(\frac{R_4}{R_2} + \frac{R_4}{R_3} + 1)$

Same as example 2.2.

2.35

$R_I = 100K\Omega \quad -10 \leq \frac{v_0}{v_i} \leq -1\ V/V$

$R_I = R_1 = 100K\Omega$

$\frac{v_0}{v_i} = \frac{-R_2}{R_1}(\frac{R_4}{R_3} + \frac{R_4}{R_2} + 1)$

$R_4 = 0 \Rightarrow \frac{v_0}{v_i} = \frac{-R_2}{R_1} = -1 \Rightarrow R_2 = 100K\Omega$

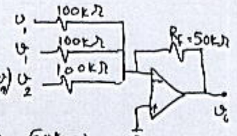
$R_4 = 10K\Omega \Rightarrow \frac{v_0}{v_i} = -10 = -1 \times (\frac{10^{K\Omega}}{R_3} + \frac{10^{K\Omega}}{100^{K\Omega}} + 1)$

$+10 = (\frac{10}{R_3} + 1.1) \Rightarrow R_3 = 1.12K\Omega$

Potentiometer in the middle: $\frac{v_0}{v_i} = -1(\frac{5}{5 + R_3} + \frac{5}{100} + 1)$
$\frac{v_0}{v_i} = -1.87 V/V$

2.36

According to eq. 2.7:



$$v_o = -\left(\frac{R_F}{R_1}v_1 + \frac{R_F}{R_2}v_2 + \frac{R_F}{R_n}v_n\right)$$

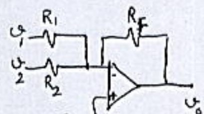
$$v_o = -\left(\frac{50k}{100k}v_1 + \frac{50k}{100k}v_1 + \frac{50k}{100k}v_2\right)$$

$$v_o = -\left(v_1 + \frac{v_2}{2}\right) \qquad v_1 = 3,\ v_2 = -3 \Rightarrow v_o = -1.5v$$

2.37

we choose the weighted summer configuration

$$v_o = -\left[4v_1 + \frac{v_2}{3}\right]$$



$i_1 = \frac{v_1}{R_1}$ $\quad i_2 = \frac{v_2}{R_2}$

$i_1, i_2 \leq 0.1\ mA$ for $v_1, v_2 = 1V$

$R_1, R_2 \geq 10k\Omega$

$\frac{R_F}{R_1} = 4$, if $R_1 = 10k\Omega \Rightarrow R_F = 40k\Omega$

$\frac{R_F}{R_2} = \frac{1}{3} \Rightarrow R_2 = 120k\Omega$

2.38

$$v_o = -(2v_1 + 4v_2 + 8v_3)$$

$R_1, R_2, R_3 \geq 10k\Omega$

$\frac{R_F}{R_1} = 2$, $\quad \frac{R_F}{R_2} = 4$ $\quad \frac{R_F}{R_3} = 8$

$R_3 = 10k\Omega \Rightarrow R_F = 80k\Omega$

$R_2 = 20k\Omega$

$R_1 = 40k\Omega$

2.39

a) $v_o = -(v_1 + 2v_2 + 3v_3)$

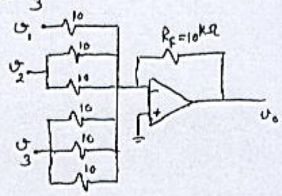
$\frac{R_F}{R_1} = 1 \Rightarrow R_1 = 10k\Omega$, $\frac{R_F}{R_2} = 2 \Rightarrow R_2 = 5k\Omega$

$\frac{R_F}{R_3} = 3 \Rightarrow R_3 = \frac{10}{3}k\Omega$

$R_{I1} = 10k\Omega$

$R_{I2} = 5k\Omega$

$R_{I3} = 3.3K$



b) $v_o = -(v_1 + v_2 + 2v_3 + 2v_4)$

$\frac{R_F}{R_1} = 1 \Rightarrow R_1 = 10k\Omega$

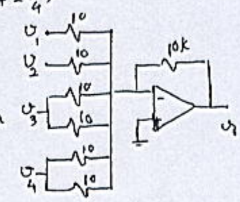
$\frac{R_F}{R_2} = 1 \Rightarrow R_2 = 10k\Omega$

$\frac{R_F}{R_3} = 2 \Rightarrow R_3 = \frac{10}{2}k\Omega$

$\frac{R_F}{R_4} = 2 \Rightarrow R_4 = \frac{10}{2}k\Omega$

$R_{I1} = R_{I2} = 10k\Omega$

$R_{I3} = R_{I4} = 5k\Omega$



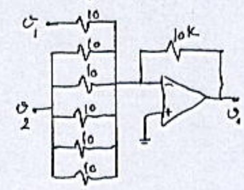
c) $v_o = -(v_1 + 5v_2)$

$R_1 = 10K$

$R_2 = \frac{10K}{5}$

$R_{I1} = 10k\Omega$

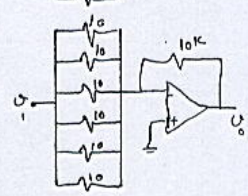
$R_{I2} = 2k\Omega$



d) $v_o = -6v_1$

$R_1 = \frac{10k}{6}$

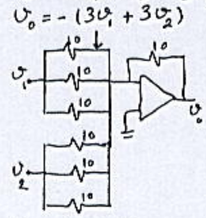
$R_{I1} = 1.67k\Omega$

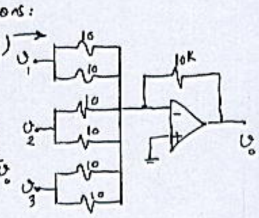


Suggested configurations:

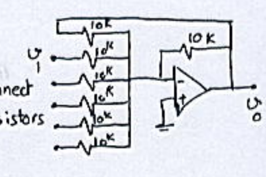
$v_o = -(2v_1 + 2v_2 + 2v_3)$

$v_o = -(3v_1 + 3v_2)$





In order to have coefficient = 0.5, connect one of the input resistors to v_o. $\frac{v_o}{v_1} = -0.5$



2.40

The output signal should be:

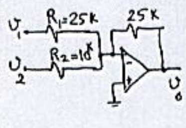
$v_o = -5\sin\omega t - 5$

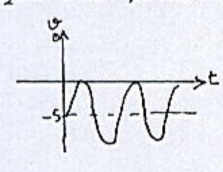
if we assume: $v_1 = 5\sin\omega t$, $v_2 = 2V$ $\Big\}\ v_o = -(v_1 + 2.5 v_2)$

In a weighted summer configuration:

$\frac{R_F}{R_1} = +1 \qquad \frac{R_F}{R_2} = 2.5$

$R_2 = 10K\Omega \Rightarrow R_F = 25K = R_1$





2.41

$v_o = v_1 + 2v_2 - 3v_3 - 4v_4$: Consider Fig. 2.11.

According to eq. 2.8 for a weighted summer circuit:

$v_o = v_1 \frac{R_a}{R_1}\frac{R_c}{R_b} + v_2 \frac{R_a}{R_2}\frac{R_c}{R_b} - v_3\frac{R_c}{R_3} - v_4\frac{R_c}{R_4}$

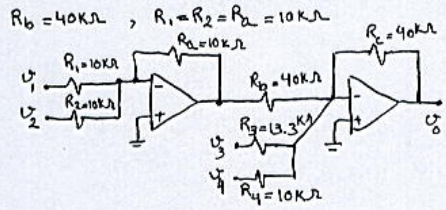
$\frac{R_a}{R_1}\frac{R_c}{R_b} = 1$, $\frac{R_a}{R_2}\frac{R_c}{R_b} = 1$, $\frac{R_c}{R_3} = 3$, $\frac{R_c}{R_4} = 4$

assume:

$R_4 = 10K\Omega \Rightarrow R_c = 40^{K\Omega} \Rightarrow R_3 = \frac{40}{3} = 13.3^{K\Omega}$

$\frac{R_a}{R_1} \times \frac{40}{R_b} = 1 \qquad \frac{R_a}{R_2} \times \frac{40}{R_b} = 1$

$R_b = 40K\Omega$, $R_1 = R_2 = R_a = 10K\Omega$



2.42

$v_1 = 3\sin(2\pi\times 60t) + 0.01\sin(2\pi\times 1000t)$

$v_2 = 3\sin(2\pi\times 60t) - 0.01\sin(2\pi\times 1000t)$

we want to have: $v_o = 10v_1 - 10v_2$

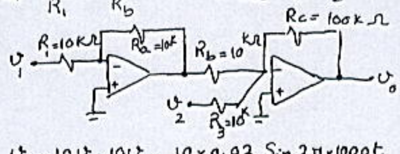
we use the circuit in Fig. 2.11.

According to Eq. 2.8:

$v_o = v_1 \frac{R_a}{R_1}\frac{R_c}{R_b} - v_3\frac{R_c}{R_3}$

$\frac{R_a}{R_1}\frac{R_c}{R_b} = 10$, $\frac{R_c}{R_3} = 10$, if $R_3 = 10^{K\Omega} \Rightarrow R_c = 100^{K\Omega}$

$\Rightarrow \frac{R_a}{R_1} \times \frac{100^{K\Omega}}{R_b} = 10 \Rightarrow R_a = R_1 = R_b = 10K\Omega$



$v_o = 10v_1 - 10v_2 = 10\times 0.02\sin 2\pi\times 1000t$

$v_o = 0.2\sin(2\pi\times 1000t) \qquad -0.2^V \leq v_o \leq 0.2^V$

2.43

This is a weighted summer circuit:

$v_o = -\left(\frac{R_F}{R_0}v_0 + \frac{R_F}{R_1}v_1 + \frac{R_F}{R_2}v_2 + \frac{R_F}{R_3}v_3\right)$

we may write: $v_0 = 5^V \times a_0$, $v_2 = 5^V \times a_2$
$v_1 = 5^V \times a_1$, $v_3 = 5^V \times a_3$

$v_o = -R_F\left(\frac{5a_0}{80K} + \frac{5}{40K}a_1 + \frac{5}{20K}a_2 + \frac{5}{10K}a_3\right)$

$v_o = -R_F\left(\frac{a_0}{16} + \frac{a_1}{8} + \frac{a_2}{4} + \frac{a_3}{2}\right)$

$v_o = -\frac{R_F}{16}\left(2^0 a_0 + 2^1 a_1 + 2^2 a_2 + 2^3 a_3\right)$

$-12^V \leq v_o \leq 0 \Rightarrow \frac{R_F}{16}\left(2^0\times 1 + 2\times 1 + 2^2\times 1 + 2^3\times 1\right) =$

$= \frac{15R_F}{16} = 12$ when $a_0 = a_1 = a_2 = a_3 = 1$ we have the peak value at v_o.

$\Rightarrow R_F = 12.8K\Omega$

2.44

a) $\frac{v_o}{v_i} = 1 = 1 + \frac{R_2}{R_1} \Rightarrow R_2 = 0, R_1 = 10K\Omega$

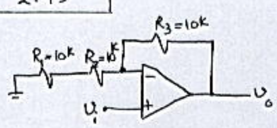
b) $\frac{v_o'}{v_i} = 2 = 1 + \frac{R_2'}{R_1} \Rightarrow R_1 = R_2 = 10K\Omega$

Cont.

c) $\frac{v_o}{v_i} = 101\ V/V = 1 + \frac{R_2}{R_1} \Rightarrow$ if $R_1 = 10^{k\Omega} \Rightarrow R_2 = 1^{M\Omega}$

d) $\frac{v_o'}{v_i} = 100\ V/V = 1 + \frac{R_2'}{R_1} \Rightarrow$ if $R_1 = 10^{k\Omega} \Rightarrow R_2 = 990\ k\Omega$

2.45

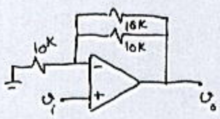


short-circuit R_2:

$\frac{v_o}{v_i} = 2$

Short circuit R_3:

$\frac{v_o}{v_i} = 1$



2.46

$v_+ = v_- = V = R \times i$, $i = 100\mu A$ when $V = 10^v$

$\Rightarrow R = \frac{10}{0.1mA} = 100k\Omega$

As indicated, i only depends on R and V and the meter resistance does not affect i.

