Homework # 4 Solution (Exercises 6.9-10 & 15)

B-6-9. The open-loop transfer function

$$G(s)H(s) = \frac{K(s+9)}{s(s^2+4s+11)}$$

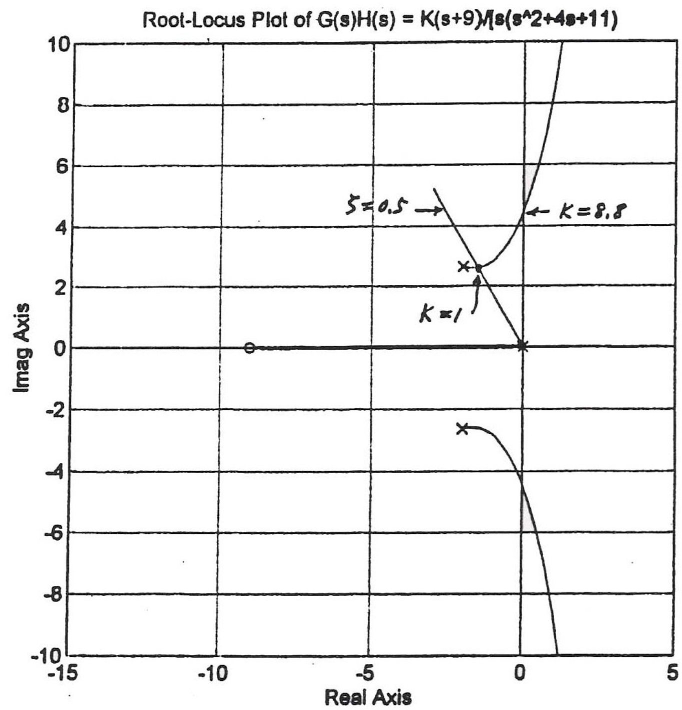
has the poles at s = 0, s = $-2 \pm j\sqrt{7}$ and the zero at s = -9. The asymptotes have angles + 90° and meet the real axis at σ_a = 2.5. The complex branches cross the imaginary axis at s = ± j 4.45. The angle of departure from the complex pole in the upper half s plane is -16.5°.

The dominant closed-loop poles having the damping ratio ζ = 0.5 can be located as the intersection of the root loci and lines from the origin having angles ± 60°. The desired dominant closed-loop poles are found to be at

$$s = -1.5 \pm j\,2.598$$

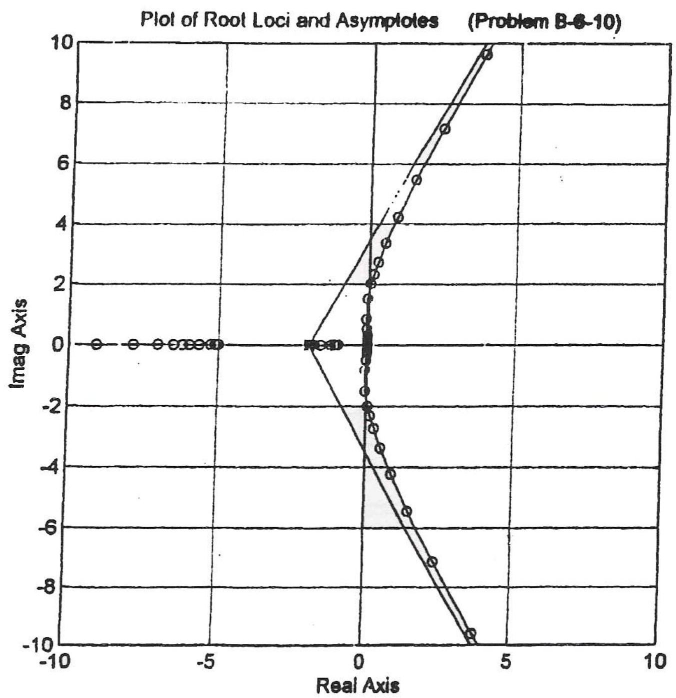
The third pole is at s = -1. The gain value corresponding to these dominant closed-loop poles is K = 1. A MATLAB program to plot the root-loci is shown below. The resulting root-locus plot is shown on the next page.

```
% ***** Root-locus plot *****

num = [0  0  1  9];
den = [1  4  11  0];
rlocus(num,den)
hold
Current plot held
x = [0,-3]; y = [0,5.196]; line(x,y);
v = [-15  5  -10  10]; axis(v); axis('square')
grid
title('Root-Locus Plot of G(s)H(s) = K(s+9)/[s(s^2+4s+11)]')
```



