Homework # 3 Solution (Exercises 5.11- 12- 13 & 27)

B-5-11.

$$\frac{C(s)}{R(s)} = \frac{16}{s^2 + (0.8 + 16k)s + 16}$$

From the characteristic polynomial, we find

$$\omega_n = 4, \quad 2\zeta\omega_n = 2 \times 0.5 \times 4 = 0.8 + 16k$$

Hence

$$k = 0.2$$

The rise time t_r is obtained from

$$t_r = \frac{\pi - \beta}{\omega_d}$$

Since

$$\omega_d = \omega_n\sqrt{1-\zeta^2} = 4\sqrt{1-0.25} = 3.46$$

$$\beta = \sin^{-1}\frac{\omega_d}{\omega_n} = \sin^{-1} 0.866 = \frac{\pi}{3}$$

we have

$$t_r = \frac{\pi - \frac{1}{3}\pi}{3.46} = 0.605 \text{ sec}$$

The peak time t_p is obtained as

$$t_p = \frac{\pi}{\omega_d} = \frac{3.14}{3.46} = 0.907 \text{ sec}$$

The maximum overshoot M_p is

$$M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} = e^{-\frac{0.5 \times 3.14}{\sqrt{1-0.25}}} = e^{-1.814} = 0.163$$

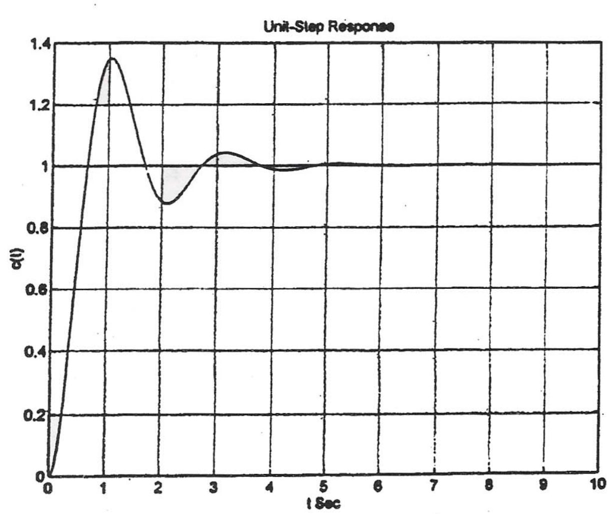
The settling time t_s is

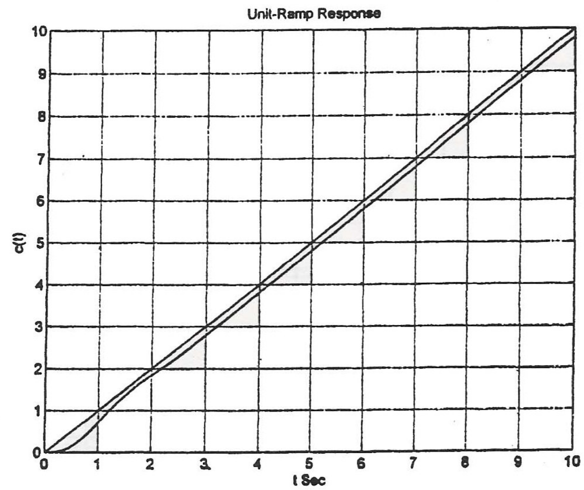
$$t_s = \frac{4}{\zeta\omega_n} = \frac{4}{0.5 \times 4} = 2 \text{ sec}$$

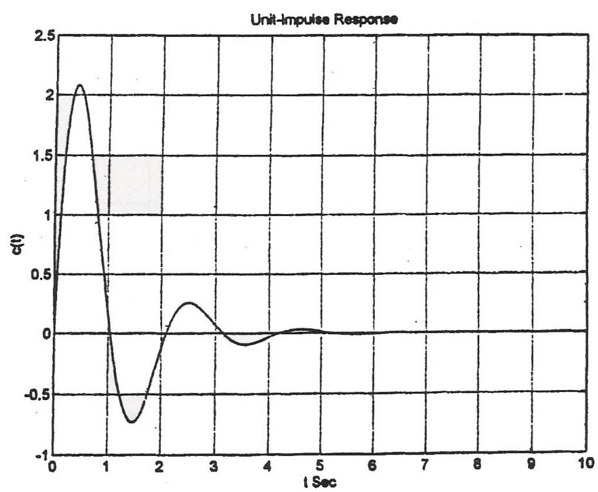
B-5-12. A MATLAB program to obtain the unit-step response, unit-ramp response, and unit-impulse response of the given system is shown on the next page.

```
% ***** Unit-step response *****

num = [0   0   10];
den = [1   2   10];
t = 0:0.02:10;
step(num,den,t)
grid
title('Unit-Step Response')
xlabel('t Sec')
ylabel('c(t)')

% ***** Unit-ramp response *****

numr = [0   0  0   10];
denr = [1   2   10   0];
c = step(numr,denr,t);
plot(t,c,'-',t,t,'--')
grid
title('Unit-Ramp Response')
xlabel('t Sec')
ylabel('c(t)')

% ***** Unit-impulse response *****

impulse(num,den,t)
grid
title('Unit-Impulse Response')
xlabel('t Sec')
ylabel('c(t)')
```

The unit-step response curve is shown below. The unit-ramp response curve and unit-impulse response curve are shown on the next page.