2.47

Refer to the circuit in P 2.47:

a) using superposition, we first set $v_{P_1} = v_{P_2} = \ldots = 0$

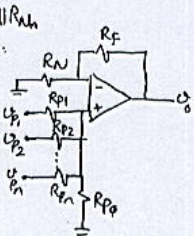
The output voltage that results in response to $v_{N1}, v_{N2}, \ldots v_{Nn}$ is:

$$v_{ON} = -\left[\frac{R_F}{R_{N1}} v_{N_1} + \frac{R_F}{R_{N2}} v_{N2} + \cdots + \frac{R_F}{R_{N_n}} v_{N_n}\right]$$

Then we set $v_{N_1} = v_{N_2} = \cdots = 0$, then:

$R_N = R_{N1} \,//\, R_{N2} \,||\, R_{N3} \,||\, \cdots \,||\, R_{Nn}$

The circuit simplifies to:



$$v_{OP} = \left(1 + \frac{R_F}{R_N}\right) \times$$

$$\left(v_{P_1} \frac{1/R_{P1}}{\frac{1}{R_{P1}} + \frac{1}{R_{P2}} + \cdots + \frac{1}{R_{Pn}}} + v_{P_2} \frac{1/R_{P2}}{\frac{1}{R_{P_1}} + \cdots + \frac{1}{R_{P_n}}} + \cdots v_{P_n} \frac{1/R_{Pn}}{1/R_{P_1} + \cdots + \frac{1}{R_{Pn}}}\right)$$

$$v_{OP} = \left(1 + \frac{R_F}{R_N}\right)\left(v_{P_1} \frac{R_P}{R_{P_1}} + v_{P_2} \frac{R_P}{R_{P_2}} + \cdots + \frac{R_P}{R_{P_n}} v_{P_n}\right)$$

where:

$$R_P = R_{P_1} \,||\, R_{P_2} \,||\, \cdots \,||\, R_{P_n}$$

when all inputs are present:

$$v_o = v_{ON} + v_{OP} = -\left(\frac{R_F}{R_{N_1}} v_{N_1} + \frac{R_F}{R_{N_2}} v_{N_2} + \cdots\right) + \left(1 + \frac{R_F}{R_N}\right)\left(\frac{R_P}{R_{P_1}} v_{N_1} + \frac{R_P}{R_{P_2}} v_{N_2} + \cdots\right)$$

b) $v_o = -2v_{N1} + v_{P_1} + 2v_{P_2}$

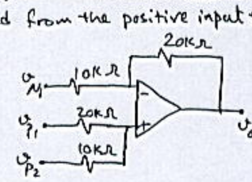
$\frac{R_F}{R_{N_1}} = 2 \quad R_{N_1} = 10K\Omega \Rightarrow R_F = 20K\Omega$

$\left(1 + \frac{R_F}{R_N}\right)\left(\frac{R_P}{R_{P_1}}\right) = 1 \Rightarrow 3\frac{R_P}{R_{P_1}} = 1 \Rightarrow R_{P_2} = \frac{R_{P1}}{2}$

$\left(1 + \frac{R_F}{R_N}\right)\left(\frac{R_P}{R_{P_2}}\right) = 2 \Rightarrow 3\frac{R_P}{R_{P_2}} = 2 \Rightarrow R_{P2} = \frac{R_{P1}}{2}$

where $R_P = \frac{R_{P_1} \times R_{P2}}{R_{P_1} + R_{P2}}$ (ignoring $R_{P\phi}$)

Note that if the results from the last 2 constraints differ, we would use an additional resistor connected from the positive input to ground. ($R_{P\phi}$)



2.48

$v_o = v_{I1} + 3v_{I2} - 2(v_{I3} + 3v_{I4})$

Refer to P2.47.

$\frac{R_F}{R_{N3}} = 2$ if $R_{N3} = 10K\Omega \Rightarrow R_F = 20K\Omega$

$\frac{R_F}{R_{N4}} = 6 \Rightarrow R_{N4} = \frac{20}{6} = 3.3^{K\Omega}$

$R_N = R_{N3} \,//\, R_{N4} = 10K \,//\, 3.3^K = 2.48^{K\Omega}$

$\left(1 + \frac{R_F}{R_N}\right) \frac{R_P}{R_{P_1}} = 1 \Rightarrow \left(1 + \frac{20}{2.48}\right)\frac{R_P}{R_{P_1}} = 1 \Rightarrow 9.06R_P = R_{P_1}$

$R_P = R_{P_1} \,//\, R_{P_2} \,//\, R_{P\phi} \Rightarrow R_P = \frac{1}{\frac{1}{R_{P1}} + \frac{1}{R_{P2}} + \frac{1}{R_{P\phi}}}$

$\left(1 + \frac{R_F}{R_N}\right)\frac{R_P}{R_{P_2}} = 3 \Rightarrow 9.06\frac{R_P}{R_{P2}} = 3 \Rightarrow R_{P_2} \simeq 3R_P$

$R_{P1} \,//\, R_{P2} = \frac{9 \times 3R_P}{9+3} = 2.25R_P$, $R_P = 2.25R_P \,//\, R_{P\phi}$

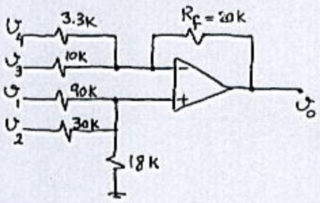
Cont.

$2.25 R_p + R_{p_0} = 2.25 R_{p_0} \Rightarrow R_{p_0} = 1.8 R_p$

if $R_p = 10 K\Omega \Rightarrow R_{p_0} = 18 K\Omega$

$R_{p_1} = 9 \times 10K = 90 K\Omega$

$R_{p_2} = 3 \times 10K = 30 K\Omega$



2.49

$$v_+ = v_I \frac{R_4}{R_3 + R_4} = v_-$$

$$\frac{v_-}{R_1} = \frac{v_o - v_-}{R_2} \Rightarrow v_o = v_-\left(1 + \frac{R_2}{R_1}\right)$$

from the two above equations :

$$\frac{v_o}{v_I} = \left(1 + \frac{R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) = \frac{1 + R_2/R_1}{1 + R_3/R_4}$$

2.50

Refer to Fig. 2.50. Setting $v_2 = 0$, we obtain the output component due to v_1 as:

$v_{o1} = -20v$

Setting $v_1 = 0$, we obtain the output component due to v_2 as:

$$v_{o2} = v_2\left(1 + \frac{20R}{R}\right)\left(\frac{20R}{20R + R}\right) = 20 v_2$$

The total output voltage is :

$v_o = v_{o1} + v_{o2} = 20(v_2 - v_1)$

For $v_1 = 10 \sin 2\pi \times 60t - 0.1 \sin(2\pi \times 1000t)$

$v_2 = 10 \sin 2\pi \times 60t + 0.1 \sin(2\pi \times 1000t)$

$v_o = 4 \sin(2\pi \times 1000t)$

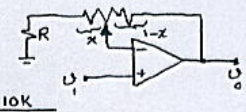
2.51

$$\frac{v_o}{v_i} = 1 + \frac{R_2}{R_1} = 1 + \frac{(1-x)}{x} = 1 + \frac{1}{x} - 1 = \frac{1}{x}$$

$0 < x \leq 1 \Rightarrow 1 \leq \frac{v_o}{v_I} \leq \infty$

if we add a resistor on the ground path :

$$\frac{v_o}{v_i} = 1 + \frac{(1-x) \times 10K}{x \times 10K + R}$$

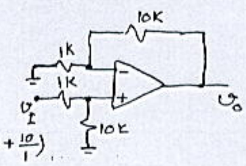


$Gain_{max} = 21$ when

$x = 0 \Rightarrow 21 = 1 + \frac{10K}{R}$

$\Rightarrow R = \frac{10K}{20} = 0.5 K\Omega$

2.52



$$v_o = v_I \frac{10}{1+10}\left(1 + \frac{10}{1}\right)$$

$v_o = 10 v_I$

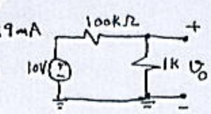
2.53

a) Source is connected directly.

$v_o = 10 \times \frac{1}{101} = 0.099V$

$i_L = \frac{v_o}{1K} = \frac{0.099}{1} = 0.099 mA$

current supplied by the source is 0.099 mA.

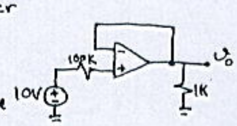


b) inserting a buffer

$v_o = 10V$

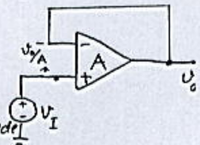
$i_L = \frac{10V}{1K} = 10mA$

current supplied by the source is 0.



The load current i_L comes from the power supply of the op-amp.

2.54



$$v_O = v_I - \frac{v_O}{A}$$

$$\frac{v_o}{v_i} = \frac{1}{1+\frac{1}{A}}$$

error of Gain magnitude

$$\frac{\left|\frac{v_O}{v_I} - 1\right|}{1} = -\frac{1}{A+1}$$

A (V/V)	1000	100	10
$\frac{v_o}{v_i}$ (V/V)	0.999	0.990	0.909
Gain Error	−0.1%	−1%	−9.1%

2.55

for an inverting amplifier:

$R_i = R_1$, $G = -\frac{R_2}{R_1}$

for a non-inverting amplifier:

$R_i = \infty$ $\quad G = 1 + \frac{R_2}{R_1}$

Case	Gain V/V	Rin	R_1	R_2
a	−10	10K	10K	100K
b	−1	100K	100K	100K
c	−2	50K	50K	100K
d	+1	∞	10K	10K
e	+2	∞	10K	10K
f	+11	∞	10K	100K
g	−0.5	10K	10K	5K

2.56

$A = 50\,V/V$ $\quad 1 + \frac{R_2}{R_1} = 10\,V/V$

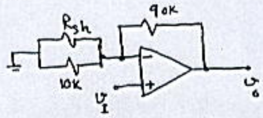
if $R_1 = 10\,k\Omega \Rightarrow R_2 = 90K$

According to Eq. 2.11: $G = \frac{v_O}{v_I} = \dfrac{1+\frac{R_2}{R_1}}{1+\frac{1+R_2/R_1}{A}}$

$$G = \frac{1+90/10}{1+\frac{1+90/10}{50}} = \frac{10}{1.2} = 8.33\,V/V$$

In order to compensate the gain drop, we can shunt a resistor with R_1.

Compensated:



$$R_{sh}: \quad 10 = \frac{1+\left(\frac{90}{10}+\frac{90}{R_{sh}}\right)}{1+\frac{1+\frac{90}{10}+\frac{90}{R_{sh}}}{50}} \Rightarrow$$

$$10 \times (510 R_{sh} + 90 R_{sh} + 900) = 50 \times (10 R_{sh} + 90 R_{sh} + 900)$$

$$100 R_{sh} = 3600 \Rightarrow \underline{R_{sh} = 36 K\Omega}$$

if $A = 100$ then :

$$G_{uncompensated} = \frac{1+\frac{90}{10}}{1+\frac{1+90/10}{100}} = \frac{10}{1.1} = 9.09\,V/V$$

$$G_{compensated} = \frac{1+\frac{90}{10}+\frac{90}{36}}{1+\frac{1+\frac{90}{10}+\frac{90}{36}}{100}} = \frac{12.5}{1.125} = 11.1\,V/V$$

2.57

$$G = \frac{G_0}{1+\frac{G_0}{A}}, \quad \frac{G_0 - G}{G_0} \times 100 = \frac{G_0/A \times 100}{1+\frac{G_0}{A}} \leq x$$

$$\text{or} \quad \frac{1+\frac{G_0}{A}}{G_0/A} \geq \frac{100}{x} \Rightarrow \frac{A}{G_0} \geq \underbrace{\left(\frac{100}{x}-1\right)}_{F}$$

$$\Rightarrow A \geq G_0 F \quad \text{where } F = \frac{100}{x} - 1 \simeq \frac{100}{x}$$

x	0.01	0.1	1	10
F	10^4	10^3	10^2	10

Thus for ,

$x = 0.01$:	G_0 (V/V)	1	10	10^2	10^3	10^4
	A (V/V)	10^4	10^5	10^6	$\underbrace{10^7 \quad 10^8}_{\text{too high to be practical}}$	
$x = 0.1$:	G_0 (V/V)	1	10	10^2	10^3	10^4
	A (V/V)	10^3	10^4	10^5	10^6	10^7
$x = 1$:	G_0 (V/V)	1	10	10^2	10^3	10^4
	A (V/V)	10^2	10^3	10^4	10^5	10^6
$x = 10$:	G_0 (V/V)	1	10	10^2	10^3	10^4
	A (V/V)	10	10^2	10^3	10^4	10^5

2.58

for non-inverting amplifier, Eq. 2.11:

$G = \frac{G_0}{1+\frac{G_0}{A}}$, $\varepsilon = \frac{G_0 - G}{G_0} \times 100$

for inverting amplifier, Eq. 2.5:

$G = \frac{G_0}{1+\frac{1-G_0}{A}}$, $\varepsilon = \frac{G_0 - G}{G_0} \times 100$

Case	G_0 (V/V)	A (V/V)	G (V/V)	ε %
a	−1	10	−0.83	16
b	1	10	0.91	9
c	−1	100	−0.98	2
d	10	10	5	50
e	−10	100	−9	10
f	−10	1000	−9.89	1.1
g	+1	2	0.67	33

2.59

Refer to fig. P2.59, when potentiometer is set to the bottom:

$v_o = v_+ = -15 + \frac{30 \times 20}{20+100+20} = -10.714V$

when set to the top: $v_o = -15 + \frac{30 \times 120}{20+100+20} = 10.714V$

$\Rightarrow -10.714 \leq v_o \leq +10.714$

pot has 20 turn, each turn: $\Delta v_o = \frac{2 \times 10.714}{20} = 1.07V$

2.60

Refer to Fig. 2.16. Notice that Similar to eq. 2.15 we have: $\frac{R_4}{R_3} = \frac{R_2}{R_1} = \frac{100}{10}$.

