CHAPTER 12 - PROBLEMS

12.1

$T(s) = \dfrac{\omega_0}{s+\omega_0} \qquad T(j\omega) = \dfrac{\omega_0}{j\omega+\omega_0}$

$|T(j\omega)| = \dfrac{\omega_0}{\sqrt{\omega_0^2+\omega^2}}$

$\phi(\omega) \triangleq \mathrm{Tan}^{-1}\left[\dfrac{\mathrm{Im}(T(j\omega))}{\mathrm{Re}(T(j\omega))}\right]$

$= -\tan^{-1} \omega/\omega_0$

$G = 20\log_{10}|T(j\omega)|$

$A = -20\log_{10}|T(j\omega)|$

ω	$\lvert T(j\omega)\rvert$ [V/V]	G [dB]	A [dB]	ϕ °
0	1	0	0	0
$0.5\omega_0$	0.8944	−0.97	0.97	−26.57
ω_0	0.7071	−3.01	3.01	−45.0
$2\omega_0$	0.4472	−6.99	6.99	−63.43
$5\omega_0$	0.1961	−14.1	14.1	−78.69
$10\omega_0$	0.0995	−20.0	20.0	−84.29
$100\omega_0$	0.010	−40.0	40.0	−89.43

12.2

$T(s) = \dfrac{1}{(s+1)(s^2+s+1)}$

$= \dfrac{1}{s^3+2s^2+2s+1}$

$T(j\omega) = \left[j(2\omega-\omega^3)+(1-2\omega^2)\right]^{-1}$

$|T(j\omega)| = \left[(2\omega-\omega^3)^2+(1-2\omega^2)^2\right.$

$= \left[4\omega^2-4\omega^4+\omega^6+1-4\omega^2+4\omega\right.$

$= \left[1+\omega^6\right]^{-1/2}$

$= \dfrac{1}{\sqrt{1+\omega^6}}$

For Phase Angle:

$\phi(\omega) = \tan^{-1}\left[\dfrac{\mathrm{Im}[T(j\omega)]}{\mathrm{Re}[T(j\omega)]}\right]$

$= -\tan^{-1}\left[\dfrac{2\omega-\omega^3}{1-2\omega^2}\right]$

For $\omega = 0.1$:

$|T(j\omega)| = (1+0.1^6)^{-1/2} \cong \underline{1}$

$\phi(\omega) = -11.5° = \underline{-0.20 \text{ rad}}$

For $\omega = 1$ rad/s:

$|T(j\omega)| = (1+1^6)^{-1/2} = 1/\sqrt{2} = \underline{0.7}$

$\phi = -\tan^{-1}(1/-1) = -135° = \underline{2.356}$

Note: $G = -3\text{dB}$

Also: $\tan^{-1}(-1) = -45°$ or $-135°$

$\tan^{-1}(-1/1) = -45°$

$\tan^{-1}(1/-1) = -135°$

For $\omega = 10$ rad/s:

$|T(j\omega)| = (1+10^6)^{-1/2} = \underline{0.001}$

$\phi = -\tan^{-1}\left[\dfrac{2(10)-10^3}{1-2(10^2)}\right]$

CONT.

$$= -\tan^{-1}\left[\frac{-980}{-199}\right]$$

$$= -\left[180^\circ + \tan^{-1}\left(\frac{980}{199}\right)\right]$$

$$= -258.5^\circ$$

$$= \underline{\underline{4.512 \text{ rad}}}$$

Now consider an input of $A \sin \omega t$ to $T(s)$. The output is then given by:

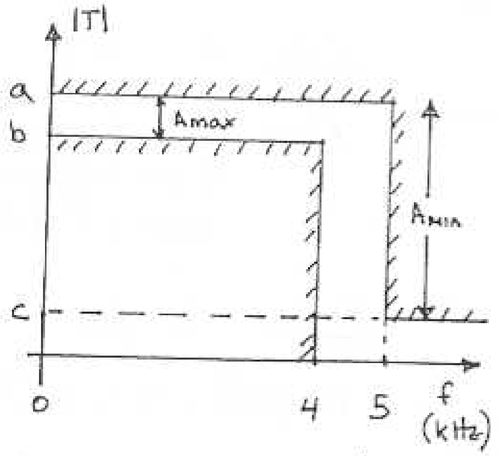
$$A\,|T(j\omega)|\,\sin(\omega t + \phi(\omega))$$

Using this result, the output to each of the following inputs will be:

INPUT	OUTPUT
$2\sin(0.1t)$	$2\sin(0.1t - 0.2)$ ie $2\times1=2$
$2\sin(1t)$	$\sqrt{2}\sin(t-2.356)$ i.e. $2\times 1/\sqrt{2}=\sqrt{2}$
$2\sin(10t)$	$2\times10^{-3}\sin(10t-4.512)$

12.3

12.4



Note $|T|$ is shown in a linear scale but Amax and Amin are in dB

From the problem

$$\frac{a}{b} = 1.1, \quad c = 0.1\%\, a \text{ or } \frac{c}{a} = 0.001$$

$$A_{max} = 20\log_{10} a - 20\log_{10} b$$

$$= 20\log_{10} a/b$$

$$= 20\log_{10}(1.1)$$

$$= \underline{\underline{0.83\text{dB}}}$$

$$A_{min} = 20\log_{10} a - 20\log_{10} c$$

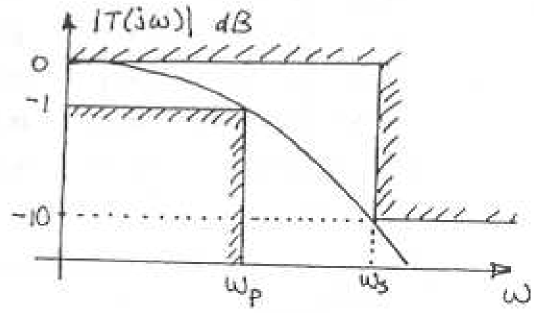
$$= 20\log_{10}(a/c)$$

$$= 20\log_{10}(0.001)$$

$$= \underline{\underline{60\text{dB}}}$$

$$\text{Selectivity} = \frac{\omega_s}{\omega_p} = \frac{2\pi 5}{2\pi 4} = \underline{\underline{1.25}}$$

12.5



$$T(s) = \frac{k}{1+s\tau}$$

If $\tau = 1s$ & the DC gain $= 1$ then $\underline{k=1}$

$$= \frac{1}{1+1s}$$

$$|T(j\omega)| = \frac{1}{\sqrt{1+\omega^2}}$$

At the passband edge:

CONT.

$$|T(j\omega_p)| = \frac{1}{\sqrt{1+\omega_p^2}} = 10^{-1/20}$$

$\therefore \omega_p = \underline{\underline{0.5088 \text{ rad/s}}}$

At the stopband edge:

$$|T(j\omega_s)| = \frac{1}{\sqrt{1+\omega_s^2}} = 10^{-10/20}$$

$\therefore \omega_s = \underline{\underline{3 \text{ rad/s}}}$

$$\text{Selectivity} = \frac{\omega_s}{\omega_p} = \frac{3}{0.5088} = \underline{5.9}$$

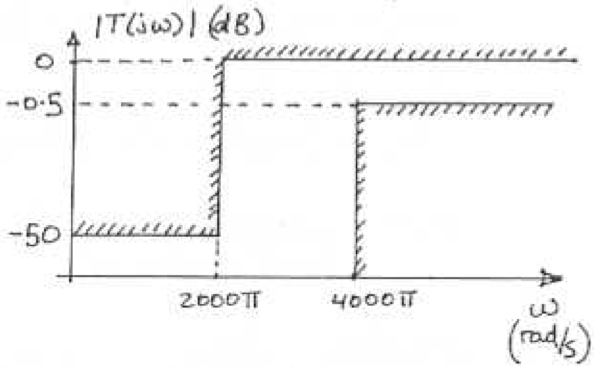
12.6

Passband is defined by: $f \geq 2\text{kHz}$

$\Rightarrow \omega_p = 2\pi(2000)$ rad/s

Stopband is defined by: $f \leq 1\text{kHz}$

$\Rightarrow \omega_s = 2\pi(1000)$ rad/s



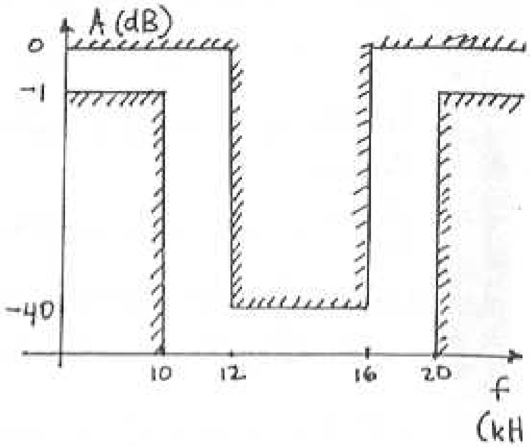
Note we assumed a maximum transmission of 0 dB.

12.7

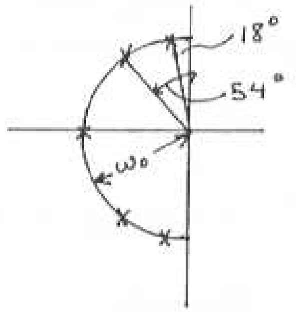
Passband: $f \in \{[0, 10\text{kHz}] \cup [20\text{kHz}, \infty]\}$

Stopband: $f \in [12\text{kHz}, 16\text{kHz}]$

$A_{max} = 1\text{dB}$, $A_{min} = 40\text{dB}$



12.8



$\omega_0 = 10^3$

Poles at 18°:

$$P_1 = \omega_0\left(-\cos(90^\circ - 18^\circ) \pm j\sin(90^\circ - 18^\circ)\right)$$
$$= \omega_0\left(-\cos 72^\circ \pm j\sin 72^\circ\right)$$
$$= \omega_0(-0.309 \pm j\,0.951)$$

Poles at 54°

$$P_2 = \omega_0\left(-\cos 36^\circ \pm j\sin 36^\circ\right)$$
$$= \omega_0(-0.809 \pm j\,0.588)$$

Poles on Real Axis

$P_3 = -\omega_0$

CONT.

Note: Given a pair of poles

$P_i = \omega_o(-\cos\alpha \pm j\sin\alpha)$,

introduces a second order term as follows:

$(s+\omega_o\cos\alpha - j\omega_o\sin\alpha)(s+\omega_o\cos\alpha + j\omega_o\sin\alpha)$

$= s^2 + s(\omega_o\cos\alpha - j\omega_o\sin\alpha + \omega_o\cos\alpha + j\omega_o\sin\alpha)$
$\quad + \omega_o^2[\cos^2\alpha + j\cos\alpha\sin\alpha - j\cos\alpha\sin\alpha + \sin^2\alpha]$

$= s^2 + s(2\omega_o\cos\alpha) + \omega_o^2$

So for P_1 we get a term:

$s^2 + s(2\omega_o 0.309) + \omega_o^2$
$\quad = s^2 + 0.618\,\omega_o s + \omega_o^2$

For P_2 we get:

$s^2 + 1.618\,\omega_o s + \omega_o^2$

For P_3: $(s+\omega_o)$

$\therefore$ The denominator of $T(s)$ is given by

$$D(s) = (s+\omega_o)(s^2 + 0.618\omega_o s + \omega_o^2)\times(s^2 + 1.618\omega_o s + \omega_o^2)$$

Case (a) - If all the zeros are @ ∞, the numerator is a constant

$|T(0)| = \frac{k}{\omega_o^5} = 1$ for unity gain at DC

$\therefore k = \omega_o^5$

$$T(s) = \frac{k}{D(s)} = \frac{\omega_o^5}{D(s)}$$

where $D(s)$ is given above.

Case (b) - For all zeros at 0, the numerator is given by ks^5

A $s = j\infty$ $\quad |T(s\to j\infty)| = \frac{k}{1} = 1$

$$\therefore T(s) = \frac{s^5}{D(s)}$$

12.9

Poles at -1 and $-0.5 \pm j0.8$ gives a denominator:

$D(s) = (s+1)(s+0.5-j0.8)(s+0.5+j0.8)$

$\quad = (s+1)(s^2 + 2(0.5)s + 0.5^2 + 0.8^2)$

$\quad = (s+1)(s^2 + s + 0.89)$

Zeros at ∞ and $\pm j2$ give a numerator:

$N(s) = k(s+j2)(s-j2) = k(s^2+4)$

Note there is one zero at ∞ because Degree(D(s)) - Degree(N(s)) = 1

$$T(s) = \frac{k(s^2+4)}{(s+1)(s^2+s+0.89)}$$

$|T(j0)| = \frac{k(4)}{0.89} = 1 \quad \because$ DC gain $= 1$

$\Rightarrow k = 0.2225$

$$\therefore T(s) = \frac{0.2225(s^2+4)}{(s+1)(s^2+s+0.89)}$$

12.10

Numerator is given by

$$a_7 (s-0)\left(s^2+(10^3)^2\right)\left(s^2+(3\times10^3)^2\right) \times \left(s^2+(6\times10^3)^2\right)$$

$$= a_7 s\left(s^2+10^6\right)\left(s^2+9\times10^6\right)\left(s^2+36\times10^6\right)$$

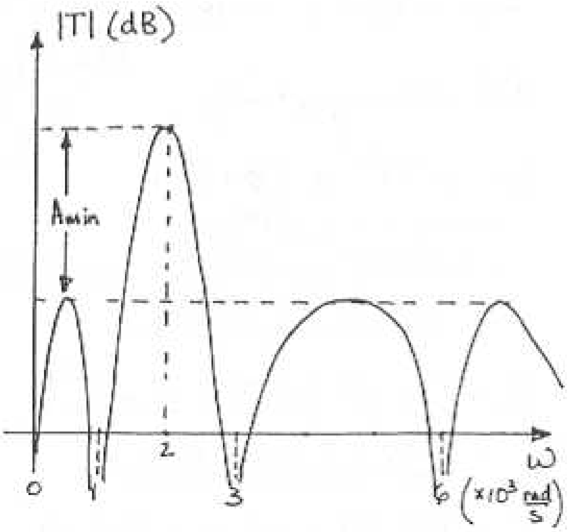
Degree of Numerator $\triangleq M = 7$

Degree of Denominator $\triangleq N$

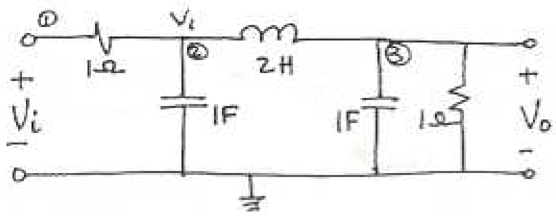
Given that there is one zero at ∞ :

$$N - M = 1 \implies \underline{\underline{N = 8}}$$

$$\therefore T(s) = \frac{a_7 s\left(s^2+10^6\right)\left(s^2+9\times10^6\right)\left(s^2+36\times10^6\right)}{s^8 + b_7 s^7 + b_6 s^6 + \cdots + b_0}$$



12.11



The easiest way to solve the circuit is to use nodal analysis at nodes ①, ②, ③

At node ③ $\quad \Sigma I = 0$

$$\frac{V_o}{1} + \frac{V_o}{1/s} + \frac{V_o - V_1}{2s} = 0$$

$$\therefore V_1 = V_o\left(2s^2 + 2s + 1\right) \qquad \text{Eq.(a)}$$

At node ② $\quad \Sigma I = 0$

$$\frac{V_1 - V_i}{1} + \frac{V_1}{1/s} + \frac{V_1 - V_o}{2s} = 0$$

$$\therefore V_1\left(2s^2+2s+1\right) = V_o + 2sV_i \qquad \text{Eq.(b)}$$

(a) → (b)

$$V_o\left(2s^2+2s+1\right)^2 = V_o + 2sV_i$$

$$V_o\left(4s^4 + s^3(4+4) + s^2(2+4+2) + s(2+2) + 1\right) = V_o + 2sV_i$$

$$\frac{V_o(s)}{V_i(s)} \triangleq T(s) = \frac{2s}{4s^4 + 8s^3 + 8s^2 + 4s}$$

$$\underline{\underline{T(s) = \frac{0.5}{s^3 + 2s^2 + 2s + 1}}}$$

Poles are given by:

$$s^3 + 2s^2 + 2s + 1 = 0$$

$$(s+1)(s^2 + s + 1) = 0$$

$\therefore$ Poles are $\quad s = -1 \quad$ and

$$\underline{\underline{s = -\frac{1}{2} \pm j\,\sqrt{3}/2}}$$

12.12

$A_{max} = 1\,dB$, $A_{min} = 20\,dB$, $\omega_s/\omega_p = 1.3$

Using:

$$A(\omega_s) = 10\log\left[1 + \epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right] \qquad \text{Eq (12.15)}$$

$$= A_{min}$$

CONT

$$\epsilon = \left[10^{1/10} - 1\right]^{1/2} = 0.5088$$

$$A_{min} = 10 \log\left[1 + \epsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$$

$$10^{A_{MIN}/10} - 1 = \epsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2N}$$

$$\log\left(10^{A_{min}/10} - 1\right) = \log\left(\epsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2N}\right)$$

$$N = \frac{\log\left[\left(10^{A_{MIN}/10} - 1\right)/\epsilon^2\right]}{2\log(\omega_s/\omega_p)}$$

$= 11.3 \Rightarrow$ choose $\underline{\underline{N = 12}}$

The actual value of stopband attenuation can be calculated using the integer value of N:

$$A(\omega_s) = 10 \log\left[1 + \epsilon^2 \left(\frac{\omega_s}{\omega_p}\right)^{2N}\right] ; \quad N = 12$$

$= \underline{\underline{27.35\,dB}}$ actual attenuation

If the stopband specs are to be met exactly we need to find Amax.

