Problem 5 (15 points)

(Car) The new Mitsubishi Lancer is being advertised in Al-Waseet for \$12,900 with credit facilities up to 7 years and an interest rate of 3.75%. You are considering buying this car with the maximum credit facility period of 7 years from M Bank. To simplify the analysis, assume annual payment and compounding periods.

(a) Assume M Bank charges interest on the outstanding balance of the car loan. That is, the annual payment will be used to cover interest on the outstanding balance and part of the car price. What would your annual payment be?

(b) Assume now that M Bank adopts a form of installment payment, where interest every year is charged based on the initial balance. That is, the annual payment will be used to cover interest on the initial balance (\$12,900) and part of the car price. What would your annual payment be?

(c) What is approximately the *actual* interest rate you are charged under the installment payment scheme in (b)?

Solution

(a) With the usual annuity formula,

$$A = 12{,}900(A/P, 3.75\%, 7) = 12{,}900 * 0.0375/(1-1.0375^{-7}) = \$2{,}129.45$$

(b) With installment payment,

$$A = 12{,}900 * (F/P, 3.75\%, 7)/7 = 12{,}900 * 1.0375^{7} / 7 = \$2{,}384.56$$

(Or $A = 12{,}900 / 7 + 12{,}900 * 0.0375 = \$2{,}326.61$)

(c) A (installment payments) = $2{,}384.56 = 12{,}900 * i_a / [1 - (1+ i_a)^{-7}]$. Therefore,

$$i_a / [1 - (1+ i_a)^{-7}] = 0.185 .$$

By trial and error, we find that $i_a \approx 6.9\%$.

(Or $i_a / [1 - (1+ i_a^{-7})] = 0.180$ and $i_a \approx 6.1\%$)

Question 4
(To buy, or not to buy) Antoon, an exuberant young engineer, is considering buying a new apartment similar to the one he is currently renting. He received an offer from his bank to finance the new apartment purchase. The bank will pay the apartment price of $45,500. In exchange, Antoon will pay the bank back in yearly installments for 20 years. He will pay $5,040/year for the first 10 years, and $2400/year for the following 10 years. His current rent is $2,520/year. He expects that rent will increase by 10% every 5 years for the next 20 years. Antoon's MARR is 3% per year (this is the interest he can earn from a saving account).

a) What should Antoon do (buy or continue renting) to maximize his wealth over the next 20 years? Assume that the new apartment will loose 30% of its $45,500 market value after 20 years.

b) What would you do if you were in Antoon's shoes? Elaborate on your answer with no more than few sentences.

Solution

a) The present worth for the renting option is

$$P_{\text{Rent}} = A_{1-5}\frac{[(1+i)^5-1]}{i(1+i)^5} + A_{6-10}\frac{[(1+i)^5-1]}{i(1+i)^5}\times\frac{1}{(1+i)^5} + A_{11-15}\frac{[(1+i)^5-1]}{i(1+i)^5}\times\frac{1}{(1+i)^{10}}$$

$$+ A_{16-20}\frac{[(1+i)^5-1]}{i(1+i)^5}\times\frac{1}{(1+i)^{15}}$$

$$= -\$42,742.06$$

(computation details below)

The present worth for the buying option is

$$P_{\text{Buy}} = A_{1-10}\frac{[(1+i)^{10}-1]}{i(1+i)^{10}} + A_{11-20}\frac{[(1+i)^{10}-1]}{i(1+i)^{10}}\times\frac{1}{(1+i)^{10}} + \frac{F_{20}}{(1+i)^{20}}$$

$$= -\$40,591.10$$

(computation details below)

<u>Antoon should buy.</u>

$$-\left[2520\frac{1.03^5-1}{0.03\left(1.03^5\right)} + (2520\cdot 1.1)\cdot\frac{1.03^5-1}{0.03\left(1.03^5\right)}\cdot\frac{1}{1.03^5} + \left(2520\cdot 1.1^2\right)\cdot\frac{1.03^5-1}{0.03\left(1.03^5\right)}\cdot\frac{1}{1.03^{10}} + \left(2520\cdot 1.1^3\right)\cdot\frac{1.03^5-1}{0.03\left(1.03^5\right)}\cdot\frac{1}{1.03^{15}}\right] = -42742.062$$

$$-5040\frac{1.03^{10}-1}{0.03\left(1.03^{10}\right)} - 2400\left[\frac{1.03^{10}-1}{0.03\left(1.03^{10}\right)}\cdot\frac{1}{1.03^{10}}\right] + \frac{45500\cdot 0.7}{1.03^{20}} = -40591.102$$

$$40591.102\cdot 1.03^{20} = 73312.045 \qquad -45091.102\cdot 1.03^{20} = -81439.546$$

b) It’s a matter of personal preferences given the factors in (c). From an economic point of view, the market value of the property plays an important role. Here, we assume that the apartment will loose 30% of its value in 20 years but we ignore the maintenance cost and property tax. A well maintained apartment may increase in value. In addition, the person’s MARR is equally important. The 3% MARR here deduced from a saving account is conservative. More risky investments such as investing in the stock market could yield higher returns.

Problem 7 (15 points)

(HB Mortgage) The Housing Bank (HB) offers home loans to Lebanese professionals. Borrowers receive the loan immediately upon buying a home, and pay back in equal monthly installments for 20 years. To encourage low-income applicants, a fixed interest rate of 6% compounded monthly is offered on loans not exceeding $120,000. A loan above $120,000 is subject to the 6% rate on the first $120,000, and the remainder part is subject to a higher interest of 8% compounded monthly. (E.g., for a $130,000 loan, the interest is 6% on the first $120,000 and 8% on the remainder $10,000). Furthermore, to help the borrower covering the many expenses associated with the purchase (e.g., taxes, fees, commissions, etc.), a 3-month "grace period" is offered. (That is, the first monthly payment is made at the end of the fourth month after receiving the loan.) However, interest at the rates above is applied to the loan during the grace period.

(a) Sa'eed, a successful EM graduate working for a growing consulting firm, applied for a $150,000 loan from HB to buy a small flat. How much is Sa'eed's monthly payment? (10 points)

(b) Sa'eed decided to go for a bigger home and now requires more money than in (a). However, HB has a stated policy that the monthly payment should not exceed one third of the borrower income. If Sa'eed's salary is $3,600 per month, what is the maximum loan he can get from HB? (10 points)

Solution

a) Saeed's monthly payment

Loan value after the 3-month grace period

$$120000\,1.005^{3} + 30000\,1.00667^{3} = 152413$$

Monthly payment on the first $120 K

$$A(P, r, n) := \frac{P \cdot r}{1 - (1 + r)^{-n}}$$

$$A(120000, 0.005, 237) = 865$$

Monthly payment on the remainder

$$A(32413, 0.00667, 237) = 273$$

Total monthly payment

$$A(120000, 0.005, 237) + A(32413, 0.00667, 237) = 1138$$

b) Maximum loan amount

$$A_{max} := \frac{3600}{3} \qquad P(A, r, n) := \frac{A}{r} \cdot \left[1 - (1 + r)^{-n}\right]$$

$$A_{max} = 1200$$

Since these exceeds 1082 from so the loan will definetly exceed 120,000. So, $865 this will be used to cover the first $120,000 of the loan value.

The remainder $1200 - 865 = 335$ will be used to cover the high-interst part of the loan

These allow for a loan value of $P(335, 0.00667, 237) = 39834$

So, the loan balance 3 months after taking it is $120000 + 39834 = 159834$

The maximum loan Saeed could take is $\dfrac{120000}{1.005^{3}} + \dfrac{39834}{1.00667^{3}} = 157265$