therefore according to 2.16:

$v_o = \frac{R_2}{R_1} v_{Id} \Rightarrow A = \frac{R_2}{R_1} = 10$ V/V

According to 2.20: $R_{id} = 2R_1 = 20k\Omega$

If $\frac{R_2}{R_1}$, $\frac{R_4}{R_3}$ were different by 1%:

$\frac{R_2}{R_1} = 0.99 \frac{R_4}{R_3}$

Refer to eq. 2.19: $A_{cm} = \frac{R_4}{R_4+R_3}\left(1 - \frac{R_2}{R_1} \cdot \frac{R_3}{R_4}\right)$

$A_{cm} = \frac{100}{100+10}(1-0.99) = 0.009$

$CMRR = 20 \log \frac{|A_d|}{|A_{cm}|}$, so let's calculate A_d

$A_d = \frac{v_o}{v_{id}}$ if we apply superposition:

$v_{o1} = -\frac{R_2}{R_1} v_{I1}$ $\quad v_{o2} = v_{I2} \frac{R_4}{R_3+R_4}\left(1+\frac{R_2}{R_1}\right)$

$v_o = v_{o2} + v_{o1} = v_{I2} \frac{R_4/R_3}{1+\frac{R_4}{R_3}}\left(1+\frac{R_2}{R_1}\right) - \frac{R_2}{R_1} v_{I1}$

Replace $\frac{R_2}{R_1} = 0.99 \frac{R_4}{R_3} \Rightarrow \frac{R_2}{R_1} = 0.99 \times \frac{100}{10} = 9.9$

$v_o = v_{I2} \frac{10}{1+10}(1+9.9) - v_{I1} 9.9 = 9.9(v_{I2} - v_{I1})$

$\frac{v_o}{v_{Id}} = 9.9 = A_d \Rightarrow CMRR = 20 \log \frac{9.9}{0.009} = 60.8$

CMRR = 60.8

2.61

If we assume $R_3 = R_1$, $R_4 = R_2$, then eq. 2.20: $R_{id} = 2R_1 \Rightarrow R_1 = \frac{20}{2} = 10k\Omega$

(Refer to Fig. 2.16)

a) $A_d = \frac{R_2}{R_1} = 1$ V/V $\Rightarrow R_2 = 10k\Omega$

$R_1 = R_2 = R_3 = R_4 = 10k\Omega$

b) $A_d = \frac{R_2}{R_1} = 2$ V/V $\Rightarrow R_2 = 20k\Omega = R_4$

$R_1 = R_3 = 10k\Omega$

c) $A_d = \frac{R_2}{R_1} = 100$ V/V $\Rightarrow R_2 = 1M\Omega = R_4$

$R_1 = R_3 = 10k\Omega$

d) $A_d = \frac{R_2}{R_1} = 0.5$ V/V $\Rightarrow R_2 = 5k\Omega = R_4$

$R_1 = R_3 = 10k\Omega$

2.62

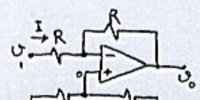
Refer to Fig P2.62:

Cont.

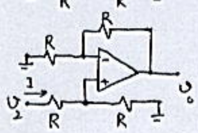
Considering that $v_- = v_+$:

$$v_1 + \frac{v_o - v_1}{2} = \frac{v_2}{2} \Rightarrow v_o = v_2 - v_1$$

v_1 only: $R_I = \frac{v_1}{I} = R$



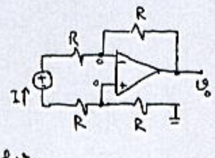
v_2 only: $R_I = \frac{v_2}{I} = 2R$



v_s between 2 terminals:

$$R_I = \frac{v}{I} = 2R$$

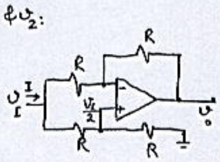
$$v_+ = v_- = 0$$



v_s connected to both v_1 & v_2:

$$R_I = \frac{v}{I} = R$$

$$v_+ = v_- = \frac{v_i}{2}$$

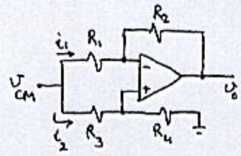


2.63

$$v_+ = v_{CM} \frac{R_4}{R_3 + R_4}$$

$$v_+ = v_-$$

$$i_2 = \frac{v_{CM}}{R_3 + R_4}$$



$$i_1 = \frac{v_{CM}}{R_1} - \frac{v_{CM} R_4}{R_3 + R_4} \times \frac{1}{R_1} = \frac{v_{CM}}{R_1} \frac{R_3}{R_3 + R_4}$$

$$i = i_1 + i_2 = \frac{v_{CM}}{R_1} \frac{R_3}{R_3 + R_4} + \frac{v_{CM}}{R_3 + R_4}$$

$$\frac{1}{R_I} = \frac{i}{v_{CM}} = \frac{1}{R_1} \frac{1}{\frac{R_4}{R_3} + 1} + \frac{1}{R_3 + R_4}$$

if we replace $\frac{R_4}{R_3}$ with $\frac{R_2}{R_1}$: $\left(\frac{R_4}{R_3} = \frac{R_2}{R_1}\right)$

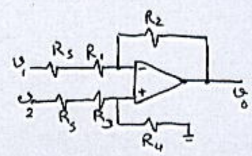
$$\frac{1}{R_I} = \frac{1}{R_1 + R_2} + \frac{1}{R_3 + R_4} \Rightarrow R_I = (R_1 + R_2) \| (R_3 + R_4)$$

2.64

In order to have an ideal differential amp:

$$\frac{R_s + R_1}{R_2} = \frac{R_s + R_3}{R_4}$$

$$\frac{R_s/R_1 + 1}{R_2/R_1} = \frac{R_s/R_3 + 1}{R_4/R_3}$$



Since $\frac{R_2}{R_1} = \frac{R_4}{R_3}$:

$$\frac{R_s}{R_1} + 1 = \frac{R_s}{R_3} + 1 \Rightarrow R_1 = R_3 \Rightarrow R_2 = R_4$$

2.65

Refer to eq. 2.19 and Fig. P2.62:

$$A_{CM} = \frac{v_o}{v_{ICM}} = \frac{R_4}{R_3 + R_4}\left(1 - \frac{R_2}{R_1}\frac{R_3}{R_4}\right)$$

The worst case is when A_{cm} has its maximum value.

$$A_{cm} = \frac{1}{\frac{R_3}{R_4} + 1}\left(1 - \frac{R_2}{R_1}\frac{R_3}{R_4}\right)$$

Max $A_{cm} \Rightarrow \frac{R_3}{R_4}$ has to be at its minimum value and also $\frac{R_2}{R_1}$ has to be minimum.

$$\frac{100-x}{100+x} \leq \frac{R_3}{R_4} \leq \frac{100+x}{100-x} \qquad \frac{100-x}{100+x} \leq \frac{R_2}{R_1} \leq \frac{100+x}{100-x}$$

so if $\frac{R_3}{R_4} = \frac{100-x}{100+x}$ & $\frac{R_2}{R_1} = \frac{100-x}{100+x}$

$$A_{cm\,Max} = \frac{1}{\frac{100-x}{100+x} + 1}\left(1 - \frac{100-x}{100+x}\frac{100-x}{100+x}\right)$$

$$A_{cm\,Max} = \frac{1}{200}\frac{(100+x)^2 - (100-x)^2}{100+x} = \frac{2x}{100+x} \simeq \frac{x}{50}$$

x	0.1	1	5
$A_{cm\,Max}$	0.002	0.02	0.1

$CMRR = 20\log\left|\frac{A_d}{A_{cm}}\right|$. Now we have to calculate A_d based on values we chose for $R_1 - R_4$ that gave us $A_{cm\,Max}$:

$$R_2 = R_3 = 100 - x \qquad R_1 = R_4 = 100 + x$$

$v_o = v_{o1} + v_{o2}$ by applying superposition

$$v_o = -\frac{R_2}{R_1} v_1 + v_2 \frac{R_4}{R_3 + R_4}\left(1 + \frac{R_2}{R_1}\right)$$

$$v_o = -\frac{100-x}{100+x} v_1 + v_2 \frac{100+x}{200}\left(1 + \frac{100-x}{100+x}\right)$$

$$v_o = -\frac{100-x}{100+x} v_1 + v_2$$

if we consider $\frac{100-x}{100+x} \simeq 1 \Rightarrow \frac{v_o}{v_{id}} \simeq 1$

Cont.

$$CMRR = 20\log\frac{A_d}{A_{cm}} = 20\log\frac{1}{x/50} = 20\log\frac{50}{x}$$

x	0.1	1	5
CMRR	54db	34db	20db

2.66

Refer to Fig. 2.16 and eq. 2.19:

$$A_{cm} = \frac{R_4}{R_3+R_4}\left(1-\frac{R_2}{R_1}\frac{R_3}{R_4}\right)$$

In order to calculate A_d, we use superposition principle:

$$v_o = v_{o1} + v_{o2} = -\frac{R_2}{R_1}v_1 + v_2\frac{R_4}{R_3+R_4}\left(1+\frac{R_2}{R_1}\right)$$

then replace $v_1 = v_{cm} - \frac{v_d}{2}$

$v_2 = v_{cm} + \frac{v_d}{2}$

$$v_o = \frac{-R_2}{R_1}v_{cm} + \frac{R_2}{R_1}v_d/2 + v_{cm}\frac{1+R_2/R_1}{1+\frac{R_3}{R_4}} + \frac{v_d}{2}\frac{1+R_2/R_1}{1+\frac{R_3}{R_4}}$$

$$v_o = \underbrace{\frac{R_2}{2R_1}\left[1+\frac{R_1/R_2+1}{R_3/R_4+1}\right]}_{A_d}v_d + \frac{R_2}{R_1}\left[-1+\frac{R_1/R_2+1}{R_3/R_4+1}\right]v_{cm}$$

$$CMRR = 20\log\left|\frac{A_d}{A_{cm}}\right| = 20\log\frac{\frac{R_2}{2R_1}\left[1+\frac{R_1/R_2+1}{R_3/R_4+1}\right]}{\frac{1}{\frac{R_3}{R_4}+1}\left(1-\frac{R_2}{R_1}\frac{R_3}{R_4}\right)}$$

$$CMRR = 20\log\left|\frac{\frac{1}{2}\frac{R_2}{R_1}\left[2+\frac{R_1}{R_2}+R_3/R_4\right]}{1-\frac{R_2}{R_1}\cdot\frac{R_3}{R_4}}\right|$$

$$CMRR = 20\log\left|\frac{1+\frac{1}{2}\frac{R_1}{R_2}+\frac{1}{2}\frac{R_3}{R_4}}{\frac{R_1}{R_2}-\frac{R_3}{R_4}}\right|$$

for worst case, or minimum CMRR we have to maximize the denominator, which means:

$R_1 = R_{1n}(1+\varepsilon)$ $\qquad R_3 = R_{3n}(1-\varepsilon)$

$R_2 = R_{2n}(1-\varepsilon)$ $\qquad R_4 = R_{4n}(1+\varepsilon)$

also: $\frac{R_{2n}}{R_{1n}} = \frac{R_{4n}}{R_{3n}} = K$

$$CMRR = 20\log\left|K\frac{1+\frac{1}{2K}\frac{1+\varepsilon}{1-\varepsilon}+\frac{1}{2K}\frac{1-\varepsilon}{1+\varepsilon}}{\frac{1+\varepsilon}{1-\varepsilon}-\frac{1-\varepsilon}{1+\varepsilon}}\right|$$

$$CMRR = 20\log\left|\frac{K(1-\varepsilon^2)+(1+\varepsilon^2)}{4\varepsilon}\right| \simeq 20\log\left|\frac{K+1}{4\varepsilon}\right|$$

for $\varepsilon^2 \ll 1$.

if $K = A_{d\,\text{ideal}} = 100$, $\varepsilon = 0.01$

$$CMRR = 20\log\frac{101}{0.04} = 68\text{db}$$

2.67

$A_d = 100$

we assume $\frac{R_2}{R_1} = \frac{R_4}{R_3}$ then $A_d = \frac{R_2}{R_1} = K$

$K = 100$

$R_{id} = 2R_1 = 20\text{k}\Omega \Rightarrow R_1 = 10\text{k}\Omega$

$CMRR = 80\text{db} = 20\log\frac{A_d}{A_{cm}} \Rightarrow \frac{A_d}{A_{cm}} = 10^4$

$\Rightarrow A_{cm} = 0.01$

$A_d = 100 = \frac{R_2}{R_1} \Rightarrow R_2 = 1\text{M}\Omega$

Refer to P2.66: $\quad CMRR = 20\log\frac{K+1}{4\varepsilon}$

$CMRR = 10^4$

$\Rightarrow \varepsilon = 10^{-2}\times 0.25$

we assumed earlier $\frac{R_2}{R_1} = \frac{R_4}{R_3}$ then

$\frac{R_4}{R_3} = 100 \Rightarrow$ if $R_3 = 10\text{k} \pm \varepsilon$

$\Rightarrow R_4 = 1\text{M}\Omega \pm \varepsilon \qquad \varepsilon = 0.25\%$

$R_2 = 1\text{M}\Omega \pm \varepsilon$

$R_1 = 10\text{k} \pm \varepsilon$

2.68

Refer to Fig. P2.68 and Eq. 2.19:

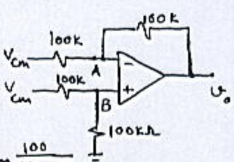
$$A_{cm} = \frac{R_4}{R_3+R_4}\left(1-\frac{R_2}{R_1}\frac{R_3}{R_4}\right) = \frac{100}{100+100}\left(1-\frac{100\cdot100}{100\cdot100}\right)$$

$\underline{A_{cm} = 0}$

Refer to 2.17: $\frac{R_2}{R_1} = \frac{R_4}{R_3}$

$\Rightarrow A_d = \frac{R_2}{R_1} = 1$

b) Since $A_{cm} = 0$, then if we apply v_{cm} to v_{I1} and v_{I2}, $v_o = 0$.