Eq. 12.15 can be rearranged to give

$$\epsilon^2 = \frac{10^{A_{min}/10} - 1}{(\omega_s/\omega_p)^{2N}} \qquad A_{min} = 20, \; N = 12$$

$= 0.1824$

$$\therefore A_{max} = 10 \log(1 + \epsilon^2)$$

$= \underline{\underline{0.73\,dB}}$

$\boxed{12.13}$

$N = 7$, $A_{max} = 3\,dB$

We want attenuation at $\omega = 1.6\,\omega_p$ or $\frac{\omega}{\omega_p} = 1.6$

$$\epsilon = \sqrt{10^{A_{max}/10} - 1} = 0.998$$

$$A(\omega) = 10 \log\left[1 + \epsilon^2 \left(\frac{\omega}{\omega_p}\right)^{2N}\right]$$

$$= 10 \log\left[1 + 0.998^2 (1.6)^{14}\right]$$

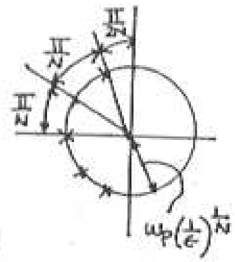
$= \underline{\underline{28.56\,dB}}$

$\boxed{12.14}$

$\omega_p = 10^3$ rad/s, $N = 5$

$A_{max} = 1\,dB \Rightarrow \epsilon = 0.5088$

Find solution graphically



$$P_1 = \omega_p \left(\frac{1}{\epsilon}\right)^{1/N} \angle \left(\frac{\pi}{2} + \frac{\pi}{2N}\right)$$

$= 873.59 \angle (6\pi/10)$

$= 873.59 \left[\cos(6\pi/10) \pm j \sin\left(\frac{6\pi}{10}\right)\right]$

$= \underline{-269.96 \pm j\,830.84}$

$$P_2 = 873.59 \angle \left[\frac{\pi}{2} + \frac{\pi}{2N} + \frac{\pi}{N}\right]$$

$= \underline{\underline{-706.75 \pm j\,513.49}}$

$P_3 = 873.59 \angle \pi = \underline{\underline{-873.59}}$

$\boxed{12.15}$

$f_p = 10\,kHz$ $\quad \frac{\omega_s}{\omega_p} = 1.5 \quad A_{MIN} = 15\,dB$

$f_s = 15\,kHz \quad A_{max} = 2\,dB$

$$\epsilon^2 = 10^{A_{max}/10} - 1 \Rightarrow \epsilon = 0.76478$$

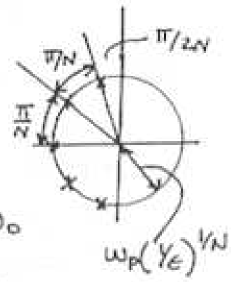
CONT.

Manipulation Eq (12.15) we get:

$$N = \frac{\log\left[\left(10^{A_{min}/10} - 1\right)/\epsilon^2\right]}{2\log(\omega_s/\omega_p)} = 4.88$$

$\therefore$ Use $\underline{\underline{N = 5}}$

Finding naturalmodes graphically :-



$$\text{radius} = \omega_p\left(\frac{1}{\epsilon}\right)^{1/N} \triangleq \omega_0$$

$$\omega_0 = 6.629 \times 10^4$$

$$P_1 = \omega_0 \measuredangle \left(\pi/2 + \pi/2N\right) = \omega_0 \measuredangle \left(6\pi/10\right)$$
$$= \omega_0\left(\cos\left(\tfrac{6\pi}{10}\right) \pm j\sin\left(\tfrac{6\pi}{10}\right)\right)$$
$$= \underline{\underline{\omega_0\left(-0.309 \pm j0.951\right)}}$$

$$P_2 = \omega_0\left(\cos\tfrac{8\pi}{10} \pm j\sin\tfrac{8\pi}{10}\right)$$
$$= \underline{\underline{\omega_0\left(-0.809 \pm j0.588\right)}}$$

$$P_3 = \omega_0\left(\cos\pi \pm j\sin\pi\right) = \underline{\underline{-\omega_0}}$$

Given a natural mode $-\alpha \pm j\beta$, the following term results

$$(s + \alpha + j\beta)(s + \alpha - j\beta)$$
$$= s^2 + 2\alpha s + \alpha^2 + \beta^2$$
$$= \underline{s^2 + 2\,\mathrm{Re}[P]\,s + |P|^2}$$

Also, note that for a Butterworth, all natural modes have a magnitude of ω_0.

P_1 yields: $s^2 + 0.618\omega_0 s + \omega_0^2$
P_2 yields: $s^2 + 1.618\omega_0 s + \omega_0^2$
P_3 yields: $s + \omega_0$

$$\therefore T(s) = \frac{k}{(s+\omega_0)(s^2 + .618\omega_0 s + \omega_0^2)} \times \frac{1}{s^2 + 1.618\omega_0 s + \omega_0^2}$$

For unity dc gain

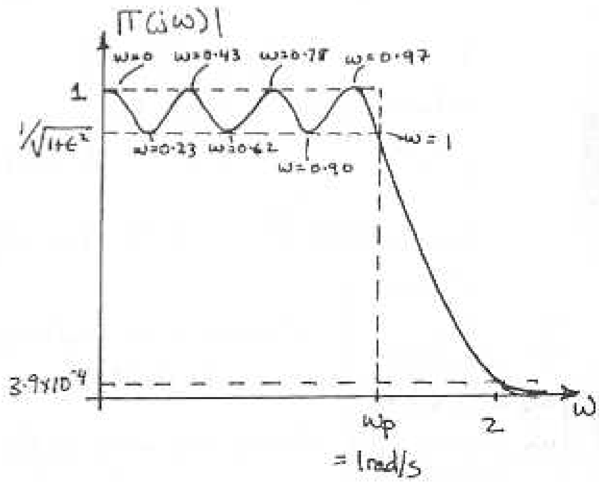
$$|T(j0)| = \frac{k}{\omega_0^5} = 1 \Rightarrow k = \omega_0^5$$

$$\therefore T(s) = \underline{\underline{\frac{\omega_0^5}{(s+\omega_0)(s^2 + .618\omega_0 s + \omega_0^2)}}} \times \underline{\underline{\frac{1}{(s^2 + 1.618\omega_0 s + \omega_0^2)}}}$$

For attenuation at 20kHz use Eq (12.15) with $\frac{\omega_s}{\omega_p} = \frac{20}{10} = 2$

$$A(\omega_s) = 10\log\left[1 + \epsilon^2\left(\frac{\omega_s}{\omega_p}\right)^{2N}\right]$$
$$= \underline{27.8\,dB}$$

12.16



Given $A_{max} = 1dB \Rightarrow \epsilon = 0.5088$

CONT.

Using Eq 12.18

$$|T(j\omega)| = \left[1+\epsilon^2 \cos^2\left(N \cos^{-1}\left(\frac{\omega}{\omega_p}\right)\right)\right]^{-1/2}$$

for $\omega \le \omega_p$

If $|T(j\omega)| = 1$

$$1 = 1+\epsilon^2 \cos^2\left(N \cos^{-1}(\omega/\omega_p)\right) \quad \omega_p = 1$$

$$N\cos^{-1}(\omega/1) = \cos^{-1}(0)$$

$$\cos^{-1}(\omega) = \frac{2i+1}{2N}\pi$$

ω's repeat after this value

$$\therefore \omega_i = \cos\left[\frac{2i+1}{2N}\pi\right] \quad i = 0, 1, \ldots, \frac{N-1}{2}$$

$$\boxed{\begin{aligned} \omega_0 &= 0.9749 \\ \omega_1 &= 0.7818 \\ \omega_2 &= 0.4339 \\ \omega_3 &= 0 \end{aligned}}$$

ω values at which $|T| = 1$

note $\omega_4 = -0.4339$
$= -\omega_2$!

If $|T| = 1/\sqrt{1+\epsilon^2}$, then

$$1/\sqrt{1+\epsilon^2} = \left[1+\epsilon^2 \cos^2\left(N \cos^{-1}(\omega/\omega_p)\right)\right]^{-\frac{1}{2}}$$

$$1 = \cos\left(N \cos^{-1}\left(\frac{\omega}{\omega_p}\right)\right)$$

$$N\cos^{-1}(\omega) = \cos^{-1}(0)$$

$$= i\pi \quad i = 0, 1, 2 \ldots$$

$$\omega_i = \cos\left[\frac{i\pi}{N}\right] \quad i = 0, 1, 2, \ldots \frac{N}{2}$$

$$\boxed{\begin{aligned} \omega_0 &= 1.0 \\ \omega_1 &= 0.9010 \\ \omega_2 &= 0.6235 \\ \omega_3 &= 0.2252 \end{aligned}}$$

ω values at which $|T| = (1+\epsilon^2)^{-1/2}$

Note $\omega_4 = -0.2252$
$= -\omega_3$!

To find $|T(j2)|$ use Eq(12.19) since $\omega > \omega_p$

$$|T(j\omega)| = \left[1+\epsilon^2 \cosh^2\left(N \cosh^{-1}\left(\frac{\omega}{\omega_p}\right)\right)\right]^{-\frac{1}{2}}$$

$$= \left[1+0.5088^2 \cosh^2\left(7 \cosh^{-1} 2\right)\right]^{-\frac{1}{2}}$$

$$= \underline{\underline{3.898 \times 10^{-4}\ \text{V/V}}}$$

$$|T|_{dB} = \underline{-68.2\ \text{dB}}$$

For roll-off consider

$$T(s) = \frac{k}{s^7 + b_6 s^6 + \ldots + b_0}$$

for $\omega \gg \omega_p \quad T(j\omega) \simeq \dfrac{k}{\omega^7}$

$\therefore$ Rolloff is $\frac{1}{2^7}$ or $20\log(1/2^7)$ per octave $= \underline{-42\ \text{dB/octave}}$.

12.17

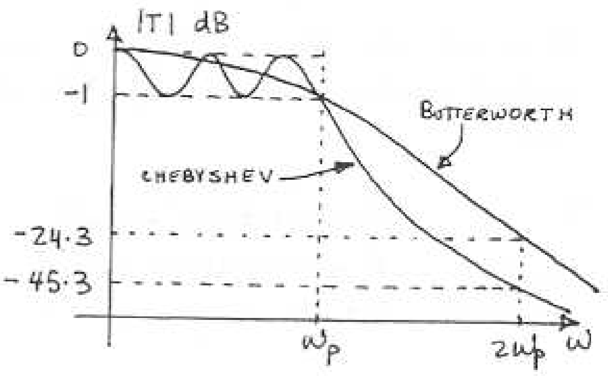
$\omega_s/\omega_p = 2 \quad A_{max} = 1\text{dB} \Rightarrow \epsilon = \sqrt{10^{\frac{A_{max}}{10}} - 1}$
$= 0.5088$

$$|T_B| = \left[1+\epsilon^2 (\omega_s/\omega_p)^{2N}\right]^{-1/2}$$

$$|T_C| = \left[1+\epsilon^2 \cosh^2\left(N \cosh^{-1}\left[\frac{\omega_s}{\omega_p}\right]\right)\right]^{-1/2}$$

$|T_B| = 6.13 \times 10^{-2} \Rightarrow \underline{-24.3\text{dB}}$

$|T_C| = 5.43 \times 10^{-3} \Rightarrow \underline{\underline{-45.3\text{dB}}}$



12.18

$f_p = 3.4\,kHz$ $\quad A_{max} = 1dB \Rightarrow \epsilon = 0.5088$

$f_s = 4\,Khz$ $\quad A_{min} = 35\,dB$

$\omega_s/\omega_p = 1.176$

Using Eq (12.22):

$$A(\omega_s) = 10\log\left[1+\epsilon^2\cosh^2\left(N\cosh^{-1}\left(\frac{\omega_s}{\omega_p}\right)\right)\right]$$

& trying different values for N

N	$A(\omega_s)$
8	28.8 dB
9	33.9 dB
10	38.98 dB

$\therefore$ Use $N=10$

Excess attenuation $= 39-35 = 4dB$

Poles are given by:

$$P_k = -\omega_p \sin\left(\frac{2k-1}{N}\cdot\frac{\pi}{2}\right)\sinh\left(\frac{1}{N}\sinh^{-1}\left(\frac{1}{\epsilon}\right)\right)$$
$$+j\,\omega_p\cos\left(\frac{2k-1}{N}\cdot\frac{\pi}{2}\right)\cosh\left(\frac{1}{N}\sinh^{-1}\left(\frac{1}{\epsilon}\right)\right)$$

for $k = 1, 2, \ldots, N$.

Since $\epsilon = 0.5088$ and $N=10$

$\sinh\left(1/N \sinh^{-1}(1/\epsilon)\right) = 0.1433$

$\cosh\left(1/N \sinh^{-1}(1/\epsilon)\right) = 1.010$

$$\therefore P_1 = \omega_p\left[-0.1433\sin\left(\tfrac{\pi}{20}\right) + j1.010\cos\left(\tfrac{\pi}{20}\right)\right]$$
$$= \omega_p(-0.0224 + j0.9978)$$

$P_2 = \omega_p(-0.0650 + j0.900)$

$P_3 = \omega_p(-0.1013 + j0.7143)$

$P_4 = \omega_p(-0.1277 + j0.4586)$

$P_5 = \omega_p(-0.1415 + j0.1580)$

Now it should be realized that the remaining poles are complex conjugates of the above.

Pole-pair P_1 & P_1^* give a factor:

$$s^2 + 2(0.0224)\omega_p s + \omega_p^2(0.0224^2 + 0.9978^2)$$
$$= \underline{s^2 + 0.0448\omega_p s + 1.023\omega_p^2}$$

i.e. this factor is from $(s-P_1)(s-P_1^*)$

P_2 yields: $\underline{s^2 + 0.130\omega_p s + 0.902\omega_p^2}$

P_3 yields: $\underline{s^2 + 0.203\omega_p s + 0.721\omega_p^2}$

P_4 yields: $\underline{s^2 + 0.255\omega_p s + 0.476\omega_p^2}$

P_5 yields: $\underline{s^2 + 0.283\omega_p s + 0.212\omega_p^2}$

Now T(s) is given by

$$T(s) = \frac{k\,\omega_p^{10}}{\epsilon\, 2^9 (s-P_1)(s-P_1^*)\cdots(s-P_5)(s-P_5^*)}$$

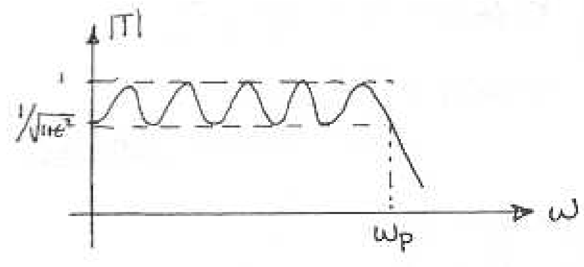
where the second order terms of the denominator are given above.

k is the dc gain

$\therefore$ We want the dc gain to be

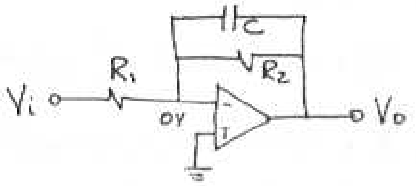
$$k = \frac{1}{1+\epsilon^2} = \underline{0.8913}$$

$\omega_p = \underline{2\pi \times 3400}$



12.19

$f_o = 10\,kHz$ DC gain $= 10$ $R_{in} = 10\,k\Omega$



$R_{in} = R_1 = \underline{\underline{10\,k\Omega}}$

DC gain $= -R_2/R_1 = -10$

$R_2 = 10R_1 = \underline{\underline{100\,k\Omega}}$

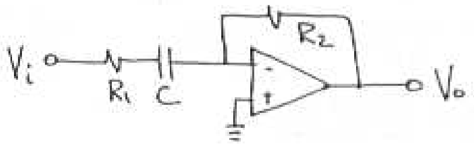
$R_2 C = 1/\omega_o$

$C = \dfrac{1}{\omega_o R_2} = \dfrac{1}{2\pi\, 10^4 \times 100\times10^3}$

$= \underline{\underline{0.159\,nF}}$

12.20

$f_o = 100\,kHz$ $R_i(\infty) = 100\,k\Omega$ $|T(\infty)| = 1$



$R_i(\infty) = R_1 = \underline{\underline{100\,k\Omega}}$

$|T(\infty)| = R_2/R_1 = 1$

$R_2 = R_1 = \underline{\underline{100\,k\Omega}}$

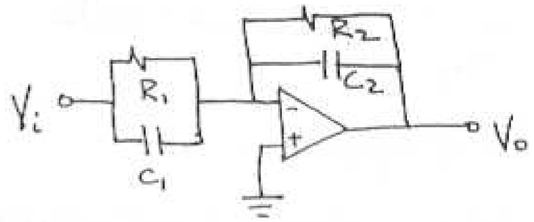
$CR_1 = 1/\omega_o$

$C = \dfrac{1}{\omega_o R_1} = \dfrac{1}{2\pi\, 100\times10^3 \times 100\times10^3}$

