Example B-9-8

Consider the system shown in Figure 9-66. It is desired to design a compensator such that the static velocity error constant is 4 $\sec^{-1}$, phase margin is 50°, and gain margin is 8 dB or more. Plot the unit-step and unit-ramp response curves of the compensated system with LabVIEW.

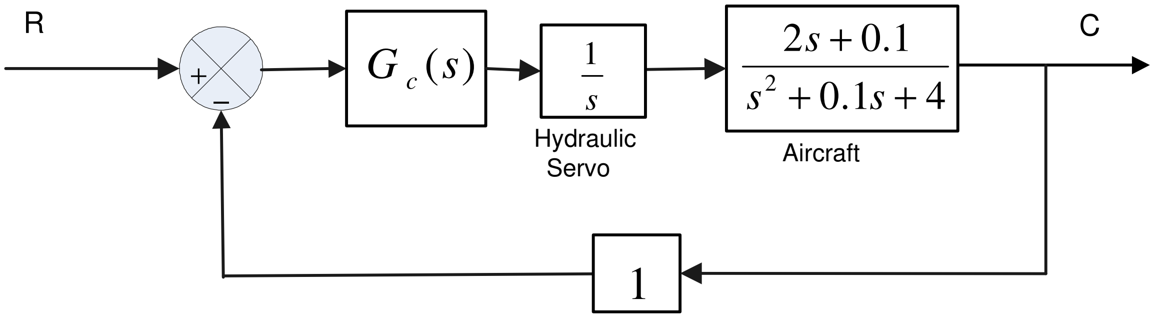


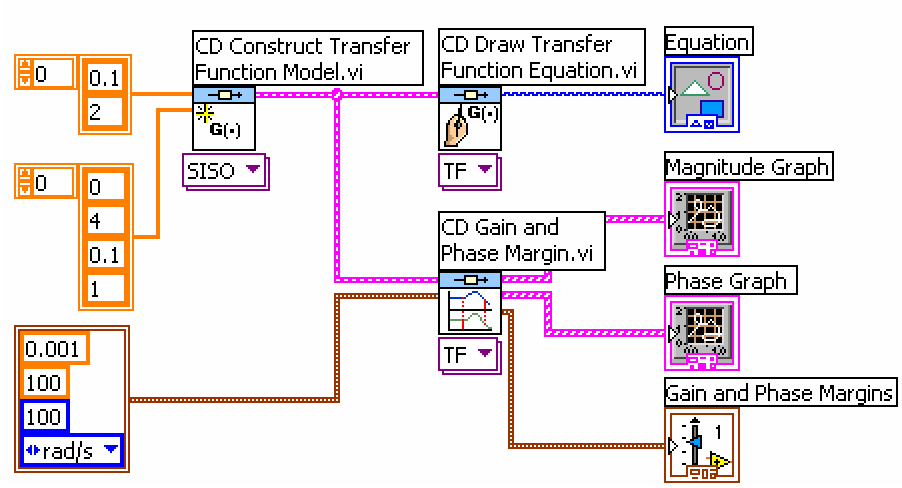
Figure 9-66. Control System

Solution:

The plant transfer function is

$$G(s) = \frac{2s + 0.1}{s(s^2 + 0.1s + 4)}$$

The plant involves a quadratic term with $\varsigma = 0.025$. This term is quite oscillatory. LabVIEW Program B-9-8(a) produces the Bode diagram of G(s) as shown in Figure 9-66(a).