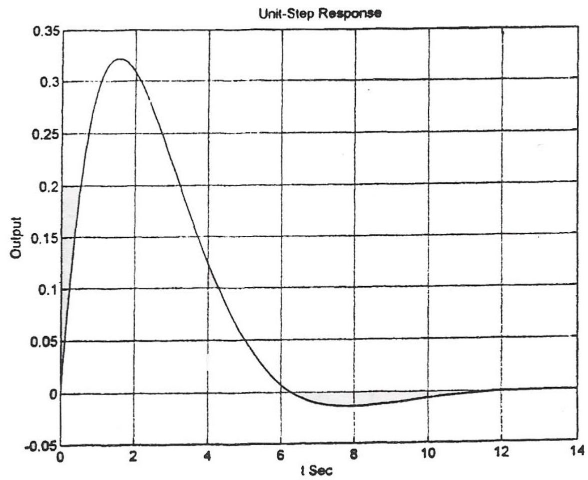






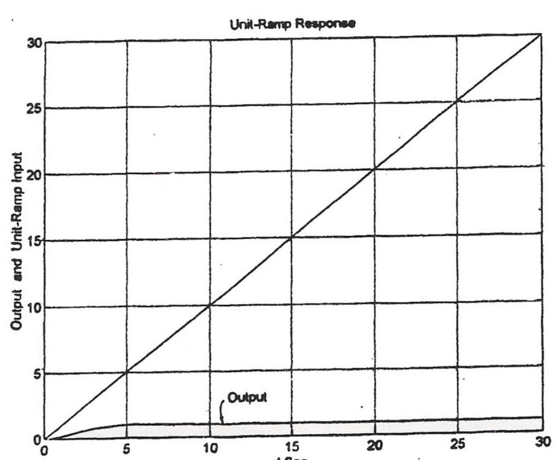
B-5-13. A MATLAB program to obtain a unit-step response of the given system is given below. The resulting unit-step response curve is shown on the next page.

```
% ***** Unit-Step Response *****

A = [-1   -0.5;1   0];
B = [0.5;0];
C = [1   0];
D = [0];
[y,x,t] = step(A,B,C,D);
plot(t,y)
grid
title('Unit-Step Response')
xlabel('t Sec')
ylabel('Output')
```



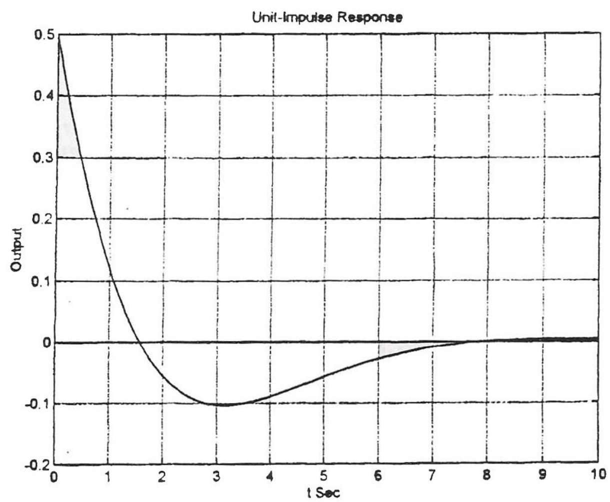
A MATLAB program to obtain a unit-ramp response of the given system is presented below, together with the unit-ramp response curve.

```
% ***** Unit-ramp response *****

A = [-1   -0.5;1   0];
B = [0.5;0];
C  = [1   0];
D = [0];

% ***** Enter matrices AA, BB, CC, and DD of the new enlarged state
% equation and output equation *****

AA = [A   zeros(2,1);C   0];
BB = [B;0];
CC = [0   0   1];
DD = [0];

% ***** Enter step-response command [z,x,t] = step(AA,BB,CC,DD) *****

[z,x,t] = step(AA,BB,CC,DD);
x3 = [0   0   1]*x'; plot(t,x3,t,t,'--')
grid
title('Unit-Ramp Response')
xlabel('t Sec')
ylabel('Output  and  Unit-Ramp Input')
text(11,3,'Output')
```



Finally, a MATLAB program to obtain a unit-impulse response of the system is given next, together with the unit-impulse response curve.

```
% ***** Unit-impulse response *****

A = [-1   -0.5;1   0];
B = [0.5;0];
C = [1   0];
D = [0];
impulse(A,B,C,D)
```



B-5-27. From Figure 5-89(b) we have

$$\frac{C(s)}{R(s)} = \frac{K}{Js^2 + KK_h s + K} = \frac{\frac{K}{J}}{s^2 + \frac{KK_h}{J} s + \frac{K}{J}}$$

By substituting K/J = 4 into this last equation, we obtain

$$\frac{C(s)}{R(s)} = \frac{4}{s^2 + 4K_h s + 4}$$

Since $\omega_n = 2$, $\zeta = 0.6$, and $2\zeta\omega_n = 4K_h$, we have

$$K_h = \frac{2\zeta\omega_n}{4} = 0.6$$