$= \underline{\underline{15.9\,nF}}$

12.21

Use general first-order circuit:



- Zero at 1kHz ; Pole at 100kHz
- $|T(0)| = 1$; $R_i(0) = 1\,k\Omega$

Thus: $R_i(DC) = \underline{\underline{R_1 = 1\,k\Omega}}$

$T(DC) = -R_2/R_1 = -1$

$R_2 = R_1 = \underline{\underline{1\,k\Omega}}$

For a pole at 100kHz

$C_2 R_2 = \dfrac{1}{\omega_o} \Rightarrow C_2 = \dfrac{1}{2\pi f_o R_2}$

$= \underline{\underline{1.59\,nF}}$

For the circuit $T(s) = \dfrac{a_1 s + a_0}{s + \omega_o}$

Thus the zero at $-a_0/a_1 = -2\pi\, 10^3$

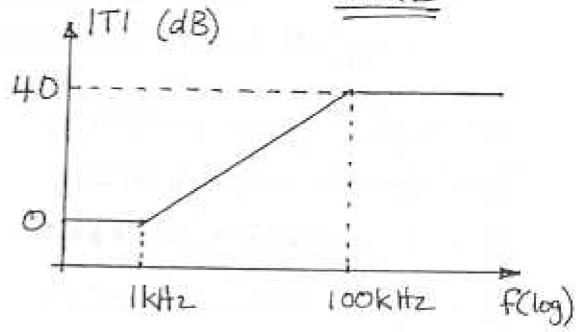
$C_1 R_1 = a_1/a_0$

$C_1 = \dfrac{1}{2\pi\, 10^3 R_1}$

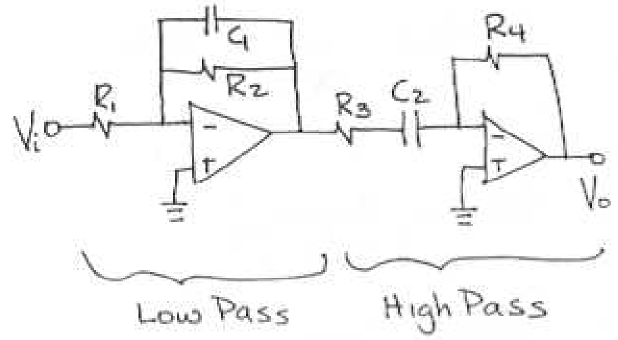
$= \underline{\underline{159\,nF}}$

High freq gain $= \dfrac{-C_1}{C_2} = \underline{\underline{-100}}$

$= \underline{\underline{40\,dB}}$



12.22



gain $= 10^{12/20} = 3.98 \approx \underline{4}$

want $R_i = R_1$ large

$\therefore R_1 = \underline{\underline{100k\Omega}}$

Total gain $= A_{LP} A_{HP} = 4$

$A_{LP} = -R_2/R_1 \implies R_2 = -A_{LP} R_1$ and $R_2 \leq 100k\Omega$

$\therefore$ make $A_{LP} = -1 \quad A_{HP} = -4$

$R_2 = \underline{\underline{100k\Omega}}$

$$R_2 C_1 = \frac{1}{\omega_{0,LP}}$$

$$C_1 = \frac{1}{2\pi f_{0,LP} R_2} = \frac{1}{2\pi (10\times10^3)\, 100\times10^3}$$

$$= \underline{\underline{0.159nF}}$$

$$\left.\begin{aligned} A_{HP} &= -R_4/R_3 = -4 \\ R_4 &= 4R_3 \end{aligned}\right\} \text{make } R_4 = \underline{\underline{100k\Omega}},\ R_3 = \underline{\underline{25k\Omega}}$$

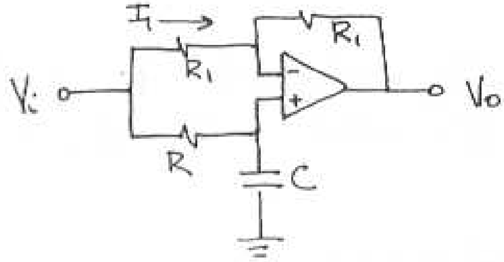
Now $R_3 C_2 = 1/\omega_{0,HP}$

$$C_2 = \frac{1}{2\pi f_{0,HP} R_3}$$

$$= \frac{1}{2\pi (100\times10^3)\, 25\times10^3}$$

$$= \underline{\underline{63.7\,nF}}$$

12.23



At +ve terminal

$$V_+ = \frac{1/sC}{1/sC + R_1} V_i$$

$$= \frac{1}{1+s\tau} V_i \qquad \underline{\tau = RC}$$

$V_- = V_+$ due to virtual short between terminals.

$$\therefore I_1 = \left(V_i - \frac{1}{1+s\tau} V_i\right)\frac{1}{R_1}$$

$$V_0 = V_- - I_1 R_1$$

$$= \frac{V_i}{1+s\tau} - \left(V_i - \frac{V_i}{1+s\tau}\right)\frac{R_1}{R_1}$$

$$\frac{V_0}{V_i} = \frac{1-(1+s\tau)+1}{1+s\tau} = \frac{1-s\tau}{1+s\tau}$$

$$= \frac{\omega_0 - s}{\omega_0 + s} \qquad \omega_0 = \frac{1}{\tau}$$

$$= \underline{\underline{-\frac{s-\omega_0}{s+\omega_0} = T(s)}}$$

$$T(j\omega) = -\frac{j\omega - \omega_0}{j\omega + \omega_0}$$

$$\phi(\omega) = 180^\circ + \mathrm{Tan}\left(\frac{\omega}{-\omega_0}\right) - \mathrm{Tan}^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$= 360^\circ - 2\tan^{-1}\left(\frac{\omega}{\omega_0}\right) \qquad \because \tan^{-1}\left(\frac{\omega}{-\omega_0}\right) = 180 - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$= -2\tan^{-1}(\omega/\omega_0)$$

Now this equation can be rearranged:

$$\frac{\omega}{\omega_0} = \tan(-\phi/2) \Leftarrow \omega_0 = \frac{1}{\tau} = \frac{1}{RC}$$

$$RC\omega = \tan(-\phi/2)$$

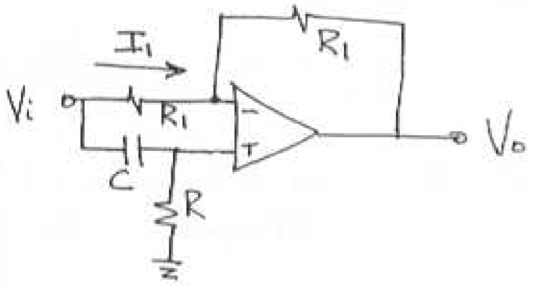
CONT.

$$\therefore R = \frac{\tan(-\phi/2)}{C\omega} = 10^4 \tan(-\phi/2)$$

$\phi = -30°, -60°, -90°, -120°, -150°$

$R = 2.68k\Omega, 5.77k\Omega, 10k\Omega, 17.32k\Omega, 37.32 k\Omega$

12.24



$$V_+ = \frac{R}{R + 1/sC} V_i = \frac{s}{s+\omega_0} V_i$$

where $\omega_0 = \frac{1}{RC}$

$$I_1 = \frac{V_i - (s/s+\omega_0)V_i}{R_1}$$

$$V_o = \frac{s}{s+\omega_0} V_i - I_1 R_1$$

$$= \frac{s}{s+\omega_0} V_i - V_i\left(1 - \frac{s}{s+\omega_0}\right)$$

$$\frac{V_o}{V_i} = \frac{2s - s - \omega_0}{s + \omega_0} = \frac{s - \omega_0}{s + \omega_0}$$

Now:

NOTE $\tan^{-1}\frac{\omega}{-\omega_0} = 180° - \tan^{-1}\frac{\omega}{\omega_0}$

$$\phi(\omega) = \tan^{-1}\left(\frac{\omega}{-\omega_0}\right) - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$= 180 - \tan^{-1}\left(\frac{\omega}{\omega_0}\right) - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

$$= 180 - 2\tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

Clearly $\phi(0) = 180°$ & $\phi(\omega \to \infty) = 0°$

12.25

Low Pass $\omega_0 = 10^3$ rad/s

$Q = 1$

DC gain $= 1$

$$T(s) = \frac{a_0}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$T(0) = a_0/\omega_0^2 = 1$$

$$a_0 = \omega_0^2 = 10^6$$

$$\therefore T(s) = \frac{10^6}{s^2 + 10^3 s + 10^6}$$

$$\omega_{max} = \omega_0\sqrt{1 - 1/2Q^2}$$

$$= \frac{\omega_0}{\sqrt{2}}$$

$$= 0.707 \text{ rad/s}$$

$$|T_{max}| = \frac{|a_0|\,Q}{\omega_0^2\sqrt{1 - \frac{1}{4Q^2}}} \Leftarrow a_0 = \omega_0^2$$

$$= \frac{|\omega_0^2|}{\omega_0^2\sqrt{3/4}}$$

$$= 2/\sqrt{3}$$

$$= 1.15 \text{ V/V}$$

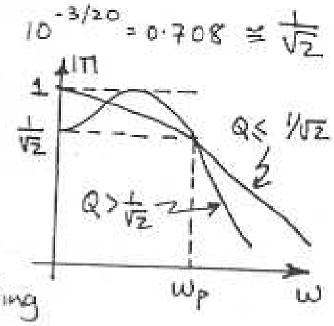
$$= 1.21 \text{ dB}$$

12.26

$\omega_p = 1$ rad/s

$A_{max} = 3$dB

$10^{-3/20} = 0.708 \cong \frac{1}{\sqrt{2}}$



There are many Q-values which may be used

$Q \leq 1/\sqrt{2}$ - no peaking

$Q > 1/\sqrt{2}$ - peaking

CONT.

Solution 1 $\quad Q \leq 1/\sqrt{2}$

For $Q = 1/\sqrt{2}$ the response is maximally flat. Because this is desirable, use: $\underline{\underline{Q = \frac{1}{\sqrt{2}}}}$

$$T(s) = \frac{a_0}{s^2 + s\omega_0\sqrt{2} + \omega_0^2}$$

$$|T(0)| = \frac{a_0}{\omega_0^2} = 1$$

$$\underline{\underline{a_0 = \omega_0^2}}$$

$$|T(j1)|^2 = \frac{\omega_0^2}{(\omega_0^2-1) + 2\omega_0^2} = \left(\frac{1}{\sqrt{2}}\right)^2$$

$$\omega_0 = 1 \text{ rad/s}$$

$$\therefore \underline{\underline{T_1(s) = \frac{1}{s^2 + \sqrt{2}s + 1}}}$$

Solution 2 $\quad Q > 1/\sqrt{2}$

From the figure: $|T(0)| = 1/\sqrt{2} = \frac{a_0}{\omega_0^2}$

$$\therefore \underline{\underline{a_0 = \omega_0^2/\sqrt{2}}}$$

$$\text{Now } |T|_{max} = \frac{|a_0|Q}{\omega_0^2\sqrt{1 - 1/4Q^2}} = 1$$

$$\frac{Q}{\sqrt{2}\sqrt{1 - 1/4Q^2}} = 1$$

$$Q = \sqrt{2}\sqrt{1 - 1/4Q^2}$$

$$\therefore Q^2 = 2\left(1 - \frac{1}{4Q^2}\right)$$

$$= 2 - \frac{1}{2Q^2}$$

$$Q^4 - 2Q^2 + \tfrac{1}{2} = 0$$

Solving for Q^2 gives:-

$$Q^2 = 1 \pm \sqrt{2}$$

ASIDE:

$\because Q > 1/\sqrt{2}$

$Q^2 > 1/2$

$4Q^2 > 2$

$\frac{1}{4Q^2} < \frac{1}{2}$

$\therefore 1 - \frac{1}{4Q^2} > 1/2$

$\therefore \left|1 - \frac{1}{4Q^2}\right| = 1 - \frac{1}{4Q^2}$

$\Rightarrow Q = 0.5412$ or 1.3066

$\because Q > \frac{1}{\sqrt{2}}$ use $\underline{\underline{Q = 1.3066}}$

Now at the passband edge

$$|T(j1)| = 1/\sqrt{2}$$

$$|T(j1)|^2 = \frac{(\omega_0^2/\sqrt{2})^2}{(\omega_0^2-1)^2 + \frac{\omega_0^2}{Q^2}} = \frac{1}{2}$$

$$\frac{\omega_0^4}{\cancel{2}} = \frac{1}{\cancel{2}}\left[\cancel{\omega_0^4} - 2\omega_0^2 + 1 + \frac{\omega_0^2}{Q^2}\right]$$

$$\omega_0^2(2 - 1/Q^2) = 1$$

$$\underline{\omega_0 = 0.841}$$

$$\therefore T_2(s) = \frac{\omega_0^2/\sqrt{2}}{s^2 + \omega_0/Q\, s + \omega_0^2}$$

$$= \underline{\underline{\frac{0.5}{s^2 + 0.644s + 0.707}}}$$

If $\omega_s = 2$

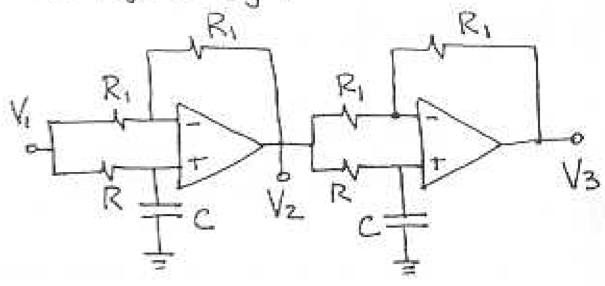
$|T_1(j2)| = 0.242 \quad |T_2(j2)| = 0.1414$

$\therefore \underline{\underline{A_{min,1} = -12.3\text{dB}}} \quad \underline{\underline{A_{MIN,2} = -17\text{dB}}}$

12.27

V_2 lags V_1 by $120°$

V_3 lags V_2 by $120°$



$\omega = 2\pi 60$ & $C = 1\,\mu F$

$$T(s) = \frac{s - \omega_0}{s + \omega_0} \qquad \omega_0 = \frac{1}{RC}$$

CONT.

$$\phi(\omega) = 180^\circ + \tan^{-1}\left(\frac{\omega}{-\omega_0}\right) - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$$

Sub: $\tan\left(\frac{\omega}{-\omega_0}\right) = 180 - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$

$$\Rightarrow \phi(\omega) = -2\tan(\omega/\omega_0)$$

Now $\phi = -120^\circ$ at $\omega = 2\pi 60$

$$-120 = -2\tan(\omega RC)$$

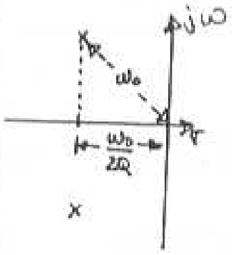
$$-60 = -\tan^{-1}(2\pi 60 \times R \times 10^{-6})$$

$$\underline{\underline{R = 4.59\ k\Omega}}$$

R_1 can be arbitrarily chosen

use $\underline{\underline{R_1 = 10k\Omega}}$

12.28



Natural Modes:

$-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$

$$\omega_0 = \sqrt{(1/2)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \underline{1.0}$$

$$\frac{\omega_0}{2Q} = \frac{1}{2} \Rightarrow \frac{\omega_0}{Q} = 1$$

$$T(s) = \frac{a_2 s^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2} = \frac{a_2 s^2}{s^2 + s + 1}$$

$$|T(j\infty)| = a_2 = 1$$

$$\therefore\ \underline{\underline{T(s) = \frac{s^2}{s^2 + s + 1}}}$$

12.29

For a 2nd-order bandpass

$$T(s) = \frac{a_1 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$T(j\omega) = \frac{j\omega a_1}{(\omega_0^2 - \omega^2) + \frac{j\omega\omega_0}{Q}}$$

$$|T(j\omega)| = \frac{a_1\omega}{\left[(\omega_0^2 - \omega^2)^2 + \frac{\omega^2\omega_0^2}{Q^2}\right]^{1/2}}$$

Part (a):

$$|T(j\omega_1)| = |T(j\omega_2)|$$

$$\frac{a_1\omega_1}{\sqrt{(\omega_0^2 - \omega_1^2)^2 + \left(\frac{\omega_1\omega_0}{Q}\right)^2}} = \frac{a_1\omega_2}{\sqrt{(\omega_0^2 - \omega_2^2)^2 + \left(\frac{\omega_2\omega_0}{Q}\right)^2}}$$

$$\omega_1^2\left[(\omega_0^2 - \omega_2^2)^2 + \cancel{\left(\frac{\omega_2\omega_0}{Q}\right)^2}\right] = \omega_2^2\left[(\omega_0^2 - \omega_1^2)^2 + \cancel{\left(\frac{\omega_1\omega_0}{Q}\right)^2}\right]$$

$$\omega_1^2(\omega_0^4 - \cancel{2\omega_0^2\omega_2^2} + \omega_2^4) =$$

$$\omega_2^2(\omega_0^4 - \cancel{2\omega_0^2\omega_1^2} + \omega_1^4)$$

$$\omega_1^2\omega_0^4 + \omega_1^2\omega_2^4 = \omega_2^2\omega_0^4 + \omega_2^2\omega_1^4$$

$$\omega_0^4(\omega_1^2 - \omega_2^2) = \omega_2^2\omega_1^4 - \omega_1^2\omega_2^4$$

$$\omega_0^4(\omega_1^2 - \omega_2^2) = \omega_2^2\omega_1^2(\omega_1^2 - \omega_2^2)$$

$$\omega_0^4 = \omega_1^2\omega_2^2$$

$$\underline{\underline{\omega_0^2 = \omega_1\omega_2}}\quad \text{Q.E.D.}$$

CONT.