Therefore, $V_A = v_{cm}\frac{100}{100+100}$

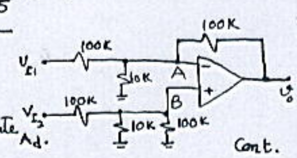
$V_A = \frac{v_{cm}}{2}$

Similarly, $v_B = \frac{v_{cm}}{2}$

We know $\quad V_A = V_B$ and $-2.5 \leq v_A \leq 2.5$

$\Rightarrow -5 \leq v_{cm} \leq 5$

c) we apply the superposition principle to calculate A_d.



Cont.

v_{o1} is the output voltage when $v_{I2}=0$
v_{o2} is the output voltage when $v_{I1}=0$

$v_o = v_{o1} + v_{o2}$

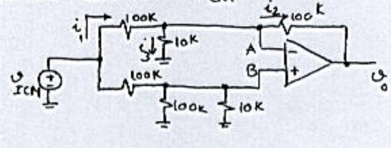
$v_{o1} = \frac{-R_2}{R_1} v_{I1} = -v_{I1}$

$v_{o2} = v_{I2} \frac{100k\Omega \| 110k}{100k \| 110k + 100} \left(1 + \frac{100k}{100k \| 110k}\right)$

$v_{o2} = v_{I2} \times 1$

$\Rightarrow v_o = v_{o1} + v_{o2} = -v_{I1} + v_{I2} \Rightarrow \underline{A_d = 1}$

Now we calculate A_{cm}:



$v_B = v_{ICM} \frac{100k \| 10k}{100k \| 10k + 100k}$, $v_A = v_B$

$i_1 = \frac{v_{ICM} - v_A}{100k}$

$v_o = v_A - 100k \times i_2$ and $i_2 = i_1 - i_3 = i_1 - \frac{v_A}{10k}$

$v_o = v_A - 100k \times i_1 + 10 \times v_A$

$v_o = v_A - v_{ICM} + v_A + 10 \times v_A$

$v_A = v_B \Rightarrow v_o = v_{ICM} \left(-1 + 12 \frac{100k \| 10^k}{100^k \| 10^k + 100^k}\right)$

$\frac{v_o}{v_{ICM}} = \underline{A_{cm} = 0}$

Now we calculate v_{ICM} range:

$-2.5 \leq v_B \leq 2.5 \Rightarrow -2.5 \leq v_{ICM} \times \frac{100k \| 10k}{100k \| 10k + 100k} < 2.5$

$-30^V \leq v_{ICM} \leq 30^V$

2.69

Refer to Fig. P2.69; we use superposition:

$v_o = v_{o1} + v_{o2}$

calculate v_{o1}: $v_+ = \frac{\beta v_{o1}}{2} = v_-$

$\frac{v_1 - \frac{\beta v_{o1}}{2}}{R} = \frac{\frac{\beta v_{o1}}{2} - v_o}{R} \Rightarrow v_{o1} = \frac{v_1}{\beta - 1}$

calculate v_{o2}:

$v_- = \frac{v_{o2}}{2} = v_+ \Rightarrow v_2 - \frac{v_{o2}}{2} = \frac{v_{o2}}{2} - \beta v_{o2}$
$\Rightarrow v_{o2} = \frac{v_2}{1-\beta}$

$v_o = v_{o1} + v_{o2} = \frac{v_1}{\beta - 1} + \frac{v_2}{1-\beta} = \frac{1}{1-\beta}(v_2 - v_1)$

$A_d = \frac{v_o}{v_2 - v_1} = \frac{1}{1-\beta}$

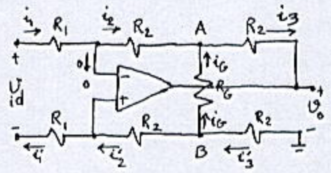
$A_d = 10 \, V/V \Rightarrow \beta = 0.9 = \frac{R5}{R_5 + R6}$

$R_{id} = 2R = 2M\Omega \Rightarrow \underline{R = 1M\Omega}$

$R_5 + R_6 \leq \frac{R}{100} \Rightarrow R_5 + R_6 \leq 10^{K\Omega}$

$\underline{R_5 = 6.8K\Omega}$ $\underline{R_6 = 680\Omega}$ $\Rightarrow \beta = \frac{6.8}{6.8 + 0.68} \approx 0.9$

2.70



$v_+ = v_-$ So we can consider v_+, v_- a virtual short: $i_1 = v_{id}/2R_1 \Rightarrow i_2 = \frac{v_{id}}{2R_1}$

$i_1' = i_2' = \frac{v_{id}}{2R_1}$

then: $i_2 R_2 + v_{AB} + i_2' R_2 = 0 \Rightarrow v_{AB} = \frac{-v_{id}}{R_1} R_2$

$i_G = \frac{v_{id}}{R_G} \times \frac{R_2}{R_1}$

$i_3 = i_2 + i_G = \frac{v_{id}}{2R_1} + \frac{v_{id}}{R_G} \frac{R_2}{R_1}$

$i_3' = i_G + i_2' = i_3$

$\Rightarrow v_o = -[i_3' R_2 + v_{BA} + i_3 R_2]$

$v_o = -[2 i_3 R_2 + v_{BA}]$

$v_o = -\left[\frac{2 v_{id}}{2R_1} R_2 + 2 v_{id} \frac{R_2}{R_1} \frac{R_2}{R_G} + \frac{v_{id}}{R_1} R_2\right]$

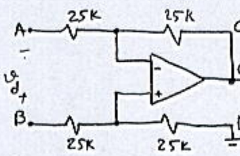
$\frac{v_o}{v_{id}} = A_d = -2 \frac{R_2}{R_1}\left[1 + \frac{R_2}{R_G}\right]$

2.71

a)
Refer to Eq. 2.17: $A_d = \frac{R_2}{R_1} = 1$. Connect C and D together.

Cont.

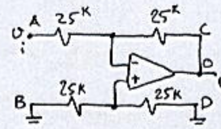
a)



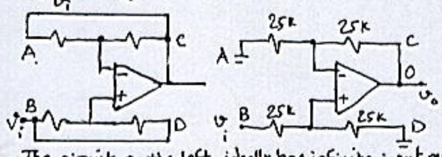
$A_d = 1$

b) $\frac{v_o}{v_i} = -1\ V/V$

i)

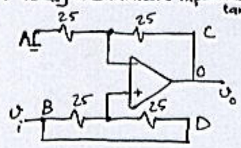


ii) $\frac{v_o}{v_i} = +1\ V/V$

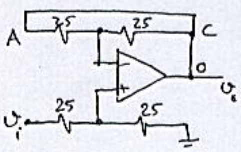


The circuit on the left ideally has infinite input resistance.

iii) $\frac{v_o}{v_i} = +2\ V/V$



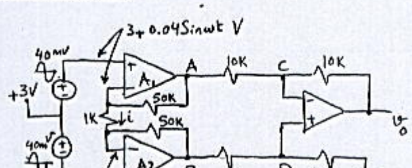
iv) $\frac{v_o}{v_i} = +\frac{1}{2}\ V/V$



$v_+ = \frac{v_i}{2} = v_o$

$\Rightarrow \frac{v_o}{v_i} = \frac{1}{2}$

2.72



$i = \frac{3+0.04\sin\omega t-(3-0.04\sin\omega t)}{1K} = 0.08\sin\omega t$, mA

$v_A = 3+0.04\sin\omega t + 50K \times i = 3 + 4.04\sin\omega t$, V

$v_B = 3-0.04\sin\omega t - 50K \times i = 3 - 4.04\sin\omega t$, V

$v_C = v_D = \frac{1}{2} v_B = 1.5 - 2.02\sin\omega t$, V

$v_o = v_B - v_A = -8.08\sin\omega t$, V

2.73

Refer to Fig. 2.20.a.

The gain of the first stage is: $(1+\frac{R_2}{R}) = 101$

If the opamps of the first stage saturate at ±14 V: $-14 \leq v_1 \leq +14V \Rightarrow -14 \leq 101\, v_{icm} \leq +14$

$\Rightarrow -0.14 \leq v_{ICM} \leq 0.14$

As explained in the text, the disadvantage of circuit in Fig. 2.20.a is that v_{ICM} is amplified by a gain equal to v_{id}, $(1+\frac{R_2}{R_1})$ in the first stage and therefore a very small v_{icm} range is acceptable to avoid saturation.

b) In Fig. 2.20b, when v_{ICM} is applied, v_- for both A_1 & A_2 is the same and therefore no current flows through $2R_1$. This means voltage at the output of A_1 and A_2 is the same as V_{ICM}.

$-14 \leq v_o \leq 14 \Rightarrow -14 \leq v_{ICM} \leq 14$

This circuit allows for bigger range of V_{icm}

2.74

$v_{i1} = v_{cm} - v_d/2$

$v_{i2} = v_{cm} + v_d/2$

Refer to fig. 2.20.a.

output of the first stage: $(1+\frac{R_2}{R_1})(v_{cm} - v_d/2)$

$v_{o1} = (1+\frac{R_2}{R_1})(v_{cm} - v_d/2)$

$v_{o2} = (1+\frac{R_2}{R_1})(v_{cm} + v_d/2)$

$v_{o2} - v_{o1} = (1+\frac{R_2}{R_1})v_d \Rightarrow A_{d(1)} = 1+\frac{R_2}{R_1}$

$\frac{v_{o2}+v_{o1}}{2} = (1+\frac{R_2}{R_1})v_{cm} \Rightarrow A_{cm(1)} = 1+\frac{R_2}{R_1}$

$CMRR = 20\log\left|\frac{A_d}{A_{cm}}\right| = 0$ (First stage)

Now Consider Fig. 2.20.b

$v_{o1} = v_{i1} + R_2 \times \frac{(v_{i1}-v_{i2})}{2R_1}$

$v_{o1} = v_{cm} - v_d/2 + \frac{R_2}{2R_1}(-v_d)$

Cont.

$v_{o1} = v_{cm} - \frac{v_d}{2}\left(1+\frac{R_2}{R_1}\right)$

$v_{o2} = v_{i2} - R_2 \times \frac{v_{i1} - v_{i2}}{2R_1} = v_{cm} + \frac{v_d}{2} + R_2 \cdot \frac{v_d}{2R_1}$

$v_{o2} = v_{cm} + \frac{v_d}{2}\left(1+\frac{R_2}{R_1}\right)$

$v_{o2} - v_{o1} = v_d\left(1+\frac{R_2}{R_1}\right) \quad \Rightarrow A_{d(1)} = 1+\frac{R_2}{R_1}$

$\frac{v_{o2}+v_{o1}}{2} = v_{cm}. \quad \Rightarrow A_{cm(1)} = 1$

$CMRR = 20\log\left|\frac{A_d}{A_{cm}}\right| = 20\log\left(1+\frac{R_2}{R_1}\right)$

In 2.20.b, the common mode voltage is not amplified and it is only propagated to the outputs of the first stage.

2.75

Refer to eq. 2.22:

$A_d = \frac{R_4}{R_3}\left(1+\frac{R_2}{R_1}\right) = \frac{100K}{100K}\left(1+\frac{100K}{5K}\right) = 21\ V/V$

$A_{cm} = 0$

$CMRR = 20\log\left|\frac{A_d}{A_{cm}}\right| = \infty$

If all resistors are $\pm 1\%$:

$A_d \cong 21$

In order to calculate A_{cm}, apply V_{cm} to both inputs and note that V_{cm} will appear at both output terminals of the first stage. Now we can evaluate v_o by analyzing the second stage as was done in problem 2.65. In P2.65 we showed that if each 100K resistor has $\pm x\%$ tolerance, A_{cm} of the differential amplifier is: $A_{cm} = \frac{v_o}{v_{cm}} = \frac{x}{50}$. Therefore the overall A_{cm} is also $\frac{x}{50}$.