LabVIEW Program B-9-8(a)

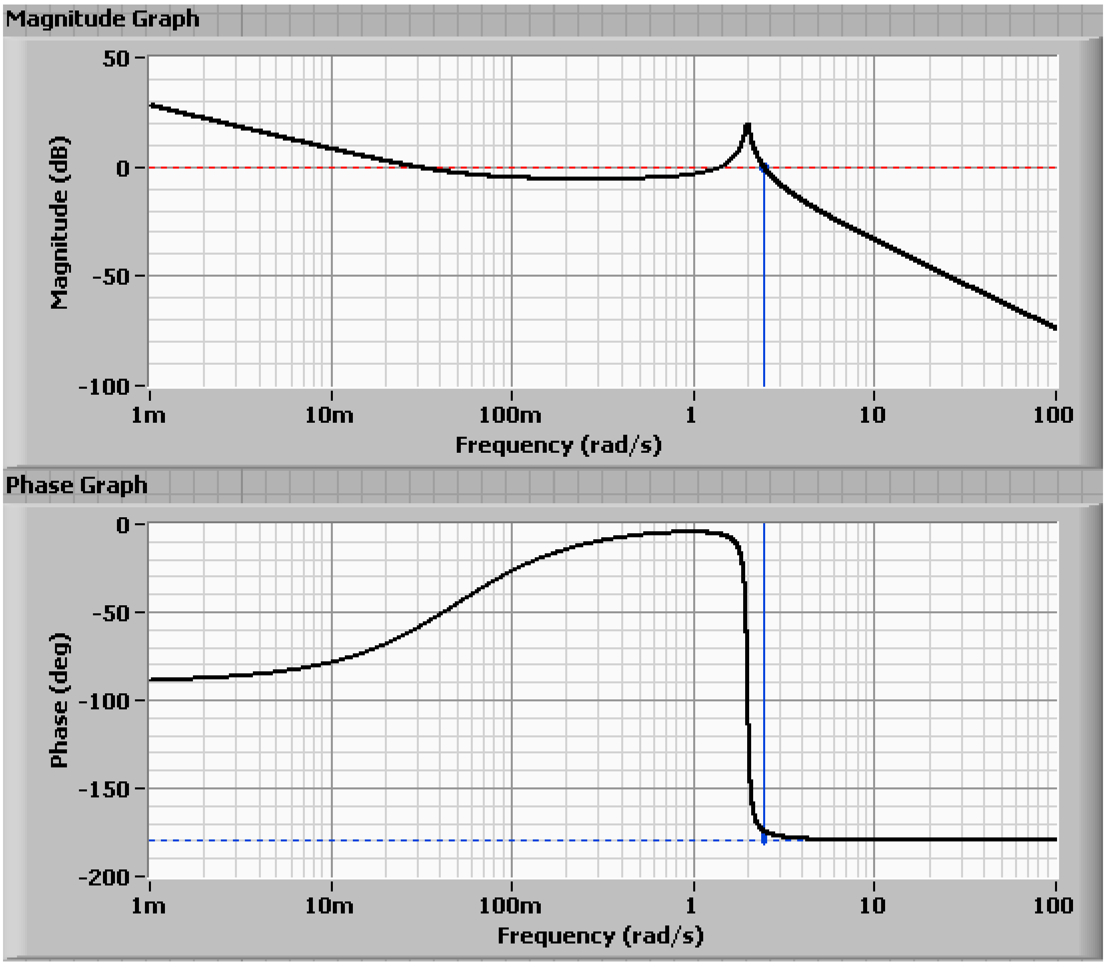


Figure 9-66(a). Bode diagram of G(s)

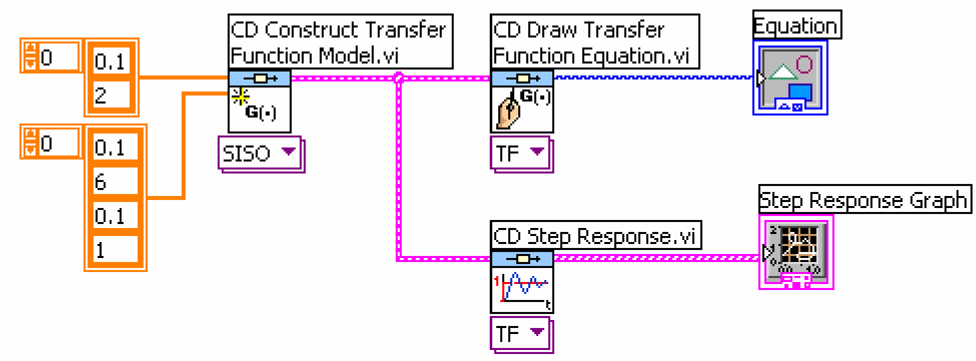
The closed-loop transfer function of the original uncompensated system is