For Fig. 12.4 : $\omega_{p1} = 8100$ rad/s
$\omega_{p2} = 10\,000$ rad/s
$A_{max} = 1$ dB

$$\omega_0^2 = (8100)(10\,000)$$

$$\omega_0 = \underline{9000 \text{ rad/s}}$$

$$|T(j\omega_{p1})| = |T(j\omega_{p2})| = 10^{-1/20} = \underline{0.8913}$$

$$|T(j\omega_0)| = \frac{\omega_0 a_1}{\sqrt{(\omega_0^2-\omega_0^2)^2 + \left(\frac{\omega_0^2}{Q}\right)^2}} = 1$$

$$\Rightarrow \frac{\omega_0 a_1}{\omega_0^2/Q} = 1$$

$$\therefore \frac{Q a_1}{\omega_0} = 1 \Rightarrow a_1 = \frac{\omega_0}{Q}$$

$$|T(j\omega_{p1})|^2 = |T(j0.9\omega_0)|^2 = 0.8913^2$$

$$\frac{(\omega_0/Q)^2(0.9\omega_0)^2}{\left(\omega_0^2 - (0.9\omega_0)^2\right)^2 + \left(\frac{0.9\omega_0}{Q}\right)^2} = 0.8913^2$$

$$\left(\frac{\omega_0}{Q}(.9\omega_0)\right)^2 = 0.8913^2\left[\left(\omega_0^2 - (0.9\omega_0)^2\right)^2 + \left(\frac{0.9\omega_0^2}{Q}\right)^2\right]$$

$$\frac{0.81\omega_0^4}{Q^2} = 0.8913^2\left[\omega_0^4(1-0.81)^2 + \frac{0.81\omega_0^4}{Q^2}\right]$$

$$\frac{0.81\omega_0^4}{Q^2}\left(1-0.8913^2\right) = 0.8913^2\omega_0^4 \times (1-0.81)^2$$

SUB $\omega_0 = 9000$ gives

$$\underline{Q = 2.41}$$

Now $a_1 = \frac{\omega_0}{Q} = 0.415\omega_0$

$$\therefore T(s) = \frac{0.415\omega_0 s}{s^2 + 0.415\omega_0 s + \omega_0^2}$$

If $\omega_{s1} = 3000$ rad/s

$$|T(j3000)| = \frac{0.415\omega_0(3000)}{\sqrt{(\omega_0^2 - 3000^2)^2 + (\omega_0 3000 \times .415)^2}}$$

$$= 0.1537$$

$$\therefore A_{MIN} = -20\log(0.1537) = \underline{16.3\text{dB}}$$

Now ω_{s1} and ω_{s2} are geometrically symmetrical about ω_0:

$$\omega_{s1}\omega_{s2} = \omega_0^2$$

$$\omega_{s2} = \frac{9000^2}{3000} = \underline{27\,000 \text{ rad/s}}$$

$\boxed{12.30}$

From exercise 12.15

$$Q = \frac{\omega_0}{BW\sqrt{10^{A/10} - 1}} \quad \begin{cases} \omega_0 = 2\pi(60) \\ BW = 2\pi 6 \\ A = 20\text{dB} \end{cases}$$

$$= \underline{1.005}$$

$$T(s) = a_2 \frac{s^2 + \omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$|T(0)| = \frac{a_2\omega_0^2}{\omega_0^2} = 1 \leftarrow \text{DC Gain}$$

CONT.

$\underline{a_2 = 1}$

$$T(s) = \frac{s^2 + (2\pi 60)^2}{s^2 + s\,\frac{2\pi 60}{1.005} + (2\pi 60)^2}$$

$$T(s) = \frac{s + 1.421\times10^5}{s^2 + 375.1s + 1.421\times10^5}$$

12.31

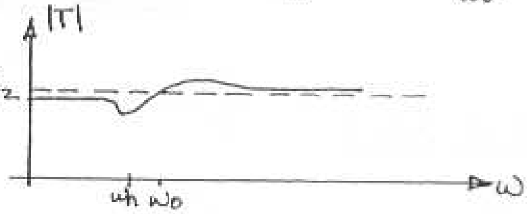
For All Pass:

$$T(s) = a_2\,\frac{s^2 - s\,\omega_0/Q + \omega_0^2}{s^2 + s\,\omega_0/Q + \omega_0^2}$$

If zero frequency < pole frequency

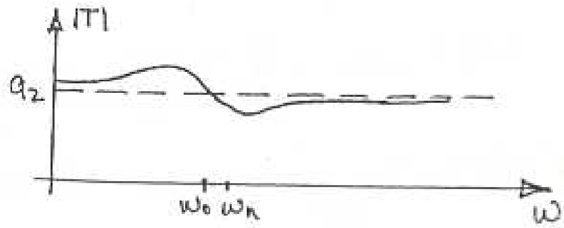
$$T(s) = a_2\,\frac{s^2 - s\,\omega_n/Q + \omega_n^2}{s^2 + s\,\omega_0/Q + \omega_0^2} \quad \omega_n < \omega_0$$

At DC: $|T| = a_2\,\frac{\omega_n^2}{\omega_0^2}$ where $\frac{\omega_n^2}{\omega_0^2} < 1$



If zero frequency > pole frequency then $\omega_n > \omega_0$

At DC: $|T| = a_2\,\omega_n^2/\omega_0^2$ where $\frac{\omega_n^2}{\omega_0^2} > 1$



12.32

$$T(s) = \frac{s^2 - s\,\omega_0/Q' + \omega_0^2}{s^2 + s\,\omega_0/Q_0 + \omega_0^2}\,a_2$$

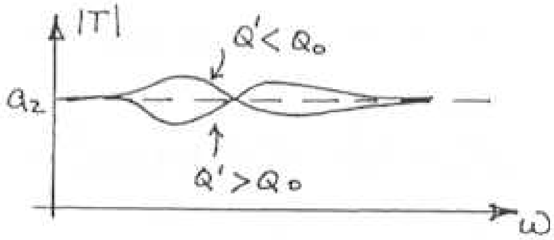
Zero Q < Pole Q $\Rightarrow Q' < Q_0$

At $\omega = \omega_0$:

$$|T| = \frac{a_2\,\omega_0^2/Q'}{\omega_0^2/Q_0} = \frac{a_2 Q_0}{Q'} > a_2$$

If $Q' > Q_0$

$$|T(j\omega_0)| = \frac{a_2 Q_0}{Q'} < a_2$$



12.33

$\omega_0 = 10^4$ rad/s, $Q = 2$, $R = 10k\Omega$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow \omega_0^2 = \frac{1}{LC} \Leftarrow L = \frac{R^2 C}{Q^2}$$

$$= \frac{Q^2}{R^2 C^2}$$

$$C = \frac{Q}{R\omega_0} = \underline{\underline{20\text{nF}}}$$

$$L = \frac{1}{C\omega_0^2} = \underline{\underline{500\text{mH}}}$$

12.34

$\omega_0 = 1/\sqrt{LC}$

If $L' = 1.01L$

$$\omega_0' = (1.01LC)^{-1/2}$$
$$= 0.9950 \frac{1}{\sqrt{LC}}$$
$$= 0.9950\,\omega_0$$

$\therefore$ $\underline{\underline{\Delta\omega_0 = -0.5\%}}$

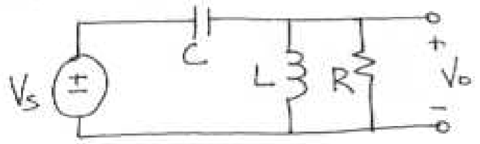
If $C' = 1.01C$

$$\omega_0' = 0.9950\,\omega_0$$

$\underline{\underline{\Delta\omega_0' = -0.5\%}}$

Changing R has no effect on ω_0

12.35



Use voltage divider rule:

$$V_o = \frac{Z_{R\parallel L}}{Z_{R\parallel L} + Z_C} V_s$$

$$\frac{V_o}{V_s} = \frac{\left(\frac{1}{R} + \frac{1}{sL}\right)^{-1}}{\left(\frac{1}{R} + \frac{1}{sL}\right)^{-1} + \frac{1}{sC}}$$

$$= \frac{sC}{\left(\frac{1}{sL} + \frac{1}{R}\right) + sC}$$

$$\therefore T(s) = \frac{V_o(s)}{V_s(s)} = \frac{s^2}{s^2 + s/RC + \frac{1}{LC}}$$

12.36

Low Pass: $\omega_0 = 10^5$, $C = 0.1\mu F$
$Q = 1/\sqrt{2}$

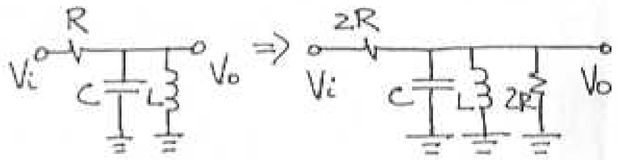
$Q = \omega_0 CR$

$$R = \frac{Q}{\omega_0 C}$$
$$= \frac{1}{\sqrt{2}\times 10^5 \times 0.1\times 10^{-6}}$$
$$= \underline{\underline{70.7\,\Omega}}$$

$\omega_0 = 1/\sqrt{LC}$

$$L = \frac{1}{\omega_0^2 C}$$
$$= \underline{\underline{1mH}}$$

12.37



$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$Q = \omega_0 CR$

$A_{MID} = 1$

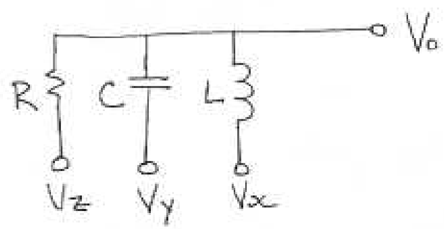
$\omega_0 = 1/\sqrt{LC}$

$Q = \omega_0 C(2R \parallel 2R)$

$= \omega_0 CR$

$$A_{MID} = \frac{2R}{2R+2R} = 1/2$$

12.38



$$V_o/V_z \Big|_{V_y = V_x = 0} = T_{BP}(s)$$

$$V_o/V_y \Big|_{V_x = V_z = 0} = T_{HP}(s)$$

CONT.

$$\left.\frac{V_o}{V_x}\right|_{V_y=V_z=0} = T_{LP}(s)$$

Using superposition

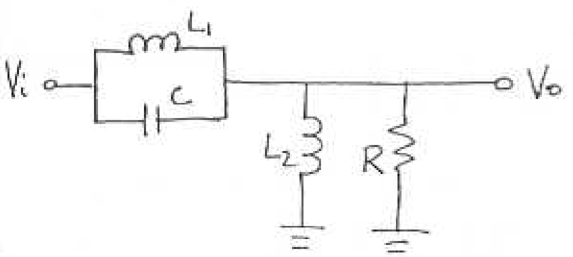
$$V_o = \frac{V_o}{V_x}V_x + \frac{V_o}{V_y}V_y + \frac{V_o}{V_z}V_z$$

$$= T_{LP}V_x + T_{HP}V_y + T_{BP}V_z$$

$$= \frac{\frac{1}{LC}V_x + s^2 V_y + \frac{s}{RC}V_z}{s^2 + s/RC + 1/LC}$$

$$\therefore V_o = V_x \frac{1/LC}{s^2 + s/RC + 1/LC} + V_y \frac{s^2}{s^2 + s/RC + 1/LC} + V_z \frac{s/RC}{s^2 + s/RC + 1/LC}$$

12.39



From Eq 12.46

$$T(s) = \frac{s^2 + 1/L_1C}{s^2 + s\,(1/CR) + \frac{1}{(L_1\|L_2)C}}$$

Required notch $\omega_n^2 = \frac{1}{L_1C} = (0.9\omega_0)^2$

but:

$$\omega_0^2 = \frac{1}{(L_1\|L_2)C} \quad \text{where } L_1\|L_2 = \frac{1}{L_1^{-1}+L_2^{-1}} = \frac{L_1L_2}{L_1+L_2}$$

$$= \frac{L_1+L_2}{L_1L_2C}$$

$$= \frac{L_1+L_2}{L_2}(.9\omega_0)^2$$

$$1 = \left(\frac{L_1}{L_2}+1\right)0.9^2$$

$$\therefore L_1/L_2 = \frac{1}{0.9^2} - 1 = \underline{\underline{0.2346}}$$

For $\omega \ll \omega_0$:-

$$|T| \cong \frac{1/L_1C}{1/(L_1\|L_2)C} = \frac{L_2}{L_1+L_2}$$

i.e. inductors dominate!

For $\omega \gg \omega_0$ L_1 & L_2 are 'open'
C is shorted

$$\underline{\underline{|T| \cong 1}}$$

12.40

$L = C_4 R_1 R_3 R_5 / R_2$

Choose $\underline{\underline{R_1 = R_2 = R_3 = R_5 = 10k\Omega}}$

$\therefore L = C_4 \times 10^8$

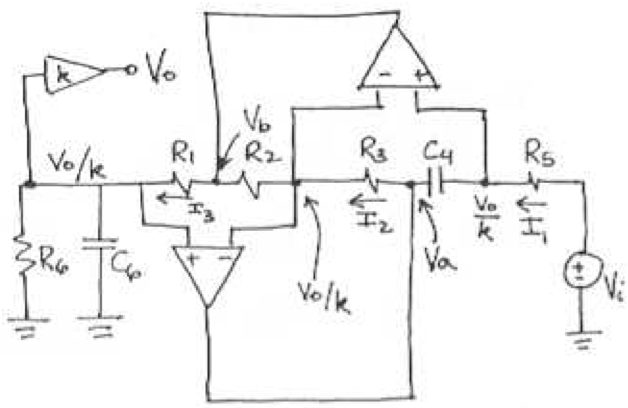
For:

$L = 10H = C_4 \times 10^8 \implies \underline{\underline{C_4 = 100nF}}$

$L = 1H \implies \underline{\underline{C_4 = 10nF}}$

$L = 0.1H \implies \underline{\underline{C_4 = 1nF}}$

12.41



Because of the virtual short at opamp input terminals, the voltages are : V_o/k

$$I_1 = \frac{V_i - V_o/k}{R_5}$$

$$V_a = V_o/k - I_1/sC_4$$

$$= V_o/k - \frac{V_i - V_o/k}{sC_4 R_5}$$

$$= \frac{V_o(sC_4R_5 + 1) - kV_i}{sC_4 R_5 k}$$

$$I_2 = \frac{V_a - V_o/k}{R_3}$$

$$= -\frac{V_i}{sC_4R_3R_5} + \frac{V_o/k}{sC_4R_3R_5}$$

$$V_b = \frac{V_o}{k} - I_2 R_2$$

$$= \frac{V_o}{k} - \frac{V_o/k - V_i}{sC_4R_3R_5/R_2}$$

$$I_3 = \frac{V_b - V_o/k}{R_1} = \frac{V_i - V_o/k}{sC_4R_1R_3R_5/R_2}$$

Now, I_3 flows only into R_6 & C_6 since for ideal opamps, $R_{IN} = \infty$!

$$\therefore \frac{V_o}{k} = I_3\left(R_6 \parallel \frac{1}{sC_6}\right)$$

Let $L \triangleq \dfrac{R_1R_3R_5C_4}{R_2}$

So: $I_3 = \dfrac{V_i - V_o/k}{sL}$ &

$$V_o/k = I_3\left(R_6 \parallel \frac{1}{sC_6}\right)$$

$$\frac{V_o}{k} = \frac{V_i - V_o/k}{sL} \cdot \frac{1}{1/R_6 + sC_6}$$

$$= \frac{V_i - V_o/k}{sL/R_6 + s^2LC_6}$$

$$V_o/k\left(1 + sL/R_6 + s^2LC_6\right) = V_i$$

$$\frac{V_o}{V_i} = \frac{k/LC_6}{s^2 + \frac{s}{R_6C_6} + \frac{1}{LC_6}}$$

Recall $L = R_1R_3R_5C_4/R_2$

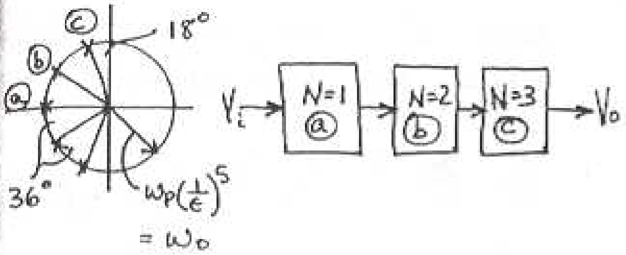
$$\therefore \frac{V_o}{V_i} = \frac{\dfrac{kR_2}{C_4R_1R_3R_5}}{s^2 + \dfrac{s}{R_6C_6} + \dfrac{R_2}{C_6C_4R_1R_3R_5}}$$

12.42

$$A_{max} = 10\log(1+\epsilon^2) = 3\text{ dB}$$

$$\therefore \epsilon = 0.998 \cong 1$$

$$\omega_o = \omega_p = 10^4$$



For circuit ⓐ use fig 12.13 (a)

DC Gain $= 1 = R_2/R_1 \Rightarrow \underline{R_1 = R_2 = 10\,k\Omega}$

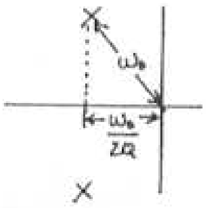
$CR_2 = 1/w_o \Rightarrow C = 1/R_2 w_o = \frac{1}{10^4 10^4}$
$= \underline{10\,nF}$

For circuit ⓑ use Fig. 12.22 (a)

$w_o = 10^4$ rad/s

$\frac{w_o}{2Q} = w_o \cos 36°$

$Q = \frac{1}{2 \cos 36} = \underline{\underline{0.618}}$



From Table 12.1

$$T(S) = \frac{\frac{kR_2}{C_4 C_6 R_1 R_3 R_5}}{s^2 + s/C_6 R_6 + \frac{R_2}{C_4 C_6 R_1 R_3 R_5}}$$

$w_o^2 = \frac{R_2}{C_4 C_6 R_1 R_3 R_5}$ Let $R_1 = R_3 = R_5 = R_2 = R$
$C_4 = C_6 = C$

$w_o^2 = \frac{1}{R^2 C^2}$ USE $\underline{\underline{C_4 = C_6 = 100\,nF}}$

$\therefore R = \frac{1}{w_o C} \Rightarrow \underline{\underline{R_1 = R_2 = R_3 = R_5 = 1\,k\Omega}}$

Now using:

$\frac{w_o}{Q} = \frac{1}{C_6 R_6}$ & $Q = 0.618$

$R_6 = \frac{Q}{C_6 w_o} = \underline{\underline{618\,\Omega}}$

For circuit ⓒ use Fig 12.22 (a)

$w_o = 10^4$ which is the same as for circuit ⓑ.