$x = 1 \Rightarrow A_{cm} = \frac{1}{50} = 0.02$

$CMRR = 20\log\frac{21}{0.02} = \underline{60db}$

If $2R_1 = 1K\Omega$: $A_d = \frac{R_4}{R_3}\left(1+\frac{R_2}{R_1}\right) = 201\ V/V$

$A_{cm} = 0.02$ unchanged

$CMRR = 20\log\frac{201}{0.02} = \underline{80db}$

Conclusion: Large CMRR can be achieved by having relatively large A_d in the first stage.

2.76

$A_{d(2)}$ of the second stage is $\frac{R_4}{R_3} = 0.5$

$R_4 = 100K\Omega$, $R_3 = 200K\Omega$

we use a series configuration of R_{1f} (Fixed) and R_1 (pot): $R_1 = 100K$ pot

Minimum gain $= 0.5\left(1+\frac{R_2}{R_1}\right) = 0.5\left(1+\frac{R_2}{\frac{100^K+R_1}{2}}\right)$

$1 \leq A_d \leq 100 \Rightarrow \quad 1 = 0.5\left(1+\frac{2R_2}{R_{1f}+100K}\right)$

$\Rightarrow R_{1f} + 100 = 2R_2 \quad (1)$

Maximum gain $= 100 = 0.5\left(1+\frac{R_2}{R_{1f}/2}\right) \Rightarrow$

$2R_2 = 199R_{1f} \quad (2)$

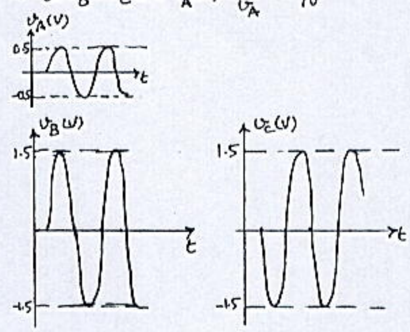
$(1),(2) \Rightarrow R_{1f} = 0.505K\Omega \cong 0.5K\Omega$

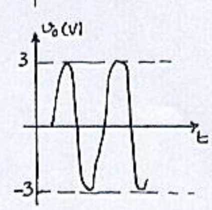
$R_2 = 50.25\ K\Omega \cong 50K\Omega$

2.77

a) $\frac{v_B}{v_A} = 1 + \frac{20}{10} = 3\ V/V$, $\frac{v_C}{v_A} = -\frac{30}{10} = -3\ V/V$

b) $v_O = v_B - v_C = 6\ V_A \Rightarrow \frac{v_O}{v_A} = 6\ V/V$





c) v_B and v_C can be $\pm 14^V$ or 28V p-p.

$-28 \leq v_O \leq 28$ or $\underline{56^V\ p\text{-}p.}$

$v_{O_{rms}} = \underline{19.8\ V} = \frac{28}{\sqrt{2}}$

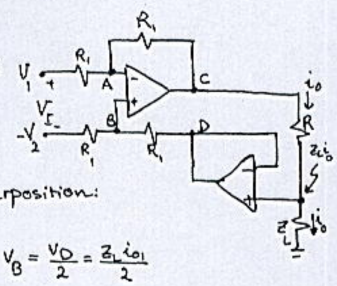
2.78

Refer to Fig. P.2.78.a.

Since the inputs of the op-amp donot draw any current, v_I appears across R:

$i_o = \frac{v_I}{R}$

Fig. P2.78.B



$V_D = Z_L i_o$

we use superposition:

$V_I = V_1 - V_2$

V_1 only: $\quad V_B = \frac{V_D}{2} = \frac{Z_L i_{o1}}{2}$

$$\frac{V_1 - \frac{Z_L i_{o1}}{2}}{R_1} = \frac{\frac{Z_L i_{o1}}{2} - i_{o1}(Z_L + R)}{R_1}$$

$\Rightarrow V_1 = i_{o1} R \Rightarrow i_{o1} = \frac{V_1}{R}$

Now if only $(-V_2)$ is applied:

$V_B = \frac{-V_2 + Z_L i_{o2}}{2}$, $V_A = \frac{i_{o2} \times (R + Z_L)}{2}$

$v_A = v_B \Rightarrow -V_2 + Z_L i_{o2} = i_{o2} R + i_{o2} Z_L$

$-V_2 = i_{o2} R \Rightarrow i_{o2} = \frac{-V_2}{R}$

The total current due to both sources is:

$i_o = i_{o1} + i_{o2} = \frac{v_1}{R} - \frac{v_2}{R} = \frac{v_I}{R}$

The circuit in Figure P2.78(a) has ideally infinite input resistance, and it requires that both terminals of Z_L be availabe, while the other circuit has finite input resistance with one side of Z_L grounded.

2.79

A_o	f_b (Hz)	f_t (Hz)
10^5	10^2	10^7
10^6	1	10^6
10^5	10^3	10^8
10^7	10^{-1}	10^6
2×10^5	10	2×10^6

eq. 2.28:

$\omega_t = A_o \omega_b$

$\Rightarrow f_t = A_o f_b$

2.80

Eq. 2.25 : $A = \frac{A_o}{1 + j\,\omega/\omega_b} \Rightarrow |A| = \frac{|A_o|}{\sqrt{1 + (\frac{f}{f_b})^2}}$

$A_o = 86$db , $A = 40$db @ $f = 100$KHZ

$20 \log \sqrt{1 + (\frac{f}{f_b})^2} = 20 \log \frac{|A_o|}{|A|} = 20 \log A_o - 20 \log A$

$= 86 - 40 = 46$db

$1 + (\frac{100\text{KHZ}}{f_b})^2 = (199.5)^2 \Rightarrow f_b = 0.501$KHZ

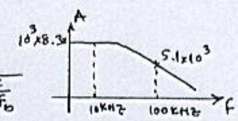
$f_b = 501$ HZ

$f_t = A_o f_b = \underbrace{1.995 \times 10^4}_{86db} \times 501 = 9.998$ MHZ $\simeq 10$MHZ

2.81

$A_o = 8.3 \times 10^3$ V/V

Eq. 2.25: $A = \frac{A_o}{1 + j\frac{f}{f_b}}$



$f_t = A_o f_b$

$5.1 \times 10^3 = \frac{8.3 \times 10^3}{\sqrt{1 + (\frac{100KH}{f_b})^2}} \Rightarrow 1 + (\frac{100\text{KHZ}}{f_b})^2 = 2.65$

$f_b = 60.7$ KHZ

$f_t = A_o f_b = 8.3 \times 10^3 \times 60.7^K = 503$ MHZ

2.82

we have:

$A_{o(db)} = 20db + A_{(db)}$ $\quad 20db = 20 \log 10 \Rightarrow A_o = 10 A$

a) $A_o = 10 \times 3 \times 10^5 = 3 \times 10^6$ HZ V/V

$A = \frac{A_o}{1 + jf/f_b} \Rightarrow |1 + jf/f_b| = \frac{A_o}{A} = 10 \Rightarrow \frac{6 \times 10^2}{f_b} = \sqrt{99}$

$\Rightarrow f_b = 60.3$ HZ

$f_t = A_o f_b = 3 \times 10^6 \times 60.3 = 180.9$ MHZ

b) $A = 50 \times 10^5$ V/V $\Rightarrow A_o = 10 \times 50 \times 10^5 = 50 \times 10^6$ V/V

$|1 + \frac{jf}{f_b}| = \frac{A_o}{A} = 10 \Rightarrow \frac{10^{HZ}}{f_b} = \sqrt{99} \Rightarrow f_b = 1$ HZ

$f_t = A_o f_b = 50$ MHZ

c) $A = 1500$ V/V $\Rightarrow A_o = 15000$ V/V

Cont.

$\left|1+\frac{jf}{f_b}\right| = 10 \Rightarrow \frac{0.1\times10^6}{f_b} = \sqrt{99} \Rightarrow f_b = 10KHz$

$f_t = 15000\times10^k = 150MHz$

d) $A_0 = 10\times100 = 1000\ V/V$

$\left|1+\frac{jf}{f_b}\right| = 10 \Rightarrow \frac{0.1\times10^8}{f_b} = \sqrt{99} \Rightarrow f_b = 10\ MHz$

$f_t = 1000\times10^{MHz} = 10GHz$

e) $A_0 = 25\ V/mV \times 10 = 25\times10^4\ V/V$

$\left|1+\frac{jf}{f_b}\right| = 10 \Rightarrow \frac{25KHz}{f_b} = \sqrt{99} \Rightarrow f_b = 2.51^{KHz}$

$f_t = A_0 f_b = 25\times10^4 \times 2.51\times10^3 = 627.5MHz$

2.83

$G_{nom} = -\frac{R_2}{R_1} = -20 \qquad A_0 = 10^4\ V/V \qquad f_b = 10^6 Hz$

Eq. 2.35 : $\omega_{3db} = \frac{\omega_t}{1+R_2/R_1} = \frac{2\pi\times10^6}{1+20} = 2\pi\times47.6\ KHz$

$f_{3db} = 47.6KHz$

Eq. 2.34 : $\frac{V_o}{V_i} \simeq \frac{-R_2/R_1}{1+\frac{s}{\omega_t/(1+\frac{R_2}{R_1})}} = \frac{-20}{1+\frac{21s}{2\pi\times10^6}}$

$f = 0.1 f_{3db} \Rightarrow \left|\frac{V_o}{V_i}\right| = \frac{-20}{\sqrt{1+(0.1)^2}} = +19.9\ V/V$

$f = 10 f_{3db} \Rightarrow \left|\frac{V_o}{V_i}\right| = \frac{-20}{\sqrt{1+100}} = 1.99\ V/V$

2.84

$1+\frac{R_2}{R_1} = 100\ V/V \quad , f_t = 20MHz$

$f_{3db} = \frac{f_t}{1+\frac{R_2}{R_1}} = 200\ KHz$

$G_{(j\omega)} = \frac{100}{1+j\,f/f_{3db}} \Rightarrow \varphi = -\tan^{-1}\frac{f}{f_{3db}} =$

$\varphi = -6^\circ \Rightarrow f = f_{3db}\times\tan 6^\circ = 21KHz$

$\varphi = -84^\circ \Rightarrow f = f_{3db}\times\tan 84^\circ = 1.9MHz$

2.85

a) $-\frac{R_2}{R_1} = -100\ V/V \quad , f_{3db} = 100KHz$

Eq. 2.35 : $\omega_t = \omega_{3db}(1+\frac{R_2}{R_1}) \Rightarrow f_t = 100^k\times101 = 10.1\ MHz$

b) $1+\frac{R_2}{R_1} = 100\ V/V \qquad f_{3db} = 100KHz$

$f_t = f_{3db}(1+\frac{R_2}{R_1}) = 10\ MHz$

c) $1+\frac{R_2}{R_1} = 2\ V/V \qquad f_{3db} = 10MHz$

$f_t = 10MHz\times2 = 20\ MHz$

d) $-\frac{R_2}{R_1} = -2\ V/V \qquad f_{3db} = 10MHz$

$f_t = 10MHz(1+2) = 30\ MHz$

e) $-\frac{R_2}{R_1} = -1000\ V/V \qquad f_{3db} = 20KHz$

$f_t = 20K(1+1000) = 20.02MHz$

f) $1+\frac{R_2}{R_1} = 1\ V/V \qquad f_{3db} = 1MHz$

$f_t = 1M\times1 = 1MHz$

g) $-\frac{R_2}{R_1} = -1 \qquad f_{3db} = 1MHz$

$f_t = 1M(1+1) = 2MHz$

2.86

$1+\frac{R_2}{R_1} = 100\ V/V \qquad f_{3db} = 8KHz$

$f_t = 8\times100 = 800KHz$

for $f_{3db} = 20KHz$: $G_0 = \frac{800}{20} = 40\ V/V$

2.87

$f_{3db} = f_t = 1MHz$

$|G| = \frac{1}{\sqrt{1+\left(\frac{f}{f_{3db}}\right)^2}} = \frac{1}{\sqrt{1+f^2}} \quad f_{in}\ MHz$

$|G| = 0.99 \Rightarrow f = 0.142MHz$

The follower behaves like a low-pass STC circuit with a time constant $\tau = \frac{1}{\omega_{3db}}$

Thus: $\tau = \frac{1}{2\pi\times10^6} = \frac{1}{2\pi}\ \mu s$

$t_r = 2.2\tau = 0.35\mu s$ (Refer to Appendix F)

2.88

$1+\frac{R_2}{R_1}=10\ V/_V \qquad R_1=1K\Omega \quad R_2=9K\Omega$

If we consider 5τ the time that it takes for the output voltage to reach 99% of its final value, then: $5\tau=100ns \Rightarrow \tau=20ns$

$\tau=\frac{1}{\omega_{3db}} \Rightarrow \omega_{3db}=50\times10^6 \Rightarrow f_{3db}=7.96MHz$

$f_t=(1+\frac{R_2}{R_1})f_{3db}=10\times7.96=79.6MHz$

2.89

a) Assume two identical stages, each with a gain function: $G=\frac{G_0}{1+j\frac{\omega}{\omega_1}}=\frac{G_0}{1+jf/f_1}$

$G=\frac{G_0}{\sqrt{1+(\frac{f}{f_1})^2}}$

Overall gain of the cascade is $\frac{G_0^2}{1+(\frac{f}{f_1})^2}$

The gain will drop by 3db when:

$1+(\frac{f_{3db}}{f_1})^2=\sqrt{2}$, Note $3db=20\log\sqrt{2}$

$f_{3db}=f_1\sqrt{\sqrt{2}-1}$

b) $40db=20\log G_0 \Rightarrow G_0=100=1+\frac{R_2}{R_1}$

$f_{3db}=\frac{f_t}{1+\frac{R_2}{R_1}}=\frac{1MHz}{100}=10KHz$

c) Each stage should have 20db gain or $1+\frac{R_2}{R_1}=10$ and therefore a 3db frequency of: $f_1=\frac{10^6}{10}=10^5\ Hz$.