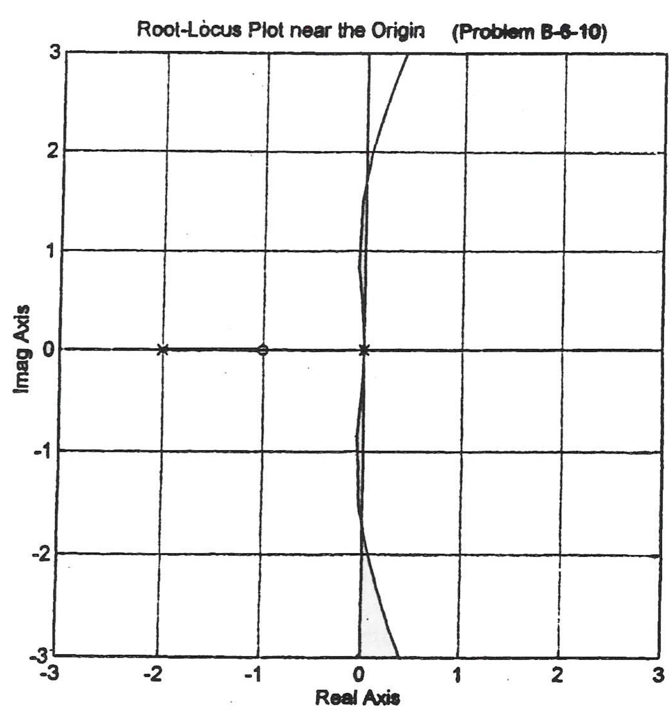
B-6-10. A MATLAB program to obtain a root-locus plot of the given system is shown below. The resulting root-locus plot is shown on the next page.

```
% ***** Root-locus plot *****

num = [0  0  0  2  2];
den = [1  7  10  0  0];
numa = [0  0  0  1];
dena = [0.5  3  6  4];
r = rlocus(num,den);
plot(r,'-')
hold
Current plot held
plot(r,'o')
rlocus(numa,dena)
v = [-10  10  -10  10]; axis(v); axis('square');
grid
title('Plot of Root Loci and Asymptotes    (Problem B-6-10)')
```



A root-locus plot near the origin can be obtained by entering the following MATLAB program into the computer. The resulting root-locus plot near the origin is shown next.

```
% ***** Root-locus plot *****

num = [0  0  0  2  2];
den = [1  7  10  0  0];
rlocus(num,den)
v = [-3  3  -3  3]; axis(v); axis('square');
grid
title('Root-Locus Plot near the Origin    (Problem B-6-10)')
```



The range of K for stability can be determined by use of Routh stability criterion. Since the closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{2K(s+1)}{s^4 + 7s^3 + 10s^2 + 2Ks + 2K}$$

the characteristic equation for the system is

$$s^4 + 7s^3 + 10s^2 + 2Ks + 2K = 0$$

The Routh array of coefficients becomes as follows:

$$\begin{array}{llll}
s^4 & 1 & 10 & 2K \\
s^3 & 7 & 2K & \\
s^2 & \dfrac{70-2K}{7} & 2K & \\
s^1 & \dfrac{\frac{(70-2K)2K}{7} - 14K}{\frac{70-2K}{7}} & 0 & \\
s^0 & 2K & &
\end{array}$$

For stability, we require

$$70 > 2K$$

$$42 - 4K > 0$$

$$K > 0$$

Thus, the range of K for stability is

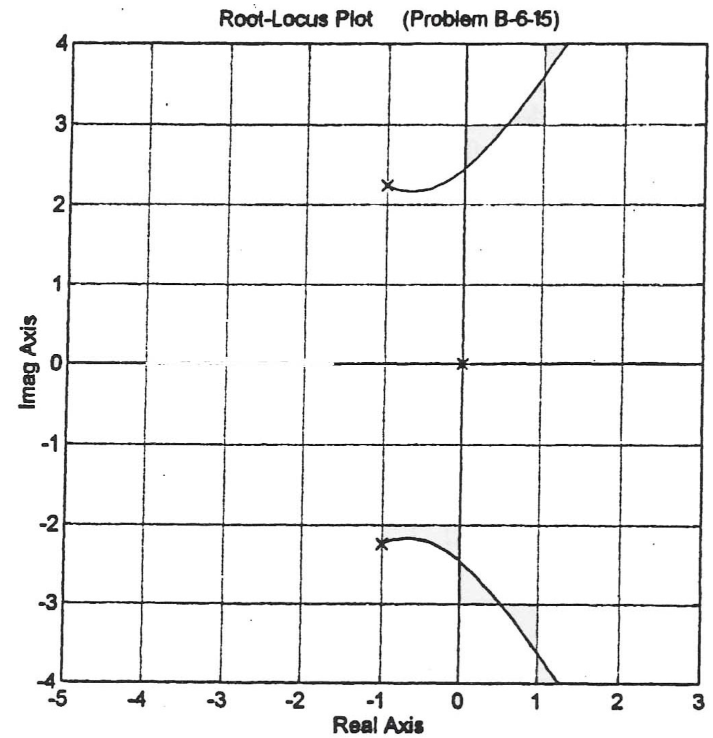
$$10.5 > K > 0$$

B-6-15. The term (s + 1) in the feedforward transfer function and the term (s + 1) in the feedback transfer function cancel each other. The reduced characteristic equation is

$$1 + G(s)H(s) = 1 + \frac{K(s+1)}{s(s^2+2s+6)} \frac{1}{s+1} = 1 + \frac{K}{s(s^2+2s+6)} = 0$$

The open-loop poles of G(s)H(s) is at s = 0 and $s = -1 \pm j\sqrt{5}$. The following MATLAB program produces the root-locus plot shown on the next page.

```
% ***** Root-locus plot *****

num = [0  0  0  1];
den = [1  2  6  0];
rlocus(num,den)

Warning: Divide by zero
v = [-5  3  -4  4]; axis(v); axis('square')
grid
title('Root-Locus Plot    (Problem B-6-15)')
```



To find the closed-loop poles when the gain K is set equal to 2, we may enter the following MATLAB program into the computer.

```
p = [1   2   6   2];
roots(p)

ans =

  -0.8147 + 2.1754i
  -0.8147 - 2.1754i
  -0.3706
```

Thus, the closed-loop poles are located at

$$s = -0.8147 \pm j\ 2.1754, \qquad s = -0.3706$$