$\therefore$ $\underline{C_4 = C_6 = 100\,nF}$
$\underline{\underline{R_1 = R_2 = R_3 = R_5 = 1\,k\Omega}}$

Now: $Q = \frac{1}{2 \cos 72°} = 1.618$

$R_6 = Q/w_o C_6 = \underline{\underline{1.618\,k\Omega}}$

12.43

$f_o = 4\,kHz$ $f_N = 5\,kHz$ $Q = 10$
now $C_4 = 10\,nF$ and $k = 1 \equiv$ dc gain
$w_o = \left[C_4 (C_{61} + C_{62}) R_1 R_3 R_5 / R_2\right]^{-1/2}$

$C_{61} + C_{62} = C_6$

Choose $\underline{\underline{C_6 = C_4 = 10\,nF}}$ &
$R_1 = R_3 = R_5 = R_2 = R$

$\therefore w_o = (C_4 C_6 R^2)^{-1/2}$

$R = \frac{1}{w_o C_4}$

$\Rightarrow \underline{\underline{R_1 = R_3 = R_5 = R_2 = 3.979\,k\Omega}}$

$w_n = (C_4 C_{61} R^2)^{-1/2}$

$C_{61} = \frac{1}{w_n^2 R^2 C_4} \Rightarrow \underline{\underline{C_{61} = 6.4\,nF}}$

& $\underline{\underline{C_{62} = 3.6\,nF}}$

$Q = R_6 \sqrt{\frac{C_{61} + C_{62}}{C_4} \cdot \frac{R_2}{R_1 R_3 R_5}}$

$= R_6 \sqrt{\frac{1}{R_1^2}} = R_6/R_1 \Rightarrow \underline{\underline{R_6 = 39.79\,k\Omega}}$

12.44

From Fig. 12.16 (g) $\phi = 180°$ at f_0 !

$\therefore$ Use $f_0 = 1\,\text{kHz}$ $\quad Q = 1$

$$\omega_0^2 = \frac{R_2}{C_4 C_6 R_1 R_3 R_5} \qquad \text{LET } \underline{\underline{C = C_4 = C_6 = 1\,\text{nF}}},\ R_1 = R_3\ R_5 = R_2 = R$$

$$= 1/C^2R^2$$

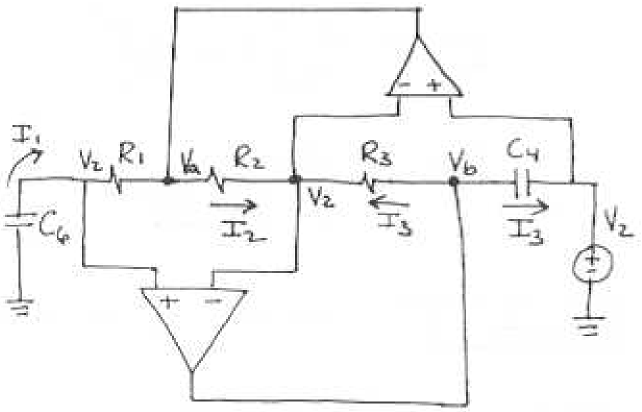
$$R = 1/\omega_0 C = \underline{\underline{159.16\,k\Omega = R_1 = R_3 = R_5 = R_2}}$$

$$\frac{\omega_0}{Q} = \frac{1}{R_6 C_6} \Rightarrow R_6 = \frac{Q}{C_6\omega_0}$$

$$= \frac{1}{10^{-9}\, 2\pi 10^3}$$

$$\therefore \underline{\underline{R_6 = 159.16\,k\Omega}}$$

12.45



Because of virtual short at opamp input terminals all nodes are at V_2 !

$$I_1 = -sC_6 V_2$$

Since no current goes into the opamp input terminals we have:

$$V_a = V_2 - I_1 R_1$$
$$= V_2 (1 + sC_6 R_1)$$

$$I_2 = (V_a - V_2)\frac{1}{R_2}$$
$$= \frac{sC_6 R_1}{R_2}$$

$$V_b = V_2 - I_2 R_3$$
$$= V_2 - \frac{sC_6 R_1 R_3}{R_2}$$

$$I_3 = (V_b - V_2)\, sC_4$$
$$= -\frac{s^2 C_4 C_6 R_1 R_3}{R_2} V_2$$

Now the voltage source sees an input impedance given by:

$$Z_{in} = -V_2/I_3 = \frac{R_2}{s^2 C_4 C_6 R_1 R_3}$$

As required.

for $s = j\omega \Rightarrow s^2 = -\omega^2$

$$Z_{in}(j\omega) = \frac{-R_2}{C_4 C_6 R_1 R_3} \cdot \frac{1}{\omega^2}$$

$$= -R(\omega)$$ ie. A PURE NEGATIVE RESISTANCE!

12.46

For LPN: See Table 12.1 & Fig. 12.22(e)

$$T(s) = k\frac{C_{61}}{C_{61}+C_{62}} \cdot \frac{s^2 + \frac{R_2}{C_4 C_{61} R_1 R_3 R_5}}{s^2 + s/C_6 R_6 + \frac{R_2}{C_4(C_{61}+C_{62}) R_1 R_3 R_5}}$$

At DC $\rightarrow s = 0$

$$T(0) = k\frac{C_{61}}{C_{61}+C_{62}} \frac{R_2/C_4 C_6 R_1 R_3 R_5}{R_2/C_4(C_{61}+C_{62}) R_1 R_3 R_5}$$

CONT.

$\Rightarrow T(0) = k \triangleq$ DC Gain !

Note that $C_{61} + C_{62}$ is the total capacitance across R_6

$\therefore C_6 = C_{61} + C_{62}$

$$\frac{\omega_n^2}{\omega_o^2} = \frac{R_2/C_4 C_{61} R_1 R_3 R_5}{R_2/C_4 (C_{61}+C_{62}) R_1 R_3 R_5}$$

$$= \frac{C_{61}}{C_{61}+C_{62}}$$

$$\frac{\omega_n^2}{\omega_o^2} = \frac{C_{61}}{C_6}$$

$$\therefore \underline{C_{61} = C_6 \left(\frac{\omega_n}{\omega_o}\right)^2 = C\left(\frac{\omega_n}{\omega_o}\right)^2}$$

Clearly from $T(s)$ above :

$$\omega_n^2 = R_2/C_4 C_{61} R_1 R_3 R_5$$

$$\Rightarrow \omega_n = \sqrt{\frac{R_2}{C_4 C_{61} R_1 R_3 R_5}}$$

$$\omega_o^2 = R_2/C_4 C_6 R_1 R_3 R_5$$

$$\Rightarrow \omega_o = \sqrt{\frac{R_2}{C_4 (C_{61}+C_{62}) R_1 R_3 R_5}}$$

12.47

For HPN : See Table 12.1 & Fig 11.22(f)

$$T(s) = k \frac{s^2 + (R_2/C_4 C_6 R_1 R_3 R_{51})}{s^2 + s/C_6 R_6 + \left(\frac{R_2}{C_4 C_6 R_1 R_3}\right)\left(\frac{1}{R_{51}} + \frac{1}{R_{52}}\right)}$$

clearly: $\omega_n = \sqrt{\dfrac{R_2}{C_4 C_6 R_1 R_3 R_{51}}}$

$$\omega_o = \sqrt{\frac{R_2}{C_4 C_6 R_1 R_3}\left(\frac{1}{R_{51}} + \frac{1}{R_{52}}\right)}$$

At high frequencies $s \to \infty$

$T(\infty) = \underline{k \triangleq \text{high freq gain}}$.

Observe that the equivalent resistance at the +ve terminal of A_1 is :

$$\frac{1}{R_5} = \frac{1}{R_{51}} + \frac{1}{R_{52}} \quad \text{AND}$$

for the resonator (table 12.1)

$$R_5 = 1/\omega_o C \Rightarrow \frac{1}{R_5} = \omega_o C$$

$$\frac{\omega_o^2}{\omega_n^2} = \frac{R_2/C_4 C_6 R_1 R_3 R_5}{R_2/C_4 C_6 R_1 R_3 R_{51}} \Rightarrow \underline{R_{51} = R_5 \frac{\omega_o^2}{\omega_n^2}}$$

Now $\dfrac{1}{R_5} = \dfrac{1}{R_5 \frac{\omega_o^2}{\omega_n^2}} + \dfrac{1}{R_{52}}$

$$\frac{1}{R_{52}} = \frac{1}{R_5}\left[1 - \frac{\omega_n^2}{\omega_o^2}\right]$$

$$R_{52} = \frac{R_5}{1 - \omega_n^2/\omega_o^2}$$

12.48

$$T(s) = \frac{0.4508\,(s^2 + 1.6996)}{(s+0.7294)(s^2 + 0.27865 + 1.0504)}$$

PART (a) Replace s with s/ω_p

$$T(s) = \frac{0.4508\left(s^2/\omega_p^2 + 1.6996\right)}{\left(\frac{s}{\omega_p} + 0.7294\right)\left(\frac{s^2}{\omega_p^2} + \frac{0.27865}{\omega_p} + 1.0504\right)}$$

CONT.

$$T(S) = \frac{0.4508\omega_p\,(s^2 + 1.6996\,\omega_p^2)}{(s + 0.7294\omega_p)(s^2 + 0.2786\omega_p s + 1.0504\,\omega_p^2)}$$

SUB $\omega_p = 10^4$ rad/s

$$T(S) = \frac{4508\,(s^2 + 1.6996\times10^8)}{(s+7294)(s^2 + 2786s + 1.0504\times10^8)}$$

Part (b)

First decompose T(s) into 1st-and 2nd-order sections with unity DC gain!

$$T_1(s) = \frac{k_1}{s+7294} \qquad T_1(0) = \frac{k_1}{7294} = 1$$

$$\Rightarrow k_1 = 7294$$

Now $k_1 k_2 = 4508 \Rightarrow k_2 = 0.6180$

$$\therefore T_2(s) = \frac{0.6180\,(s^2 + 1.6996\times10^8)}{s^2 + 2786s + 1.0504\times10^8}$$

As a check:

$$T_2(0) = \frac{0.6180(1.6996\times10^8)}{1.0504\times10^8} = 1.000$$

As EXPECTED!

$\therefore T(S) = T_1(s) \cdot T_2(s)$

For first-order section use Fig 12.13(a)

$\omega_0 = 7294$ rad/s DC Gain = 1

Let $C = 10nF$

$R_1 = R_2 = \frac{1}{\omega_0 C} \Rightarrow R_1 = R_2 = 13.71k\Omega$

For second-order section

$\omega_n^2 = 1.6996\times10^8 \Rightarrow \omega_n = 13.037\times10^3$

$\omega_0^2 = 1.0504\times10^8 \Rightarrow \omega_0 = 10.249\times10^3$

$\frac{\omega_0}{Q} = 2786 \Rightarrow Q = 3.6787$

For LPN use table 12.1 and Fig 12.22(e)

Make $R_1 = R_2 = R_3 = R_5 = R$ and
$C = C_4 = C_6 = 10nF$

$R = \frac{1}{\omega_0 C} = 9.757k\Omega = R_1 = R_2 = R_3 = R_5$

$\omega_n^2 = \frac{1}{C R^2 C_{61}} \Rightarrow C_{61} = 6.18nF$

$C_{62} = C - C_{61}$
$= 3.82nF$

$$Q = R_6\sqrt{\frac{C_6}{C_4}\frac{R_2}{R_1R_3R_5}} = \frac{R_6}{\sqrt{R^2}} = \frac{R_6}{R}$$

$\therefore R_6 = RQ = (9.757\times10^3)(3.6787)$
$= 35.89k\Omega$

12.49

$f_0 = 1kHz$

The 3dB bandwidth for a 2^{nd} order filter is given by:

$B = \omega_0/Q \Rightarrow Q = \frac{2\pi10^3}{2\pi50} = 20$

Choose $C = 10nF$

$R = \frac{1}{\omega_0 C} = 15.92k\Omega$

Use $R_1 = R_f = 10k\Omega$

CONT.

$$\frac{R_3}{R_2} = 2Q-1 = 39$$

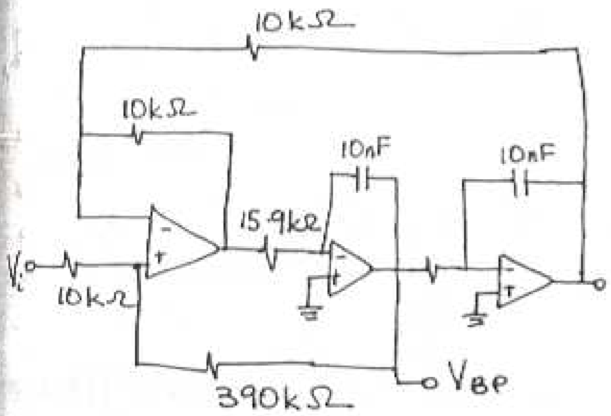
choose $\underline{R_2 = 10k\Omega}$ $\underline{R_3 = 390k\Omega}$

Now $T(s) = \dfrac{-k\omega_0 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$

$\Rightarrow |T(j\omega_0)| = \dfrac{k\omega_0^2}{\omega_0^2/Q} = kQ$

but $k = 2 - 1/Q = 1.95$

$\therefore$ Centre-freq gain $= kQ = \underline{39}$



12.50

$R_L = R_H = R_B/Q \Rightarrow R_B = QR_H$

$R_L = R_H$

Using Eq 12.66 :

$$\frac{V_o}{V_i} = -k\,\frac{\frac{R_F}{R_H}s^2 - s\left(\frac{R_F}{R_B}\right)\omega_0 + \left(\frac{R_F}{R_L}\right)\omega_0^2}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$= -k\frac{R_F}{R_H}\;\frac{s^2 - \frac{\omega_0}{Q}s + \omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

Flat Gain $= \underline{-k\, R_F/R_H}$

Part (b) – $\omega_0 = 10^4$ rad/s $Q = 2$ Flat Gain $= 10$

choose $\underline{C = 10nF} \Rightarrow R = \dfrac{1}{\omega_0 C} = \underline{10k\Omega}$

choose $\underline{R_F = R_1 = 10k\Omega}$

$\dfrac{R_3}{R_2} = 2Q-1 = 3 \Rightarrow \underline{R_2 = 10k\Omega}$

$\underline{R_3 = 30k\Omega}$

Now $k = 2 - 1/Q = 1.5$

$\therefore$ Flat Gain $= 10 = (1.5)\dfrac{R_F}{R_H}$

$\therefore \dfrac{R_H}{R_F} = 0.15$

choose $\underline{R_F = 100k\Omega}$

$\underline{R_H = R_L = 15k\Omega}$

$R_B = QR_H = \underline{30k\Omega}$

12.51

Note ω_n does not depend on R or C

From eq. 12.67 :

$$\frac{R_H}{R_L} = \left(\frac{\omega_n}{\omega_0}\right)^2$$

$\therefore \omega_n = \omega_0\sqrt{\dfrac{R_H}{R_L}}$ Nominally $R_H = R_L \pm 1\%$

Thus :

$\omega_n' = \omega_0\sqrt{\dfrac{1.01}{0.99}} = 1.01\omega_0$ $\omega_n'' = \omega_0\sqrt{\dfrac{0.99}{1.01}} = 0.99\omega_0$

$\therefore$ ω_n can deviate from ω_0 by $\underline{\pm 1\%}$

12.52

Use Tow Thomas to realize a LPN (Fig 12.26)

$\omega_0 = 10^4 \quad \omega_n = 1.2\omega_0 \quad Q = 10 \quad$ DC Gain $= 1$

$C = 10nF \quad r = 20k\Omega$

$R = \frac{1}{\omega_0 C} = \underline{\underline{10k\Omega}}$

from 12.16 (e):

DC Gain $= a_2 \frac{\omega_n^2}{\omega_0^2} = 1$

$a_2 \frac{1.2^2\omega_0^2}{\omega_0^2} = 1$

$a_2 = \frac{1}{1.2^2} = $ HF. Gain

$C_1 = C a_2 = \frac{10\times10^{-9}}{1.2^2} = \underline{\underline{6.94nF}}$

$R_2 = \frac{R(\omega_0/\omega_n)^2}{\text{HF Gain}} = R\left(\frac{1}{1.2}\right)^2 \times (1.2)^2$

$= R = \underline{\underline{10k\Omega}}$

$\underline{\underline{R_1 = R_3 = \infty}}$

$$Q_z = \frac{\sqrt{\frac{1}{C^2RR_2}\frac{C}{C_1}}}{\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right)\left(\frac{C}{C_1}\right)}$$

$$= \underline{\underline{\frac{1}{\sqrt{RR_2}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right)\sqrt{\frac{C}{C_1}}}}}$$

For All Pass $R_1 \to \infty$

To adjust Q_z, trim r or R_2 (independent of ω_z!)