The overall $f_{3db}=10^5\sqrt{\sqrt{2}-1}=64.35KHz$ which is 6 times greater than the bandwidth achieved using single op-amp. (case b above)

2.90

$f_t=100\times5=500MHz$. if single op-amp is used.

with op-amp that has only $f_t=40MHz$, the possible closed loop gain at 5MHz is:

$|A|=\frac{40}{5}=8\ V/_V$.

To obtain an overall gain of 100, three such amplifiers cascaded, would be required. Now, if each of the 3 stages, has a low-frequency (d) closed loop gain K, then its 3db frequency will be $\frac{40}{K}$ MHz. Thus for each stage the closed loop gain is: $|G|=\frac{K}{\sqrt{1+\left(\frac{f}{\frac{40}{K}}\right)^2}}$

which at f=5MHz becomes:

$|G_{5MHz}|=\frac{K}{\sqrt{1+(\frac{K}{8})^2}}$.

The overall gain of 100: $100=\left[\frac{K}{\sqrt{1+(\frac{K}{8})^2}}\right]^3$

$K=5.7$

Thus for each cascade stage: $f_{3db}=\frac{40}{5.7}$

$f_{3db}=7MHz$

The 3-db frequency of the overall amplifiers, f_1, can be calculated as:

$\left[\frac{5.7}{\sqrt{1+(\frac{f}{7})^2}}\right]^3=\frac{(5.7)^3}{\sqrt{2}} \Rightarrow f_1=3.6MHz$

2.91

a) $\frac{R_2}{R_1}=K \qquad f_{3db}=\frac{f_t}{1+\frac{R_2}{R_1}}=\frac{f_t}{1+K}$

$GBP=Gain\times f_{3db}$

$GBP=K\frac{f_t}{1+K}$

b) $1+\frac{R_2}{R_1}=K \qquad f_{3db}=\frac{f_t}{K}$

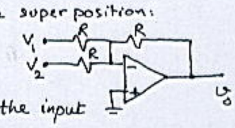
$GBP=K\frac{f_t}{K}=f_t$

The non-inverting amplifier realizes a higher GBP and it's independent of K.

2.92

To find f_{3db} we use superposition:

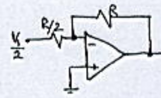
Set V2=0



Now using Thevenin's Theorem to simplify the input circuit results in:

Cont.

$$\frac{V_o}{V_i/2} = \frac{-R/R/2}{1+s\,\frac{1+R/R/2}{\omega_t}}$$



which gives:

$$\frac{V_o}{V_i} = \frac{-1}{1+s/(\omega_t/3)}$$

Thevenin's equivalent

$f_{3db} = \frac{f_t}{3}$. Similar results can be obtained for $\frac{V_o}{V_2}$.

2.93

The peak value of the largest possible sine wave that can be applied at the input without output clipping is: $\frac{\pm 12V}{100} = 0.12V = 120mV$

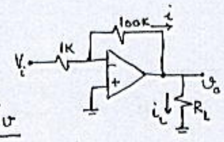
rms value $= \frac{120}{\sqrt{2}} = 85mV$

2.94

a) $R_L = 1K\Omega$

for $V_{omax} = 10^V$: $V_p = \frac{10}{100}$

$\underline{V_p = 0.1V}$



when output is at its peak, $i_L = \frac{10}{1K} = 10mA$

$i = \frac{10}{100K} = 0.1mA$. therefore $i_o = 10+0.1 = 10.1\,mA$ is well under $i_{omax} = 20mA$.

b) $R_L = 100\Omega$

If output is at its peak: $i_L = \frac{10^V}{0.1} = 100mA$ which exceeds $i_{omax} = 20mA$. Therefore V_o cannot go as high as 10^V. instead:

$20\,mA = \frac{V_o}{100\Omega} + \frac{V_o}{100K} \Rightarrow V_o = \frac{20}{10.01} = 2^V$

$V_p = \frac{2}{100} = 0.02V = \underline{20mV}$

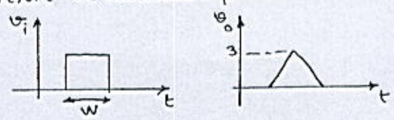
c) $R_L = ?$ $\quad i_{omax} = 20mA = \frac{10^V}{R_{Lmin}} + \frac{10^V}{100^k}$

$20 - 0.1 = \frac{10}{R_{Lmin}} \Rightarrow R_{Lmin} = \underline{502\Omega}$

2.95

The output is triangular with the slew rate of $20^V/\mu s$. In order to reach $3V$, it takes $\frac{3}{20}\,\mu s = 0.15\mu s = 150ns$.

Therefore the minimum pulse width is 150^{ns}.



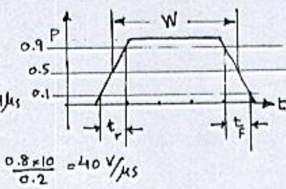
2.96

$W = 2\mu s$

$t_r + t_f = 0.2W = 0.4\mu s$

$t_r = t_f = 0.2\,\mu s$

$SR = \frac{(0.9-0.1)P}{t_r} = \frac{0.8\times 10}{0.2} = 40\,V/\mu s$



2.97

Slope of the triangle wave $= \frac{20V}{T/2} = SR$

Thus $\frac{20}{T}\times 2 = 10V/\mu s$

$\Rightarrow T = 4\,\mu s$ or $f = \frac{1}{T} = 250KHZ$

For a Sine wave $v_o = \hat{v}_o \sin(2\pi \times 250\times 10^3 t)$

$\left.\frac{dv_o}{dt}\right|_{max} = 2\pi\times 250\times 10^3\,\hat{v}_o = SR$

$\Rightarrow \hat{v}_o = \frac{10\times 10^6}{2\pi\times 10^3\times 250} = 6.37V$

2.98

$v_o = 10\sin\omega t \Rightarrow \frac{dv_o}{dt} = 10\omega\cos\omega t \Rightarrow \left.\frac{dv_o}{dt}\right|_{max} = 10\omega$

The highest frequency at which this output is possible is that for which:

$\left.\frac{dv_o}{dt}\right|_{max} = SR \Rightarrow 10\omega_{max} = 60\times 10^{+6} \Rightarrow \omega_{max} = 6\pi 10^5$

$\Rightarrow f_{max} = 45.5\,KHZ$.

2.99

a) $v_i = 0.5$, $V_o = 10\times 0.5 = 5V$

Cont.

Output distortion will be due to slew Rate limitation and will occur at the frequency for which $\frac{dv_o}{dt}\Big|_{max} = SR$

$\omega_{max} \times 5 = \frac{1}{10^{-6}} = 2\times10^{5}$ rad/s $\Rightarrow f_{max} = 31.8$ KHZ

b) The output will distort at the value of V_i that results in $\frac{dv_o}{dt}\Big|_{max} = SR$.

$v_o = 10\, v_i \sin 2\pi \times 20\times10^{3}$

$\frac{dv_o}{dt}\Big|_{max} = 10\, v_i \times 2\pi \times 20\times10^{3}$

Thus $v_i = \frac{1/10^{-6}}{10\times2\pi\times20\times10^{3}} = 0.795\ V$

c) $v_i = 50\ mV$ $V_o = 500\ mV = 0.5\ V$

Slew rate begins at the frequency for which $\omega\times0.5 = SR$

which gives $\omega = \frac{1/10^{-6}}{0.5} = 2\times10^{6}$ rad/s or $f = 318.3$ KHZ

However the small signal 3db frequency is $f_{3db} = \frac{f_t}{1+\frac{R_2}{R_1}} = \frac{2\times10^{6}}{10} = 200$ KHZ

Thus the useful frequency range is limited at 200KHZ.

d) for $f = 5$ KHZ, the slew Rate limitation occurs at the value of v_i given by

$\omega\times10\, V_i = SR \Rightarrow V_i = \frac{1/10^{-6}}{2\pi\times5\times10^{3}\times10} = 3.18\ V$

Such an input voltage, however would ideally result in an output of 31.8V which exceeds V_{omax}. Thus $V_{imax} = \frac{V_{omax}}{10} = 1\ V$ peak.

2.100

$V_o = V_{os}(1+\frac{R_2}{R_1}) \Rightarrow -0.3 = V_{os}(1+\frac{100}{1}) \simeq 3\ mV$

2.101

$V_{os} = \pm2\ mV$

$V_o = 0.01 \sin\omega t \times 200 + V_{os}\times200 = 2\sin\omega t \pm 0.4\ V$

2.102

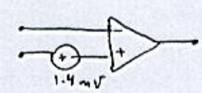
Output DC offset, $V_{os} = 3\ mV\times1000 = 3\ V$

Therefore the maximum amplitude of an input sinusoid is the one that results in an output peak amplitude of $13-3 = 10V \Rightarrow v_i = \frac{10}{1000} = 10\ mV$

If the amplifier is capacity coupled, then: $v_{imax} = \frac{13}{1000} = 13\ mV$

2.103

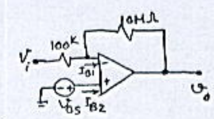
$V_{os} = \frac{1.4}{100} = 1.4\ mV$



2.104

a) $I_B = (I_{B1} + I_{B2})/2$

open input:



$V_o = V_+ + R_2 I_{B_1} = V_{os} + R_2 I_{B1}$

$9.31 = V_{os} + 10000\, I_{B1}$ ①

input connected to ground:

$V_o = V_+ + R_2(I_{B1} + \frac{V_{os}}{R_1}) = V_{os}(1+\frac{R_2}{R_1}) + R_2 I_{B1}$

$9.09 = V_{os}\times101 + 10000\, I_{B1}$ ②

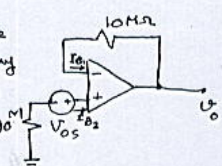
①, ② $\Rightarrow 100 V_{os} = -0.22 \Rightarrow V_{os} = -2.2\ mV$

$\Rightarrow I_{B_1} = 930\ nA$

$I_B \simeq I_{B_1} = 930\ nA$

b) $V_{os} = -2.2\ mV$

c) In this case, Since R is too Large, we may ignore V_{os} compare to the voltage drop across R.

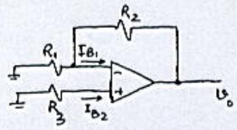


$V_{os} \ll R I_B$, Also Eq. 2.46 holds: $R_3 = R_1 \| R_2$

therefore from Eq. 2.47: $V_o = I_{os}\times R_2 \Rightarrow I_{os} = \frac{-0.8}{10^{4}}$

$I_{os} = -80\ nA$

2.105



$R_2 = 100K\Omega$

$R_1 = \frac{100K\Omega}{9}$

$R_3 = 5K\Omega$

$I_{B_1} = 1 \pm 0.05\ \mu A \quad , v_{os} = 0$

$I_{B_2} = 1 \mp 0.05\ \mu A$

From Eq. 2.45 : $V_o = -I_{B_2} R_3 + R_2 (I_{B_1} - I_{B_2} \frac{R_3}{R_1})$

a) For $I_{B_1} = 1.05\mu A$, $I_{B_2} = 0.95\mu A$

$V_o = -0.95 \times 5 + 100 (1.05 - 0.95 \times \frac{5}{100} \times 9) = 57.5mV$

b) For $I_{B_1} = 0.95\mu A$, $I_{B_2} = 1.05\mu A$

$V_o = -1.05 \times 5 + 100 (0.95 - 1.05 \times \frac{5}{100} \times 9) = 42.5mV$

$\Rightarrow 42.5^{mV} \leq V_o \leq 57.5^{mV}$

From the discussion in the text we know that to minimize the dc output voltage resulting from the input bias current, we should make the total DC resistance in the inputs of the op-amp equal. Currently, the negative input sees a resistance of $R_1 \| R_2 = \frac{100}{9} \| 100 = 10K\Omega$ while the positive input terminal sees $5K\Omega$ source resistance. Therefore we should add $\underline{\underline{5K\Omega}}$ series resistor to the positive input terminal to make the effective Resistance $5K\Omega + 5K\Omega = 10^{K\Omega}$

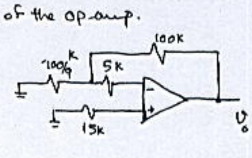
The resulting V_o can be found as follows:

$V_o = -I_{B_2} \times 10 + 100 (I_{B_1} - I_{B_2} \frac{10}{100/9}) = (I_{B_1} - I_{B_2}) \times 100$

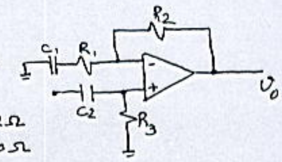
$V_o = I_{os} \times 100 = \pm 0.1 \times 100 = \pm 10mV$

$V_o = \pm 10mV$

If the signal source resistance is $15K\Omega$, then the resistances can be equalized by adding a $5K\Omega$ resistor in series with the negative input load of the op-amp.