Now $\omega_0 = \frac{1}{CR}$ so do not trim R or C!

Note if we trim R_2 or C_1 to adjust ω_z, this will also affect Q_z. So the options are:

For ω_z: (a) trim R_2 AND (r or R_3) to maintain the value of Q_z

OR

(b) trim C_1, and r or R_3

Prefer not to trim a capacitor so use (a)!

12.53

For all pass:

$$T(s) = \frac{-s^2\left(\frac{C_1}{C}\right) + s\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right) + \frac{1}{C^2RR_2}}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$\omega_z^2 = \frac{1}{C^2RR_2}\cdot\frac{C}{C_1} \Rightarrow \underline{\underline{\omega_z = \frac{1}{C\sqrt{RR_2}}\cdot\sqrt{\frac{C}{C_1}}}}$$

$$Q_z = \frac{\omega_z}{\frac{1}{C}\left(\frac{1}{R_1} - \frac{r}{RR_3}\right)\frac{C}{C_1}}$$

12.54

$$T(s) = \frac{0.4508(s^2 + 1.6996)}{(s + 0.7294)(s^2 + 0.27865s + 1.0504)}$$

Part (a) Replace s with s/ω_p

$\omega_p = 10^4$ rad/s.

$$T(s) = \frac{0.4508\left(s^2/\omega_p^2 + 1.6996\right)}{\left(\frac{s}{\omega_p} + 0.7294\right)\left(\frac{s^2}{\omega_p^2} + \frac{0.27865}{\omega_p} + 1.0504\right)}$$

CONT.

$$T(s) = \frac{0.4508\,\omega_p\left(s^2 + 1.6996\,\omega_p^2\right)}{\left(s+0.7294\,\omega_p\right)\left(s^2+0.2786\,\omega_p s + 1.0504\,\omega_p^2\right)}$$

$$= \frac{4508\left(s^2 + 1.6996\times10^8\right)}{\left(s+7294\right)\left(s^2+2786s+1.0504\times10^8\right)}$$

For First Order Section use Fig 12.13(a)

$\omega_0 = 7294$ DC gain $=1$

choose $\underline{C = 10nF}$

$R_1 = R_2 = \frac{1}{\omega_0 C} \Rightarrow \underline{R_1 = R_2 = 13.71k\Omega}$

For Second Order Section - use Fig 12.26

$\omega_n^2 = 1.6996\times10^8 \Rightarrow \omega_n = 13.037\times10^3$

$\omega_0^2 = 1.0504\times10^8 \Rightarrow \omega_0 = 10.249\times10^3$

$\frac{\omega_0}{Q} = 2786 \Rightarrow Q = 3.6787$

DC gain $=1$

For Tow Thomas LPN use table 12.2

choose $\underline{C = 10nF}$

$R = \frac{1}{\omega_0 C} = \frac{1}{10.249\times10^3 \cdot 10\times10^{-9}}$

$= \underline{9.757k\Omega}$

choose $\underline{r = 20k\Omega}$

Now from Fig 12.16 (e):

$T(0) = a_2\frac{\omega_n^2}{\omega_0^2} = 1 \Rightarrow a_2 = \frac{\omega_0^2}{\omega_n^2} = 0.618$

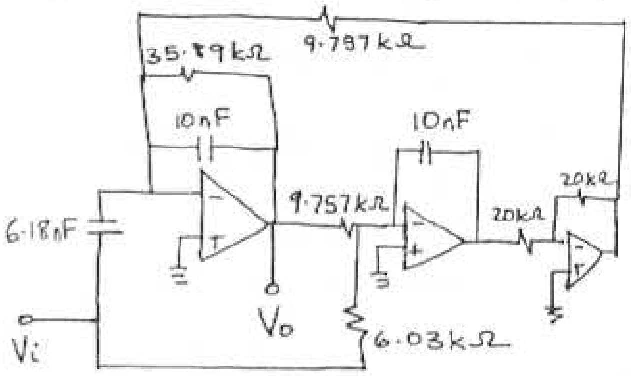
$\therefore$ HF gain $= a_2 = 0.618$

$C_1 = C \times \text{HF gain} \Rightarrow \underline{C_1 = 6.18nF}$

$R_2 = R\left(\omega_0/\omega_n\right)^2$

$R_2 = 0.618R \Rightarrow \underline{R_2 = 6.03k\Omega}$

$\underline{R_1 = R_3 = \infty}$ $QR = \underline{35.89k\Omega}$



12.55

Make $C_1 = C_2 = 1nF = C$

$\omega_0 = \frac{1}{\sqrt{C_1C_2R_3R_4}}$ $Q = \left[\frac{\sqrt{C_1C_2R_3R_4}}{R_3}\left(\frac{1}{C_1}+\frac{1}{C_2}\right)\right]^{-1}$

Let $R_3 = R$ $m = 4Q^2$

$R_4 = \frac{R}{m}$ $= 4/2 = 2$

$\therefore \omega_0 = \frac{1}{\sqrt{C^2 R^2/2}} = \frac{\sqrt{2}}{RC} = 10^4$

$\Rightarrow R = \frac{\sqrt{2}}{10^4 10^{-9}} = \underline{141.42k\Omega}$

$\underline{R_3 = 141.4k\Omega}$

$R_4 = \frac{R_3}{2} \Rightarrow \underline{R_4 = 70.7k\Omega}$

12.56

For Fig 12.28(a)

$$t(s) = \frac{s^2 + s\left(\frac{1}{C_1} + \frac{1}{C_2}\right)\frac{1}{R_3} + \frac{1}{C_1C_2R_3R_4}}{s^2 + s\left(\frac{1}{C_1R_3} + \frac{1}{C_1R_4} + \frac{1}{C_2R_3}\right) + \frac{1}{C_1C_2R_3R_4}}$$

But $C_1 = C_2 = C$ & $R_3 = R_4 = R$, $RC = \tau$

$$\therefore t(s) = \frac{s^2 + s\,2/RC + 1/R^2C^2}{s^2 + s\,3/RC + \frac{1}{R^2C^2}}$$

$$= \frac{s^2 + s\,2/\tau + 1/\tau^2}{s^2 + s\frac{3}{\tau} + 1/\tau^2}$$

Zeros defined by $\omega_z = 1/\tau$
$Q_z = \frac{1}{2}$

$\Rightarrow$ Double Root at $s = -1/\tau$

Poles of $t(s)$ are given by the quadratic formula:

$$s = \frac{-3}{2\tau} \pm \frac{\sqrt{5}}{2\tau} = \frac{-3 \pm \sqrt{5}}{2\tau}$$

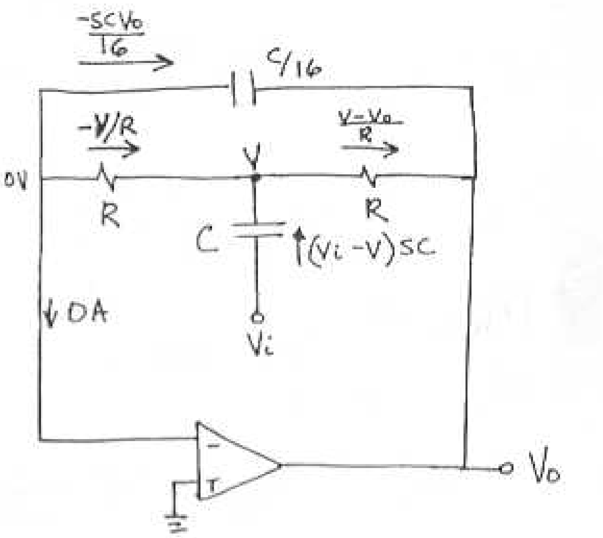
i.e. two roots on the negative real axis

If the network is placed in the negative feedback path of an ideal amplifier ($A = \infty$) then the poles are given by the zeros of $t(s)$:

Closed loop poles:

$$s = -1/\tau \quad (\text{multiplicity} = 2)$$

12.57



Note first $-\frac{sCV_o}{16} = -\frac{-V}{R}$

$$V = -\frac{sCRV_o}{16}$$

ΣI at V

$$-\frac{V}{R} + sC(V_i - V) - \frac{V - V_o}{R} = 0$$

$$\frac{sCV_o\cancel{R}}{\cancel{R}\,16} + sCV_i + s^2\frac{C^2R\,V_o}{16} + \frac{sCV_o}{16} + \frac{V_o}{R} = 0$$

mult by: $16R$ and let $RC = \tau$

$$s\tau V_o + 16\tau V_i s + s^2\tau^2 V_o + s\tau V_o + 16V_o = 0$$

$$V_o\left[s^2\tau^2 + s \times 2\tau + 16\right] = -16 s\tau V_i$$

$$\therefore \frac{V_o}{V_i} = -\frac{16 s\tau}{s^2\tau^2 + 2\tau s + 16}$$

$$\therefore T(s) = \frac{s\,16/RC}{s^2 + s\,2/RC + 16/R^2C^2}$$

CONT.

Let $\omega_0^2 = \frac{16}{(RC)^2} \Rightarrow \omega_0 = \frac{4}{RC}$

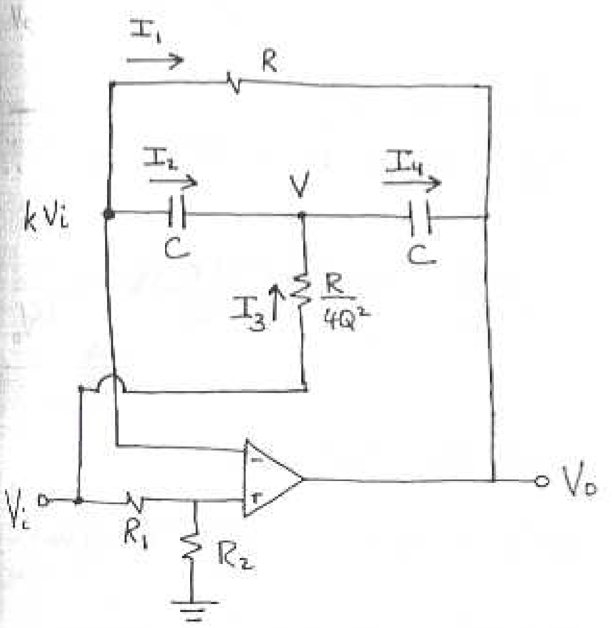
$\frac{\omega_0}{Q} = \frac{2}{RC} \Rightarrow Q = \frac{RC\omega_0}{2} = \underline{\underline{2}}$

$\therefore \frac{V_o}{V_i} = \frac{-4\omega_0 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$

$\left.\begin{array}{l} |T|_{s=0} = 0 \\ |T|_{s=\infty} = 0 \end{array}\right\}$ Bandpass

$|T(j\omega_0)| = 4/\tfrac{1}{2} = \underline{\underline{8\,\frac{V}{V}}}$ ← CENTRE FREQ GAIN

12.58



$RC = 2Q/\omega_0$

$k = \frac{R_2}{R_1 + R_2}$

$V_{+ve} = V_{-ve} = kV_i$ due to virtual short

$I_1 = -I_2$

$\frac{kV_i - V_o}{R} = \frac{V - kV_i}{1}(sC)$

$V = \frac{1}{sCR}\left(kV_i - V_o + sCRkV_i\right)$

ΣI at $V = 0$

$I_2 + I_3 - I_4 = 0$

$sC(kV_i - V) + \frac{4Q^2}{R}(V_i - V) - sC(V - V_o) = 0$

$sC\left(kV_i - \frac{kV_i}{sCR} + \frac{V_o}{sCR} - kV_i\right)$

$+ \frac{4Q^2}{R}\left(V_i - \frac{kV_i}{sCR} + \frac{V_o}{sCR} - kV_i\right)$

$- sC\left(\frac{kV_i}{sCR} - \frac{V_o}{sCR} + kV_i - V_o\right) = 0$

$\Rightarrow$

$-\frac{kV_i}{\cancel{R}} + \frac{V_o}{\cancel{R}}$

$+ \frac{4Q^2}{\cancel{R}}\left(V_i - \frac{kV_i}{sCR} + \frac{V_o}{sCR} - kV_i\right)$

$- \frac{sC}{\cancel{R}}\left(\frac{kV_i}{sC} - \frac{V_o}{sC} + kRV_i - V_oR\right)$

$= 0$

$\Rightarrow$ SUB $CR = \frac{2Q}{\omega_0}$ & $R = \frac{2Q}{C\omega_0}$

$-kV_i + V_o$

$+ 4Q^2V_i - \frac{{}^{2}\cancel{4}Q^{\cancel{2}}kV_i\,\omega_0}{s\cancel{2}\cancel{Q}} + \frac{V_o\,\omega_0\,\cancel{4}Q^{\cancel{2}}{}^{2}}{s\cancel{2}\cancel{Q}} - 4Q^2kV_i$

$- kV_i + V_o - sk\,2Q/\omega_0\,V_i + sV_o\frac{2Q}{\omega_0} = 0$

$$V_o\left[1+\frac{2Q\omega_0}{s}+1+\frac{2Qs}{\omega_0}\right]$$

$$=V_i\left[k-4Q^2+\frac{2kQ\omega_0}{s}+4Q^2k+k+\frac{2kQs}{\omega_0}\right]$$

$$\Rightarrow V_o\left[s^2\frac{2Q}{\omega_0}+2s+2Q\omega_0\right]=V_i\left[s^2\frac{2kQ}{\omega_0}+s(4Q^2k-4Q^2+2k)+2kQ\omega_0\right]$$

$$\Rightarrow \frac{V_o}{V_i}=\frac{s^2\frac{2kQ}{\omega_0}+s(4Q^2k-4Q^2+2k)+2kQ\omega_0}{s^2\frac{2Q}{\omega_0}+2s+2Q\omega_0}$$

$$=k\,\frac{s^2+s\frac{\omega_0}{Q}\left(2Q^2-\frac{2Q^2}{k}+1\right)+\omega_0^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$$

Recall $k=\frac{R_2}{R_1+R_2}$ and $\frac{1}{k}=1+\frac{R_1}{R_2}$

$$\Rightarrow \frac{V_o}{V_i}=\left(\frac{R_2}{R_1+R_2}\right)\frac{s^2+s\frac{\omega_0}{Q}\left(1-\frac{R_1}{R_2}\cdot 2Q^2\right)+\omega_0^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$$

$$\therefore T(s)=\frac{R_2}{R_1+R_2}\,\frac{s^2+s\frac{\omega_0}{Q}\left(1-\frac{2Q^2R_1}{R_2}\right)+\omega_0^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$$

For All Pass

we want $T(s)\propto\dfrac{s^2+\frac{\omega_0}{Q}(-1)s+\omega_0^2}{s^2+s\frac{\omega_0}{Q}+\omega_0^2}$

$$\Rightarrow 1-\frac{2Q^2R_1}{R_2}=-1$$

$$2Q^2\frac{R_1}{R_2}=2$$

$$\frac{R_1}{R_2}=\frac{1}{Q^2}$$

$$\therefore \underline{\underline{\frac{R_2}{R_1}=Q^2}} \quad \& \quad \frac{R_2}{R_1+R_2}=\frac{R_2/R_1}{1+R_2/R_1}=\underline{\underline{\frac{Q^2}{1+Q^2}}}$$

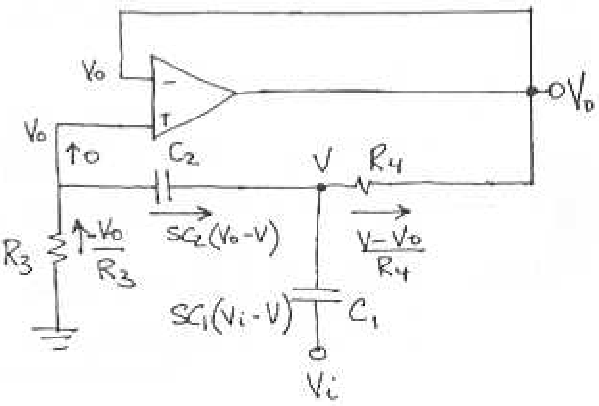
For Notch:

$$1-2Q^2\frac{R_1}{R_2}=0$$

$$\frac{R_1}{R_2}=\frac{1}{2Q^2}$$

$$\underline{\underline{\frac{R_2}{R_1}=2Q^2}} \quad \& \quad \underline{\underline{\frac{R_2}{R_1+R_2}=\frac{2Q^2}{1+2Q^2}}}$$

12.59



$\because$ No current can't flow into the terminal

$$-\frac{V_o}{R_3}=sC_2(V_o-V)$$

$$V=V_o\left(1+\frac{1}{sC_2R_3}\right)$$

CONT.

ΣI @ $V = 0$

$$-\frac{V_o}{R_3} + \frac{V_i - V}{1} sC_1 = \frac{V - V_o}{R_4}$$

$$V_o\left[-\frac{1}{R_3} + \frac{1}{R_4}\right] + V\left[-sC_1 - \frac{1}{R_4}\right] = -sC_1V_i$$

$$V_o[R_4 - R_3] + V[sC_1R_3R_4 + R_3] = -V_i sC_1R_3R_4$$

$$V_o(R_4 - R_3) + V_o\left(1 + \frac{1}{sC_2R_3}\right)(sC_1R_3R_4 + R_3)$$
$$= sC_1R_3R_4V_i$$

$$V_o\left(R_4 - \cancel{R_3} + sC_1R_3R_4 + \cancel{R_3} + \frac{C_1}{C_2}R_4 + \frac{1}{sC_2}\right)$$
$$= sC_1R_3R_4V_i$$

$$V_o\left(s^2C_1C_2R_3R_4 + sC_1R_4 + sC_2R_4 + 1\right)$$
$$= s^2C_1R_3R_4C_2V_i$$

$$\therefore \frac{V_o}{V_i} = \frac{s^2C_1C_2R_3R_4}{s^2C_1C_2R_3R_4 + sC_1R_4 + sC_2R_4 + 1}$$

$$= \frac{s^2}{s^2 + s\left(\frac{1}{C_2R_3} + \frac{1}{C_1R_3}\right) + \frac{1}{C_1C_2R_3R_4}}$$

Note $\left.\begin{array}{l}|T(0)| = 0 \\ |T(\infty)| = 1\end{array}\right\}$ ∴ High Pass, High Freq Gain $= 1\frac{V}{V}$

3dB freq $= 10^3$ rad/s, $Q = \frac{1}{\sqrt{2}}$ for max flat.

$\therefore \omega_0 = 10^3 \frac{rad}{s}$ $\quad C_1 = C_2 = 10nF$

clearly $\omega_0^2 = \frac{1}{C_1C_2R_3R_4}$ and

$$\frac{\omega_0}{Q} = \frac{1}{C_2R_3} + \frac{1}{C_1R_3} = \frac{C_1 + C_2}{C_1C_2R_3}$$

$$= \frac{2C}{C^2R_3} = \frac{2}{CR_3} = \sqrt{2} \times 10^3$$

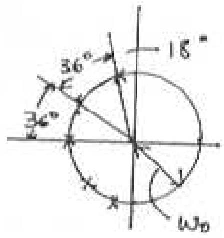
$$R_3 = \frac{2}{10\times10^{-9}\times10^3\times\sqrt{2}}$$

$$\underline{\underline{R_3 = 141.4\ k\Omega}}$$

$$R_4 = \frac{1}{\omega_0^2 C_1C_2R_3} \Rightarrow \underline{\underline{R_4 = 70.7k\Omega}}$$

$\boxed{12.60}$

$A_{max} = 3dB$



$$\epsilon = \left(10^{3/10} - 1\right)^{\frac{1}{2}} \cong 1$$

$$\omega_0 = \omega_p\left(\frac{1}{\epsilon}\right)^{1/N} = \omega_p = 2\pi 5000 = 10^4\pi$$

$$Q_1 = \frac{1}{2\cos 36} = 0.618$$

$$Q_2 = \frac{1}{2\cos 72} =$$

For first order section:

$\omega_0 = 10^4\pi$ $\quad$ dc gain $= 1$

From 12.13 (a)

$$\underline{\underline{R_1 = R_2 = 10k\Omega}}$$

$$C = \frac{1}{\omega_0 R_2} = \frac{1}{10^4\pi 10^4} = \underline{3.18\ nF}$$

Second-Order Section $Q = 0.618$:

from 12.34 (c): $m = 4Q^2 = 1.528$

CONT.

$RC = \frac{2Q}{\omega_0}$ let $\underline{\underline{R_1 = R_2 = 10\,k\Omega}}$

$C = \frac{2Q}{\omega_0 R} \Rightarrow$ $\underline{\underline{C_4 = C = 3.93\,nF}}$

$\underline{\underline{C_3 = \frac{C}{m} = 2.57\,nF}}$

Second Order Section $Q = 1.618$:

$C = \frac{2Q}{\omega_0 R}$ $m = 4Q^2 = 10.472$

$= 10.3\,nF \Rightarrow$ $\underline{\underline{R_1 = R_2 = 10\,k\Omega}}$

$\underline{\underline{C_4 = C = 10.3\,nF}}$

$\underline{\underline{C_3 = \frac{C}{m} = 0.984\,nF}}$

12.61

For a bandpass filter

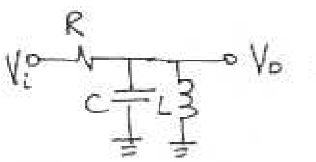
$$t(s) = \frac{\omega_0/Q\, s}{s^2 + s\,\omega_0/Q + \omega_0^2}$$

centre freq. gain = 1

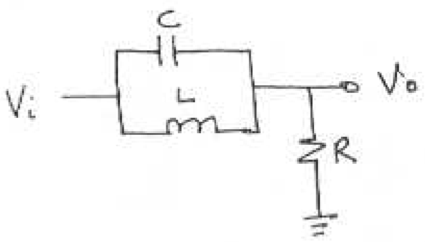
complementary transfer function:

$$t' = 1 - t = \frac{s^2 + \omega_0^2}{s^2 + s\,\omega_0/Q + \omega_0^2} \equiv \text{NOTCH!}$$

From FIG 12.18(d)



$\Rightarrow$ INTERCHANGE V_i & gnd to get:



this is the same as Fig 12.18(e)

12.62

for Fig 12.18(d):

$$T(s) = \frac{s/RC}{s^2 + s/RC + 1/LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad Q = R\sqrt{\frac{C}{L}}$$

FOR ω_0

$$\frac{\partial \omega_0}{\partial L} = \frac{\partial (LC)^{-1/2}}{\partial L} = -\frac{1}{2}L^{-3/2}C^{-1/2} = \frac{-\omega_0}{2L}$$

$$\frac{\partial \omega_0}{\partial C} = -\frac{\omega_0}{2C}$$

$$\frac{\partial \omega_0}{\partial R} = 0$$

$$\therefore S_L^{\omega_0} = \frac{\partial \omega_0}{\partial L}\frac{L}{\omega_0} = \underline{\underline{-1/2}}$$

$$S_C^{\omega_0} = \frac{\partial \omega_0}{\partial C} \times \frac{C}{\omega_0} = \underline{\underline{\frac{-1}{2}}}$$

$$S_R^{\omega_0} = \frac{\partial \omega_0}{\partial R}\frac{R}{\omega_0} = \underline{\underline{0}}$$

FOR Q

$$\frac{\partial Q}{\partial L} = \frac{R\sqrt{C}}{L\sqrt{L}}\left(\frac{-1}{2}\right)$$

$$\frac{\partial Q}{\partial C} = \frac{1}{2}\frac{R}{\sqrt{LC}} = \frac{1}{2}\frac{R\sqrt{C}}{C\sqrt{L}} = \frac{Q}{2C}$$

$$\frac{\partial Q}{\partial R} = \sqrt{C/L} = \frac{R}{R}\sqrt{C/L} = Q/R$$

CONT.

$$S_L^Q = \frac{-Q}{2L} \times \frac{L}{Q} = -\frac{1}{2}$$

$$S_C^Q = \frac{Q}{2C} \times \frac{C}{Q} = \frac{1}{2}$$

$$S_R^Q = \frac{Q}{R} \cdot \frac{R}{Q} = 1$$

12.63

$y = uv$

$$S_x^y = \frac{\partial(uv)}{\partial x} \frac{x}{uv}$$

$$= v\frac{\partial u}{\partial x} \frac{x}{uv} + u\frac{\partial v}{\partial x} \frac{x}{uv}$$

$$= \frac{\partial u}{\partial x} \frac{x}{u} + \frac{\partial v}{\partial x} \frac{x}{v}$$

$$= S_x^u + S_x^v$$

Part (b) $y = u/v$

$$S_x^y = \frac{\partial y}{\partial x} \cdot \frac{x}{y} = \frac{\partial(u/v)}{\partial x} \frac{xv}{u}$$

$$= \frac{1}{v} \frac{\partial u}{\partial x} \frac{vx}{u} + \frac{-u}{v^2} \frac{\partial v}{\partial x} \cdot \frac{xv}{u}$$

$$= \frac{\partial u}{\partial x} \frac{x}{u} - \frac{\partial v}{\partial x} \frac{x}{v}$$

$$= S_x^u - S_x^v$$

Part (c) $y = ku$

$$S_x^y = \frac{\partial y}{\partial x} \cdot \frac{x}{y} = \frac{\partial(ku)}{\partial x} \frac{x}{ku}$$

$$= k\frac{\partial u}{\partial x} \frac{x}{ku}$$

$$= \frac{\partial u}{\partial x} \frac{x}{u}$$

$$= S_x^u$$

Part (d) $y = u^n$

$$S_x^y = \frac{\partial y}{\partial x} \cdot \frac{x}{y} = \frac{\partial(u^n)}{\partial x} \frac{x}{u^n}$$

$$= nu^{n-1} \frac{\partial u}{\partial x} \cdot \frac{x}{u^n}$$

$$= n \frac{\partial u}{\partial x} \frac{x}{u} = n S_x^u$$

Part (e) $y = f_1(u)$ $u = f_2(x)$

$$S_x^y = \frac{\partial y}{\partial x} \frac{x}{y} = \frac{\partial f_1(u)}{\partial x} \frac{x}{f_1(u)}$$

$$= \frac{\partial f_1(u)}{\partial x} \frac{\partial u}{\partial x} \cdot \frac{x}{f_1(u)}$$

$$= \frac{\partial f_1}{\partial u} \cdot \frac{\partial f_2}{\partial x} \cdot \frac{x}{f_1} \cdot \frac{u}{u}$$

But $u = f_2$

$$\therefore S_x^y = \frac{\partial f_1}{\partial u} \cdot \frac{\partial f_2}{\partial x} \cdot \frac{x}{f_1} \cdot \frac{u}{f_2}$$

$$= \frac{\partial f_1}{\partial u} \cdot \frac{u}{f_1} \cdot \frac{\partial f_2}{\partial x} \frac{x}{f_2}$$

$$= S_u^{f_1} \cdot S_x^{f_2}$$

$$= S_u^y \, S_x^u$$

12.64

Since the characteristic equation of Fig. 12.33 (b) is the same as for Fig. 12.29 the poles are given by Eq 12.86:

$$s^2 + s\frac{\omega_0}{Q}\left[1 + \frac{2Q^2}{A+1}\right] + \omega_0^2 = 0$$

This is because 12.33 (b) is based on the complementary transform of 12.29 and hence the pole

locations are preserved!
Now the actual ω_0 and Q are given by:

$$\omega_{0,a} = \omega_0 \quad \text{and} \quad Q_a = \frac{Q}{1 + \frac{2Q^2}{(A+1)}}$$

Thus from Ex. 12.3

$$S_A^{\omega_{0,a}} = 0$$

$$S_A^{Q_a} = \frac{A}{A+1} \quad \frac{2Q^2/(A+1)}{1 + 2Q^2/(A+1)}$$

$$\therefore S_A^{Q_a} \cong \frac{2Q^2}{A}$$

12.65

If $R_1 = R_2$, then from (12.77) & (12.78)

$$\omega_0 = \frac{1}{\sqrt{C_3 C_4 R_1 R_2}}$$

$$Q = \frac{1}{\sqrt{C_3 C_4 R_1 R_2}\left(\frac{1}{C_4}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

$$\frac{\partial \omega_0}{\partial C_3} = \frac{-1}{2C_3\sqrt{C_3 C_4 R_1 R_2}}$$

$$\frac{\partial \omega_0}{\partial C_3} = \frac{-1}{2C_3\sqrt{C_3 C_4 R_1 R_2}} = -\frac{\omega_0}{2C_3}$$

$$S_{C_3}^{\omega_0} = \frac{\partial \omega_0}{\partial C_3} \frac{C_3}{\omega_0} = -\frac{1}{2}$$

clearly $S_{C_3}^{\omega_0} = S_{C_4}^{\omega_0} = S_{R_1}^{\omega_0} = S_{R_2}^{\omega_0} = -1/2$

$$\frac{\partial Q}{\partial C_3} = \frac{-1}{2C_3\sqrt{C_3 C_4 R_1 R_2}\left(\frac{1}{C_4}\right)\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} = \frac{-Q}{2C_3}$$

$$\therefore S_{C_3}^{Q} = -1/2$$

$$\frac{\partial Q}{\partial C_4} = \frac{Q}{2C_4} \Rightarrow S_{C_4}^{Q} = +1/2$$

$$\frac{\partial Q}{\partial R_1} = \frac{1/\sqrt{R_1} - \sqrt{R_1}/R_2}{R_1\left(\frac{1}{\sqrt{R_1}} + \sqrt{R_1}/R_2\right)} \cdot \frac{Q}{2}$$

$$= \frac{\sqrt{R_2/R_1} - \sqrt{R_1/R_2}}{R_1\left(\sqrt{\frac{R_2}{R_1}} + \sqrt{\frac{R_1}{R_2}}\right)} \cdot \frac{Q}{2}$$

$$\therefore S_{R_1}^{Q} = \frac{\sqrt{R_2/R_1} - \sqrt{R_1/R_2}}{\sqrt{R_2/R_1} + \sqrt{R_1/R_2}}$$

if $R_1 = R_2 \Rightarrow S_{R_1}^{Q} = 0$ &

$$S_{R_2}^{Q} = 0$$

12.66

From table 12.1

$$\omega_0 = \frac{1}{\sqrt{C_4 C_6 R_1 R_3 R_5 / R_2}}$$

$$Q = R_6 \sqrt{\frac{C_6}{C_4} \frac{R_2}{R_1 R_3 R_5}}$$

$$\frac{\partial \omega_0}{\partial C_4} = \frac{-\omega_0}{2C_4}$$

$$\therefore S_{C_4}^{\omega_0} = \frac{-\omega_0}{2C_4} \times \frac{C_4}{\omega_0} = -1/2$$

Similarly $S_{C_6}^{\omega_0} = S_{R_1}^{\omega_0} = S_{R_3}^{\omega_0} = S_{R_5}^{\omega_0} = -\frac{1}{2}$

$$\frac{\partial \omega}{\partial R_2} = \frac{\omega_0}{2R_2} \Rightarrow S_{R_2}^{\omega_0} = \frac{1}{2}$$

Now for Q:

$$\frac{\partial Q}{\partial R_6} = \frac{Q}{R_6} \Rightarrow S_{R_6}^{Q} = \frac{\partial Q}{\partial R_6} \frac{R_6}{Q} = +1$$

CONT.

$$\frac{\partial Q}{\partial C_6} = \frac{Q}{2C_6} \Rightarrow S_{C_6}^{Q} = S_{R_2}^{Q} = +\frac{1}{2}$$

$$\frac{\partial Q}{\partial C_4} = \frac{-Q}{2C_4} \Rightarrow S_{C_4}^{Q} = S_{R_1,R_3,R_5}^{Q} = -\frac{1}{2}$$

12.67

$$R_{eq} = \frac{T_c}{C_1} = \frac{1/100\times10^3}{C_1}$$

for $1pF \rightarrow R_{eq} = 10^{-5}/10^{-12} = \underline{10M\Omega}$

for $10pF \rightarrow R_{eq} = 10^{-5}/10\times10^{-12} = \underline{1M\Omega}$

12.68

charge transferred $\Rightarrow Q = CV$

$= 10^{-12}(1)$

$= \underline{1pC}$

For $f_0 = 100kHz$, average current is given by:

$$I_{AVE} = \frac{Q}{T} = 1pC \times \frac{1}{100\times10^3}$$

$$= \underline{0.1\mu A}$$

For each clock cycle, the output will change by the same amount as the change in voltage across C_2!

$$\therefore \Delta V = Q/C_2 = \frac{1pC}{10pF} = \underline{0.1V}$$

For $\Delta V = 0.1V$ for each clock cycle, the amplifier will saturate in

$$\#\text{cycles} = \frac{10V}{0.1V} = \underline{100 \text{ cycles}}$$

$$\text{slope} = \frac{\Delta V}{\Delta t} = \frac{10V}{(100\text{ cycles})(1/100\times10^3)}$$

$$= \underline{10^4 \frac{V}{s}}$$

12.69

$f_c = 400kHz$ $f_0 = 10kHz$ $Q = 20$

$C_1 = C_2 = 20pF = C$

$C_3 = C_4 = \omega_0 T_c C = 2\pi(10^4)\frac{1}{400\times10^3}20\times10^{-12}$

$= \underline{3.14pF}$

$$C_5 = \frac{\omega_0 T_c C}{Q}$$

$$= \frac{C_3}{Q} = \underline{0.157pF}$$

$$C_6 = \frac{\omega_0 T_c C}{Q} \times \text{centre frequency gain}$$

$$= \underline{0.157\ pF}$$

Note that the clock frequency has doubled. Hence the period, T_c, is halved. Therefore, for the same integrating capacitors, the resistors (switched capacitors) will change by the factor of 2. So compensate for this by changing the switched caps by a factor of 1/2.