2.106



$R_2 = R_3 = 100K\Omega$

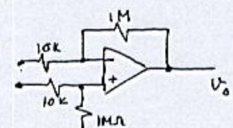
$1 + \frac{R_2}{R_1} = 200$

$R_1 = \frac{100K}{199} = 502\Omega \simeq 500\Omega$

$\frac{1}{R_1 C_1} = 2\pi \times 100 \Rightarrow C_1 = \frac{1}{500 \times 2\pi \times 100} = 3.18\ \mu F$

$\frac{1}{R_3 C_2} = 2\pi \times 10 \Rightarrow C_2 = \frac{1}{100^k \times 2\pi \times 10} = 0.16\ \mu F$

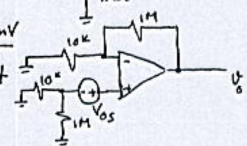
2.107



The output component due to V_{os} is :

$V_{o1} = V_{os} (1 + \frac{1M}{10K})$

$V_{o1} = 4(1 + 100) = \underline{404mV}$



The output component due to I_B or input bias current is :

$I_{B_1} = I_B + \frac{I_{os}}{2} \quad , \quad I_{B_2} = I_B - \frac{I_{os}}{2}$

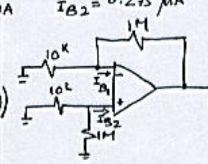
$I_{B_1} = 0.3 + \frac{0.05}{2} = 0.325\mu A \qquad I_{B_2} = 0.275\ \mu A$

$V_+ = -I_{B_2} \times (10^K \| 1M)$

$V_+ = -2.72mV$

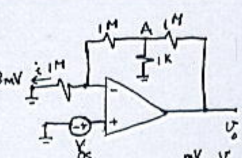
$V_{o2} = V_+ + (1^M \times (I_{B_1} + \frac{V_+}{10K}))$

$\underline{V_{o2} = 50\ mV}$



The worst case (largest) DC offset voltage at the output is $404 + 50 = \underline{454mV}$

2.108



$V_- = V_+ = V_{os} \Rightarrow V_A = 2V_{os} = 8mV$

$i = \frac{V_{os}}{1M} = V_{os}\ (\mu A)$

$V_o = V_A + 1M \times (i + \frac{V_A}{1K})$

$V_o = 2V_{os} + 1^M (\frac{V_{os}}{1M} + \frac{2V_{os}}{1K}) = 2003V_{os} = 2003 \times 4^{mV} = 8^V$

$\underline{V_o = 8V}$

Cont.

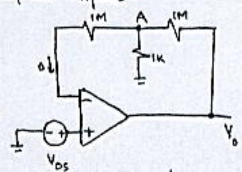
for capacitively coupled input:

$V_+ = V_- = V_{os}$

$V_A = V_{os}$

$V_o = V_A + 1M \times \frac{V_{os}}{1^K}$

$V_o = 1001\, V_{os} = \underline{\pm 4.004^V}$



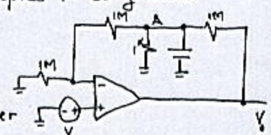
for capacitively coupled 1^K to ground:

$V_+ = V_- = V_{os}$

$V_A = 2V_{os}$

$V_o = 3V_{os} = \pm 12mV$

This is much smaller than capacitvely coupled input case.



2.109

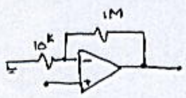
At $0°C$, we expect $\pm 10 \times 25 \times 1000^{\mu} = \pm 250mV$

At $75°C$, we expect $\pm 10 \times 50 \times 1000^{\mu} = \pm 500mV$

We expect these quantities to have opposite polarities.

2.110

$100 = 1 + \frac{R_2}{R_1} \Rightarrow R_1 = 10.1^{K\Omega}$



a) $V_o = 100 \times 10^{-9} \times 1 \times 10^6 = 0.1^V$

b) largest output offset is:

$V_o = 1^{mV} \times 100 + 0.1^V = 200mV = \underline{0.2^V}$

c) for bias current compensation we connect a resistor R_3 in series with the positive input terminal of the op-amp, with: $R_3 = R_1 \| R_2$

$I_{os} = \frac{100}{10} = 10nA$ $\qquad R_3 = 10.1^K \| 1^M = 10K\Omega$

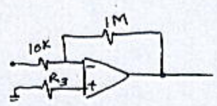
The offset current alone results in an output offset voltage of $I_{os} \times R_2 = 10 \times 10^{-9} \times 1 \times 10^6 = 10^{mV}$

d) $V_o = 100mV + 10mV = \underline{110\ mV}$

2.111

$R_3 = R_1 \| R_2 = 9.9^{K\Omega}$

(Refer to 2.46)

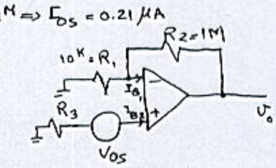


$V_o = I_{os} R_2$ Eq. 2.47

$V_o = 0.21 = I_{os} \times 1^M \Rightarrow I_{os} = 0.21\,\mu A$

If $V_{os} = 1mV$

$V_+ = -I_{B2} R_3 \mp V_{os}$



$$I_{B_1} = \frac{R_3 I_{B2} \pm V_{os}}{R_1} + \frac{0.21 + R_3 I_{B2} \pm V_{os}}{R_2}$$

$$I_{B_1} = R_3 I_{B_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \pm V_{os}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\frac{1}{R_3} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow I_{B_1} - I_{B_2} = \pm V_{os}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$\Rightarrow I_{os} = \pm \frac{1^{mV}}{9.9^K} = \pm 0.1\,\mu A$$

If we apply the same current as I_{os} to the other end of R_3, then it will cancel out the offset current effect on the output. $\underline{\pm 0.1\mu A}$

Now if we use $\pm 15V$ supplies:

2.112

$$\frac{v_o}{v_i} = -\frac{1}{sCR} = \frac{-1}{j\omega CR} = \frac{1}{-j\omega \times 10 \times 10^{-9} \times 100 \times 10^3}$$

$$\frac{v_o}{v_i} = -\frac{10^3}{j\omega}$$

a) $\frac{v_o}{v_i} = 1 \Rightarrow \omega = 1^{KRad/s} \Rightarrow \underline{f = 159HZ}$

b) $\frac{1}{j}$ indicates $90°$ lag, but since its $\frac{-1}{j}$, it results in output leading the input by $90°$

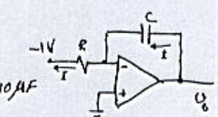
c) $\frac{v_o}{v_i} = \frac{-10^3}{j\omega}$ if frequency if lowered by a factor of 10, then the output would increase by a factor of 10.

Cont.

d) The phase doesnot change and the output still leads the input by $90°$

2.113

$R_{in} = R = 100K\Omega$



$CR = 1s \Rightarrow C = \frac{1}{100\times10^3} = 10\mu F$

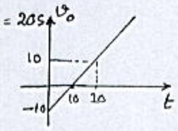
with a $-1V$ dc input applied, the capacitor charges with a constant current:

$I = \frac{1V}{R} = 0.01\,mA$ and its voltage rises linearly:

$v_o(t) = -10 + \frac{1}{C}\int_0^t I dt = -10 + \frac{I}{C}t = -10 + \frac{t}{RC}$

the voltage reaches $0V$ at $t = 10RC = 10s$

and it reaches $10V$ at $t = 20s$



2.114

$|T| = \frac{1}{\omega RC}$ If $|T| = 100V/V$ for $f = 1^{KHZ}$, then for $|T| = 1V/V$, f has to be $1^K \times 100 = 100\,KHZ$.

Also $RC = \frac{1}{\omega T} = \frac{1}{2\pi\times1^K\times100} = \underline{1.59\mu s}$

2.115

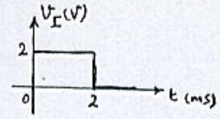
$R_{in} = R$, Thus $R = 100k\Omega$.

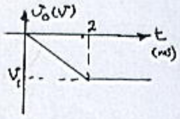
$|T| = \frac{1}{\omega RC} = 1$ at $\omega = \frac{1}{RC}$.

$\omega = 1000 = \frac{1}{RC} \Rightarrow C = \frac{1}{1000\times100^K} = 10^{nF}$

with a 2V-2ms pulse at the input, the output falls linearly until t=2ms at which $v_o = V_1$, $v_o = \frac{-I}{C}t = \frac{-2}{RC}t = -2t$ Volts where t in ms

Thus $\underline{V_1 = -4V}$





with $V_I = 2\sin 1000t$ applied at the input,

$v_o(t) = 2 \times \frac{1}{1000\times10^{-3}}\sin(1000t + 90°)$

$v_o(t) = 2\sin(1000t + 90°)$

2.116

$R_{in} = R = \underline{20k\Omega}$

$|T| = \frac{1}{\omega RC} = 1$ at $\omega = 2\pi\times10^{KHZ} \Rightarrow C = \frac{1}{2\pi\times10^K\times20^K}$

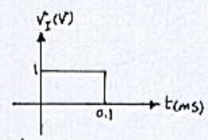
$\underline{C = 0.796\,nF}$

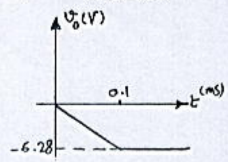
Refer to discussion in page 110:

$\frac{V_o}{V_i} = \frac{R_F/R}{1+sCR_F}$ and the finite dc gain is $\frac{-R_F}{R}$. There fore for 40db gain or equivalently $100\,V/V$ we have: $\frac{-R_F}{R} = -100\,V/V$

$\Rightarrow R_F = 100\times20K = 2M\Omega$

The corner frequency $\frac{1}{CR_F}$ is: $\frac{1}{0.796^n\times2^M} = 628^{HZ}$





a) when no R_F

$v_o(t) = \frac{-1}{RC}\int_0^t 1.dt = -62.8t$ $\quad 0 \leq t \leq 0.1ms$

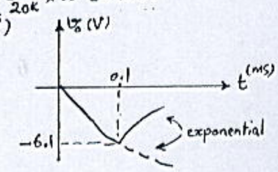
$v_o(0.1) = -6.28V$

b) with R_F: $v_o(t) = V_o(\infty)(1 - e^{-t/CR_F})$

(Refer to pg. 112)

$v_o(\infty) = -I\times R_F = -\frac{1V}{20k}\times2^M = -100V$

$v_o(t) = -100(1 - e^{-t/1.6})$

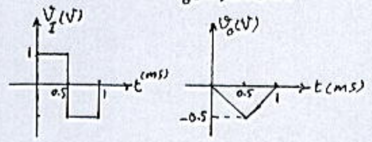


2.117

For $0 \leq t \leq 0.5\,ms$: $v_O(t) = v_O(0) - \frac{1}{RC}\int_0^t v_I\,dt$

$v_O(t) = 0 - \frac{t}{RC} = -\frac{t}{1\,ms}$

$v_O(0.5) = -0.5V$



For $0.5 \leq t \leq 1\,ms$: $v_O(t) = v_O(0.5) - \frac{1}{RC}\int_{0.5}^t -1\,dt$

$v_O(t) = -0.5 + \frac{1}{RC}(t - 0.5)$

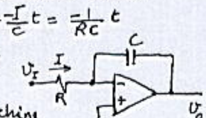
$v_O(1^{ms}) = -0.5 + \frac{0.5}{1} = 0V$

Another way of thinking about this circuit is as follows:

For $0 \leq t \leq 0.5\,ms$ a current $I = \frac{1V}{R}$ flows through R and C in the direction indicated on the diagram. At time t we write:

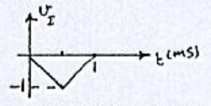
$I.t = -C\,v_O(t) \Rightarrow v_O(t) = \frac{-I}{C}t = \frac{-1}{RC}t$

which indicates that the output voltage is linearly decreased, reaching $-0.5V$ at $t = 0.5ms$.



Then for $0.5 \leq t \leq 1^{ms}$, the current flows in the opposite direction and v_O rises linearly reaching $0V$ at $t = 1ms$.