12.70

Ex 12-31 for $Q = 40$ $f_c = 200kHz$

$f_0 = 10kHz$

$C_1 = C_2 = 20pF = C$

$C_3 = C_4 = \omega_0 T_c C$

$= 2\pi(10^4)\left(\frac{1}{200\times10^3}\right)20\times10^{-12}$

$= \underline{6.28pF}$

CONT.

$$C_5 = \frac{\omega_0 T_c C}{Q} = \frac{C_3}{Q} = \underline{\underline{0.157\,pF}}$$

$$C_6 = \frac{\omega_0 T_c C}{Q} = C_5 = \underline{\underline{0.157\,pF}}$$

12.71

$\omega_0 = 10^4$, $Q = 1/\sqrt{2}$, $f_c = 100\,kHz$

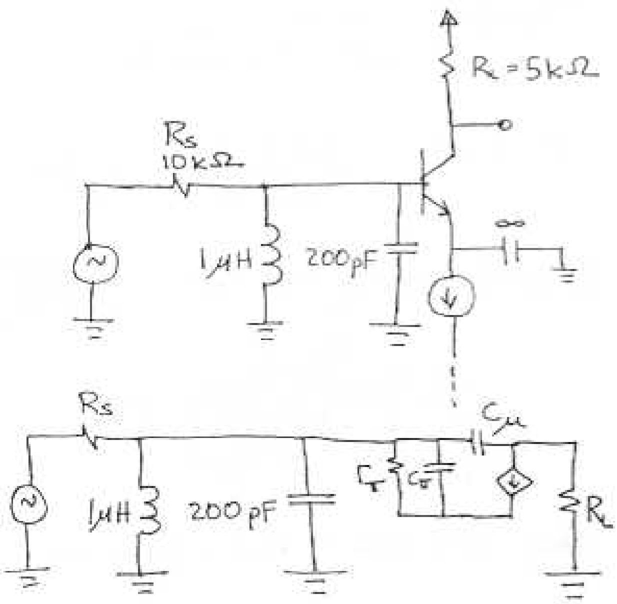
DC gain $\Rightarrow \frac{R_4}{R_6} \Rightarrow \frac{C_6}{C_4} = 1$

$C_1 = C_2 = 10\,pF$

$$C_3 = C_4 = C_6 = \omega_0 T_c C = 10^4\left(\frac{1}{100\times10^3}\right)10\times10^{-12} = \underline{1\,pF}$$

$$C_5 = C_4/Q = \underline{\underline{1.41\,pF}}$$

12.72



$r_e = 25\Omega$, $C_\mu = 1pF$, $C_\pi = 10pF$, $\beta = 200$

$r_\pi = (\beta+1) r_e = 5.025\,k\Omega$

From base to collector

$$\frac{V_c}{V_b} = -\frac{\beta}{\beta+1} \cdot \frac{R_L}{r_e} = -199 = k$$

Total capacitance at base

$$C_T = C_\pi + 200p + C_\mu(1-k) \quad \text{Miller Effect}$$

$$= 10 + 200 + 1(1+199)$$

$$= 410\,pF$$

$$\therefore \omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-6}\times 410\times10^{-12}}} = \underline{\underline{49.4\times10^6\ rad/s}}$$

Centre frequency gain =

$$\frac{r_\pi}{R_s + r_\pi}\cdot k = \frac{5.025}{10+5.025}\times -199 = \underline{\underline{-66.6\ V/V}}$$

$$BW = \frac{1}{RC} = \frac{1}{(R_s \| r_\pi)\,410\,pF} = \underline{\underline{729\times10^3\ \frac{rad}{s}}}$$

$$Q = \frac{\omega_0}{BW} = 49.4/0.7293 = \underline{\underline{67.7}}$$

12.73

$$Q_0 = \frac{R_p}{\omega_0 L} \Rightarrow R_p = Q_0 \omega_0 L$$
$$= 200(2\pi 10^6)(10\times10^{-6})$$
$$= \underline{\underline{12.57\,k\Omega}}$$

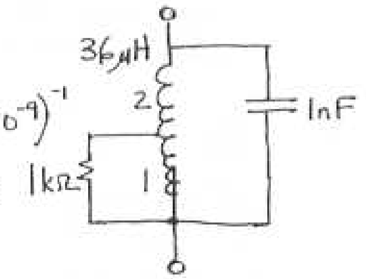
$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{\omega_0^2 L}$$
$$= \frac{1}{(2\pi 10^6)^2\, 10\times10^{-6}}$$
$$= \underline{\underline{2.533\,nF}}$$

$$B = \frac{1}{RC} \qquad R_T = \frac{1}{(2\pi\times10\times10^3)(2.533\times10^{-9})}$$
$$= 6.283\,k\Omega$$

$$\therefore \frac{1}{R_1} + \frac{1}{R_P} = \frac{1}{R_T}$$

$$\Rightarrow R_1 = \underline{\underline{12.57\,k\Omega}} \qquad \text{i.e. } R_1 // R_p = R_T$$

12.74

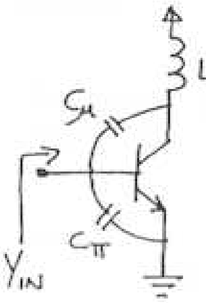


$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$
$$= \left(2\pi\sqrt{(36\times10^{-6})\,10^{-9}}\right)^{-1}$$
$$= \underline{\underline{838.8\,kHz}}$$

$$R_p = n^2 R$$
$$= 9\,(1k\Omega)$$
$$= 9k\Omega$$

$$Q = R_P/\omega_0 L$$
$$= \frac{9\times10^3}{2\pi\; 838.8\times10^3\times 36\times10^{-6}}$$
$$= \underline{\underline{47.4}}$$

12.75



for $\omega C_\mu \ll \frac{1}{\omega L}$

$$\therefore \omega^2 \ll \frac{1}{LC_\mu}$$

ie well below resonance

$$\therefore \text{gain} = -g_m(j\omega L)$$

$$\therefore Y_{in} = \frac{1}{r_\pi} + j\omega C_\pi + j\omega C_\mu(1 + g_m j\omega L)$$

$$= \left(\frac{1}{r_\pi} - \omega^2 g_m C_\mu L\right) + j\omega\left(C_\pi + C_\mu\right)$$

AS REQUIRED!

12.76

From Fig 12.16 (c):

$$T(s) = \frac{a_1 s}{s^2 + s\frac{\omega_0}{Q} + \omega_0^2}$$

$$T(j\omega) = \frac{j a_1 \omega}{\omega_0^2 - \omega^2 + j\frac{\omega\omega_0}{Q}}$$

$$T(j\omega_0) = \frac{j a_1 \omega_0}{j\omega_0^2/Q} = \frac{a_1 Q}{\omega_0}$$

$$|T(j\omega)| = a_1 \omega \left[(\omega_0^2 - \omega^2)^2 + \left(\frac{\omega\omega_0}{Q}\right)^2\right]^{-\frac{1}{2}}$$

$$= \frac{a_1 \omega \; Q/\omega\omega_0}{\sqrt{1 + Q^2\left(\frac{\omega_0^2 - \omega^2}{\omega_0 \omega}\right)^2}}$$

Now $\omega = \omega_0 + \delta\omega$, $\frac{\delta\omega}{\omega_0} \ll 1$

and $\omega^2 \simeq \omega_0^2\left(1 + 2\frac{\delta\omega}{\omega_0}\right)$

CONT.

so $\omega_0^2-\omega^2 = -2\delta\omega\omega_0$

$\therefore \quad |T(j\omega)| \cong \dfrac{a_1 Q/\omega_0}{\sqrt{1+Q^2\left(\frac{2\delta\omega}{\omega}\right)^2}}$

for $Q >> 1$: $Q^2\left(\frac{2\delta\omega}{\omega}\right)^2 \cong Q^2\left(\frac{2\delta\omega}{\omega_0}\right)$

$\therefore \omega \approx \omega_0$!

$$\Rightarrow |T(j\omega)| \cong \frac{|T(j\omega_0)|}{\sqrt{1+Q^2\left(\frac{2\delta\omega}{\omega_0}\right)^2}} = \frac{|T(j\omega_0)|}{\sqrt{1+4\frac{Q^2\delta\omega}{\omega_0}}}$$

For N bandpass sections, synchronously tuned in cascade, half power is given by:

$$\left(\frac{1}{\sqrt{1+4Q^2\left(\frac{\delta\omega}{\omega_0}\right)^2}}\right)^N = \frac{1}{\sqrt{2}}$$

$$\left(1+4Q^2\left(\frac{\delta\omega}{\omega_0}\right)^2\right)^N = 2$$

$$4Q^2\left(\frac{\delta\omega}{\omega_0}\right)^2 = 2^{1/N}-1$$

$$\delta\omega = \frac{\omega_0}{2Q}\sqrt{2^{1/N}-1}$$

$\therefore$ Bandwidth:

$$\underline{\underline{B = 2\delta\omega = \frac{\omega_0}{Q}\sqrt{2^{1/N}-1}}}$$

12.77

For first order low pass:

$T(s) = \dfrac{\omega_0'}{s+\omega_0'} \qquad |T(j\omega)| = \dfrac{\omega_0'}{\sqrt{\omega^2+\omega_0'^2}}$

for a bandpass response around ω_0 with $\omega_0' = \frac{\omega_0}{2Q}$:

$$|T(j\omega)| \cong \frac{\omega_0/2Q}{(\delta\omega)^2+\left(\frac{\omega_0}{2Q}\right)^2} = \frac{\omega_0/2Q}{\frac{\omega_0}{2Q}\sqrt{\left(\frac{2Q}{\omega_0}\right)^2(\delta\omega)^2+1}} = \frac{1}{\sqrt{1+4Q^2(\delta\omega/\omega_0)^2}}$$

Now at $\omega=\omega_0$ or $\delta\omega=0$ $|T(j\omega_0)|=1$, then

$$T(j\omega) \simeq \frac{|T(j\omega_0)|}{\sqrt{1+4Q^2\left(\frac{\delta\omega}{\omega_0}\right)^2}}$$

Part (b)

For N synchronously tuned sections in cascade; 3dB bandwidth is given by:

$$\left(|T|/|T_0|\right)^N = \frac{1}{\sqrt{2}}$$

$$\left(|T|/|T_0|\right)^2 = \frac{1}{2^{1/N}} \quad \text{OR}$$

$$1+4Q^2\left(\frac{\delta\omega}{\omega_0}\right)^2 = 2^{1/N} \quad \text{OR}$$

$$2\delta\omega = \frac{\omega_0}{Q}\sqrt{2^{1/N}-1} \qquad (12.110)$$

CONT.

Thus: $|T(j\omega)|_{overall} = |T(j\omega)|^N$

$$= \frac{|T(j\omega_0)|_{overall}}{\left[1+4Q^2\left(\frac{\delta\omega}{\omega_0}\right)^2\right]^{N/2}}$$

NOTE $Q = \frac{\omega_0}{B}\sqrt{2^{1/N}-1}$

$$= \frac{|T(j\omega_0)|_{overall}}{\left(1+4\frac{\omega_0^2}{B^2}\left(2^{1/N}-1\right)\left(\frac{\delta\omega}{\omega_0}\right)^2\right)^{\frac{N}{2}}}$$

$$= \frac{|T(j\omega_0)|_{overall}}{\left[1+4\left(2^{1/N}-1\right)\left(\frac{\delta\omega}{B}\right)^2\right]^{\frac{N}{2}}}$$

Part (c) (i)

for bandwidth = 2B, ie $\delta\omega = \pm B$

$$Att = -20\log\left(1+4\left(2^{1/N}-1\right)(1)\right)^{-N/2}$$

$$= -10N\log\left(1+2^{2+1/N}-4\right)$$

$$= 10N\log\left(2^{2+1/N}-3\right)$$

N =	1	2	3	4	5
Att (dB)	6.70	8.49	9.28	9.79	10.13

Part (c) (ii)

3 dB bandwidth $\delta\omega = \pm B/2$

30dB bandwidth $\frac{\delta\omega}{B} = x$

$$-30 = -20\frac{N}{2}\log\left(1+4\left(2^{1/N}-1\right)x^2\right)$$

$$3 = N\log\left(1+4\left(2^{1/N}-1\right)x^2\right)$$

$$x = \left[\frac{10^{3/N}-1}{4\left(2^{1/N}-1\right)}\right]^{1/2}$$

Ratio of 30dB to 3dB

$$BW = \frac{2Bx}{B} = 2x$$

N =	1	2	3	4	5
Ratio =	31.6	8.6	5.7		4.5

12.78

See fig 12.48 and Eq 12.115 and 12.116

(a) For the narrowband approximation variation of Ω around 0 is equivalent to $\delta\omega$ around ω_0. Thus, a low-pass maximally flat filter of bandwidth B/2 and order N for which $|T| = \left[\left(1+\Omega/_{B/2}\right)^{2N}\right]^{-\frac{1}{2}}$

is transformed to a band-pass maximally flat filter of bandwidth B/2 and order 2N, and centre frequency ω_0, for which:

$$|T| = \left(1+\left(\frac{\delta\omega}{B/2}\right)^{2N}\right)^{-1/2}$$

(b) For bandwidth 2B, $\delta\omega = B$ &

$$|T| = \left(1+\left(\frac{B}{B/2}\right)^{2N}\right)^{-1/2}$$

$$= \left(1+2^{2N}\right)^{-1/2} \quad \text{thus:}$$

N	1	2	3	4	5
$\lvert T\rvert$	0.447	0.242	0.124	0.062	0.031
$\lvert T\rvert_{dB}$	-6.99	-12.3	-18.1	-24.1	-30.1

For 30 dB bandwidth,

$-30 = 20\log x \Rightarrow x = 10^{-3/2}$
$= \frac{1}{31.6}$

$\therefore \quad 1 + \left(\frac{\delta' \omega}{B/2}\right)^{2N} = (31.6)^2$

$\left(\frac{\delta' \omega}{B/2}\right)^{2N} = 999 - 1 = 998$

Now the ratio of 30dB to 3dB bandwidths is

$\text{ratio} = \frac{2\delta'\omega}{B} = \frac{\delta'\omega}{B/2} = 998^{\frac{1}{2N}}$

N	1	2	3	4	5
ratio	31.6	5.62	3.16	2.37	1.99

12.79

$A_{max} = 3\text{dB} \Rightarrow \epsilon = \sqrt{10^{A_{max}/10} - 1} \approx 1$

Poles of low pass prototype are given by FIG 12.10(c)

Poles: $-\omega_p$, $\omega_p\left(-\tfrac{1}{2} \pm j\sqrt{3}/2\right)$

Make $\omega_p = B/2$

$\Rightarrow$ poles: $\left\{-\frac{B}{2}, \ +\frac{B}{2}\left(-\frac{1}{2} \pm j\frac{\sqrt{3}}{2}\right)\right\}$

Using the low-pass to bandpass transformation:

Poles of the band pass filter:

$-\frac{B}{2} \pm j\omega_0$,

$-\frac{B}{4} \pm j\left(\frac{\sqrt{3}}{4}B + \omega_0\right)$ and

$-\frac{B}{4} \pm j\left(\frac{\sqrt{3}}{4}B - \omega_0\right)$

For the three circuits:

① $\omega_{01} = \omega_0 \qquad B_1 = B \qquad Q_1 = \omega_0/B$

② $\omega_{02} \simeq \frac{\sqrt{3}}{4}B + \omega_0 \qquad B_2 = \frac{B}{2} \qquad Q_2 \simeq \frac{2\omega_0}{B}$

③ $\omega_{03} \simeq \frac{\sqrt{3}}{4}B - \omega_0 \qquad B_3 = \frac{B}{2} \qquad Q_3 \simeq \frac{2\omega_0}{B}$