For $v_I = \pm 2V$: we obtain the following waveform: (assuming time constant is the same)



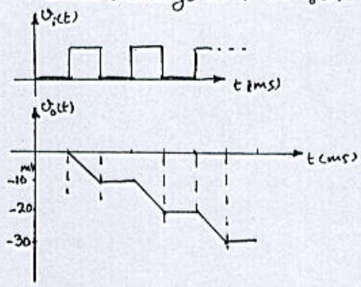
If RC is also doubled, then the waveform becomes the same as the first case where $v_I = \pm 1V$ and $RC = 1^{ms}$.

2.118

Each pulse lowers the output voltage by:

$\Delta v_O = \frac{1}{RC}\int_0^{10\mu s} 1 \cdot dt = \frac{10\mu s}{RC} = \frac{10\mu s}{1\,ms} = 10\,mV$

Therefore a total of 100 pulses are required to cause a change of 1V in $v_O(t)$.



2.119

Refer to Fig. P2.119.

$\frac{V_o}{V_i} = -\frac{Z_2}{Z_1} = -\frac{Y_1}{Y_2} = -\frac{1/R_1}{1/R_2 + SC} = -\frac{R_2/R_1}{1 + SCR_2}$

which is an STC LP circuit with a dc gain of $-\frac{R_2}{R_1}$ and a 3-db frequency $\omega_0 = \frac{1}{CR_2}$.

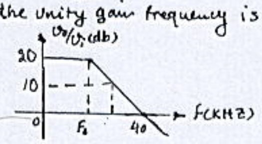
The input resistance equal to R_1. So for:

$R_i = 1K \Rightarrow R_1 = 1K\Omega$ and for dc-gain of 20db or 10: $\frac{R_2}{R_1} = 10 \Rightarrow R_2 = 10\,K\Omega$

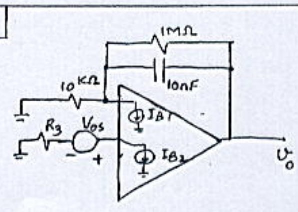
for 3db frequency of 4KHz: $\omega_0 = 2\pi \times 4 \times 10^3 = \frac{1}{CR_2}$

$\Rightarrow C \approx 4nF$

the unity gain frequency is (0db) is 40 KHz

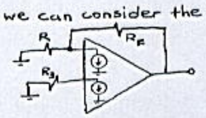


2.120



Cont.

a) To compensate for the effect of dc bias current I_B, we can consider the following model



Similar to the discussion leading to equation (2.46) we have: $R_3 = R \| R_F = 10k\Omega \| 1M\Omega \Rightarrow R_3 = 9.9k\Omega$

(b) As discussed in Section 2.8.2 the dc output voltage of the integrator when the input is grounded is: $V_o = V_{os}(1 + \frac{R_F}{R}) + I_{os} R_F$

$V_o = 3mV(1 + \frac{1M\Omega}{10k\Omega}) + 10nA \times 1M\Omega = 0.303V + 0.01V$

$V_o = 0.313V$

2.121

$\frac{v_o}{v_i}(s) = -SRC = -S \times 0.01 \times 10^{-6} \times 10 \times 10^{3} = -10^{-4}S$

$\frac{v_o}{v_i}(j\omega) = -j\omega \times 10^{-4} \Rightarrow \left|\frac{v_o}{v_i}\right| = -\omega \times 10^{-4} \Rightarrow$

$\left|\frac{v_o}{v_i}\right| = 1$ when $\omega = 10^4$ Rad/s or $f = 1.59KHZ$

For an input 10 times this frequency, the output will be 10 times as large as the input: 10V peak-to-peak. The (-j) indicates that the output lags the input by 90°. Thus $v_o(t) = -5\sin(10^5 t + 90°)$ volts

2.122

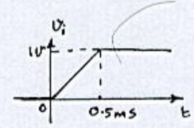
$v_o = -CR\frac{dv_i}{dt}$

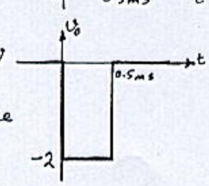
therefore:

for $0 \leq t \leq 0.5$:

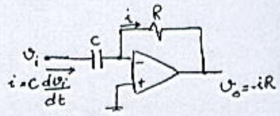
$v_o = -1ms \times \frac{1V}{0.5ms} = -2V$

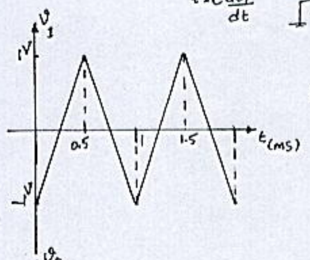
and $v_o = 0$ otherwise



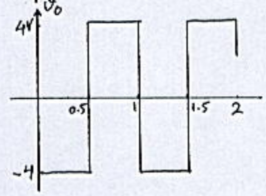


2.123





Slope $= \frac{2V}{0.5ms} = 4\,V/ms$



$C\frac{dv_i}{dt} = 0.1 \times 10^{-6} \times \frac{4}{10^{-3}} = 0.4mA$

Thus the peak value of the output square wave is $0.4mA \times 10k\Omega = 4V$. The frequency of the output is the same as the input (1KHZ).

The average value of the output is 0.

To increase the value of the output to 10V, R has to be increased to $\frac{10}{4} = 2.5$, i.e $25k\Omega$.

When a 1-KHz, 1V peak input sine wave is applied $v_i = \sin(2\pi \times 1000t)$ a sinusoidal signal appears at the output. It can be determined by one of the following methods:

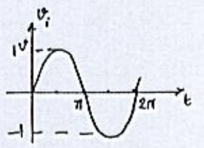
a) $v_o(t) = -RC\frac{dv_i}{dt} = -0.1 \times 10^{-6} \times 10 \times 10^{3}\frac{dv_i}{dt} = -10^{-3}\frac{dv_i}{dt}$

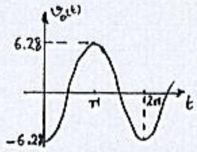
$v_o(t) = -10^{-3} \times 2\pi \times 1000 \times \cos(2\pi \times 1000t)$

$v_o(t) = -2\pi\cos(2\pi \times 1000t)$

Thus the peak amplitude is 6.28V and the negative peaks occur at $t = 0, \frac{2\pi}{2\pi \times 1000}, \ldots$

Cont.





b) $\frac{v_o}{v_i} = -SRC \Rightarrow \frac{v_o}{v_i}(j\omega) = -j\omega RC \Rightarrow v_o(t) = -j\omega RC\, v_i(t)$

the output is inverted and has $90°$ phase shift, due to $(-j)$ factor.

$v_o(t) = -(\omega RC) \times 1 \sin(2\pi \times 1000t + 90°)$

$v_o(t) = -6.28 \sin(2\pi \times 1000t + 90°)$

$v_o(t) = -6.28 \cos(2\pi \times 1000t)$

Same as before.

c) The peaks of the output waveform are equal to $RC \times$(maximum slope of input wave). Since the maximum slope occurs at the zero crossings, its value is $2\pi \times 1000$. Thus the peak output $= 2\pi \times 1000 \times RC = 6.28V$

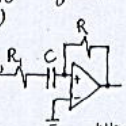
The negative peak occurs at $\omega t = 0, 2\pi, \ldots$

2.124

$RC = 10^{-3}s$ when $C = 10^{nF} \Rightarrow R = 100K\Omega$

$\frac{v_o}{v_i} = -SRC \quad \frac{v_o}{v_i}(j\omega) = -j\omega RC \quad \varphi = -90°$ always

$\left|\frac{v_o}{v_i}\right| = 1 \Rightarrow \omega_{unity} = \frac{1}{RC} = 1\,^{krad}/_{s}$. Gain is 10 times the unity gain, when the frequency is 10 times the unity gain frequency. Similarly for $\omega = \frac{1}{10}\,^{krad}/_{s}$, gain is $0.1\,^{V}/_{V}$. (for $\omega = 10\,krad/s$, gain $= 10\,^{V}/_{V}$)



for high frequencies C is short-circuited,

$\frac{V_O}{V_i} = \frac{-R}{R_1} = -100 \Rightarrow R_1 = 1K\Omega$

$\frac{v_o}{v_i} = \frac{-RCS}{R_1CS+1} = \frac{-10^{-3}S}{10^{-5}S+1} \Rightarrow \omega_{3db} = 100\,Krad/s$ or $f_{3db} = 15.9\,KHz$

for unity gain: $|10^{-3}S| = |10^{-5}S + 1| \Rightarrow \omega_H = 1.01\,Krad/s$

if $\omega = 10.1\,Krad/s$: $\left|\frac{v_o}{v_i}\right| = \frac{10.1}{1.01} = 10, \quad \varphi = -95.77°$

2.125

Refer to Fig. p2.125:

$$\frac{v_o}{v_i} = -\frac{Z_2}{Z_1} = \frac{-R_2}{R_1 + \frac{1}{SC}} = \frac{-\left(\frac{R_2}{R_1}\right)S}{S + \frac{1}{R_1C}}$$

which is the transfer function of an STC HP filter with a high frequency gain $K = -\frac{R_2}{R_1}$ and a 3-db frequency $\omega_0 = \frac{1}{R_1C}$

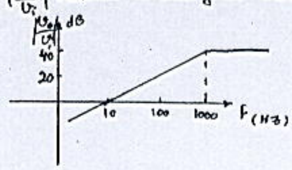
The high-frequency input impedance approaches R_1. (as $\frac{1}{j\omega C}$ becomes neglibibly small) So we can select $R_1 = 10K\Omega$

To Obtain a high-frequency gain of 40db (i.e. 100) : $\frac{R_2}{R_1} = 100 \Rightarrow R_2 = 1M\Omega$.

For a 3-db frequency of 1000 Hz:

$\frac{1}{R_1C} = 2\pi \times 1000 \Rightarrow C = 15.9nF$

from the Bode-diagram below, we see that $\left|\frac{v_o}{v_i}\right|$ reduces to unity at $f = 0.01 f_0 = 10\,Hz$



2.126

Refer to the circuit in Fig. P2.126:

$$\frac{v_o}{v_i} = -\frac{Z_2}{Z_1} = -\frac{1}{Z_1 Y_2} = -\frac{1}{\left(R_1 + \frac{1}{SC_1}\right)\left(\frac{1}{R_2} + SC_2\right)}$$

$$\frac{v_o}{v_i} = -\frac{R_2/R_1}{\left(1 + \frac{1}{R_1C_1S}\right)(1 + SR_2C_2)}$$

$$\frac{v_o}{v_i}(j\omega) = \frac{-R_2/R_1}{\left(1 + \frac{1}{j\omega R_1C_1}\right)(1 + j\omega R_2C_2)} = \frac{-R_2/R_1}{\left(1 + \frac{\omega_1}{j\omega}\right)\left(1 + j\frac{\omega}{\omega_2}\right)}$$

where $\omega_1 = \frac{1}{R_1C_1}$, $\omega_2 = \frac{1}{R_2C_2}$

a) for $\omega \ll \omega_1 \ll \omega_2$

$$\frac{v_o}{v_i}(j\omega) \propto \frac{-R_2/R_1}{\left(1 + \frac{\omega_1}{j\omega}\right)} \simeq \frac{-R_2/R_1}{\omega_1/j\omega} = -j\frac{R_2}{R_1}\frac{\omega}{\omega_1}$$

Cont.

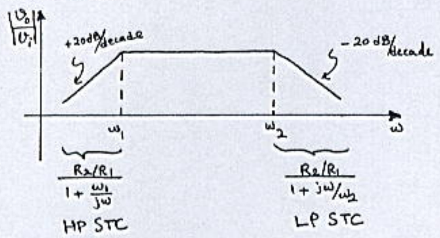
b) for $\omega_1 \ll \omega \ll \omega_2$

$$\frac{v_o}{v_i}(j\omega) \simeq \frac{-R_2}{R_1}$$

c) for $\omega \gg \omega_2$ and $\omega_2 \gg \omega_1$:

$$\frac{v_o}{v_i}(j\omega) \simeq \frac{-R_2/R_1}{1 + j\omega/\omega_2} \simeq \frac{-R_2/R_1}{j\,\omega/\omega_2} = j\left(\frac{R_2}{R_1}\right)\left(\frac{\omega_2}{\omega}\right)$$

from the results of a), b) and c) we can draw the Bode-plot:



Design: $\frac{R_2}{R_1} = 1000$ (60dB gain in the mid-frequency range)

R_{in} for $\omega \gg \omega_1$ $= \underline{R_1 = 1K\Omega} \Rightarrow \underline{R_2 = 1M\Omega}$

$f_1 = 100HZ \Rightarrow \omega_1 = 2\pi \times 100 = \frac{1}{R_1 C_1} \Rightarrow C_1 = 1.59\mu F$

$f_2 = 10KHZ \Rightarrow \omega_2 = 2\pi \times 10 \times 10^3 = \frac{1}{R_2 C_2} \Rightarrow C_2 = 15.9\,pF$