$$\frac{C(s)}{R(s)} = \frac{2s + 0.1}{s^3 + 0.1s^2 + 6s + 0.1}$$

The closed-loop poles of the uncompensated system are

$$s = -0.0417 + j2.4489$$
$$s = -0.0417 - j2.4489$$
$$s = -0.0167$$

LabVIEW Program B-9-8(b) is used to obtain the unit-step response of this original, uncompensated system. The resulting unit-step response curve is shown in Figure 9-66(b).



LabVIEW Program B-9-8(b)

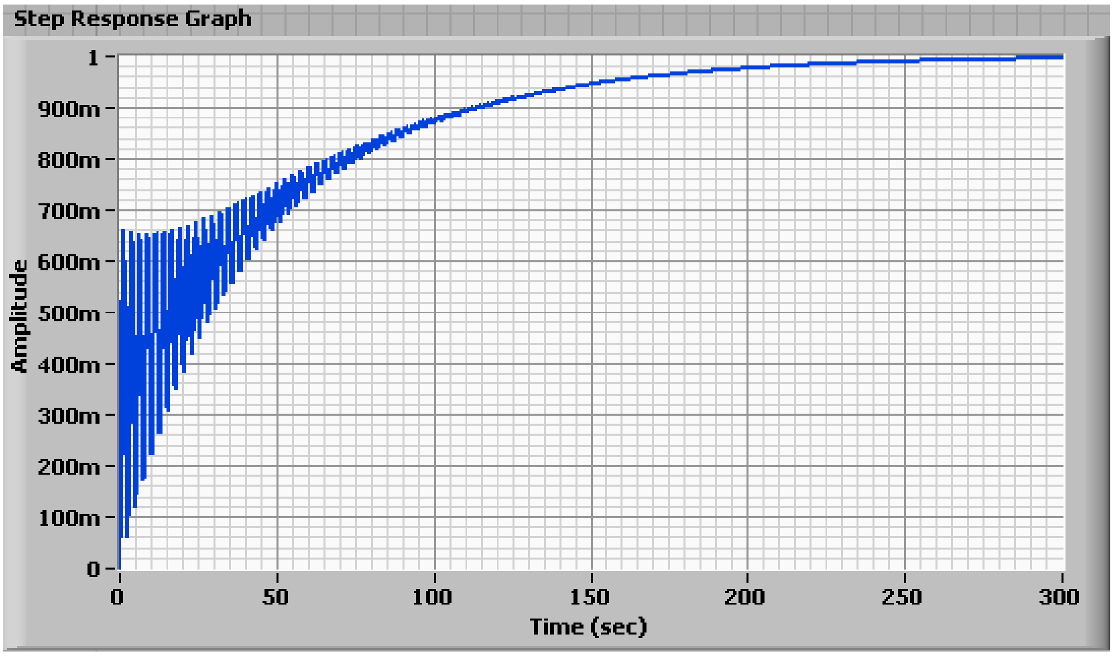


Figure 9-66(b). Unit-step response of uncompensated system

To design a compensator for such a system, it is desirable to cancel the zero of the plant, since it is located very close to the origin. It is some-times useful to include double zero and double pole in the compensator. So, we may choose the compensator to be

$$G_c(s) = K_c \frac{(s+2)^2}{(s+10)^2} \frac{s+a}{2s+0.1} \ .$$

Where we have chosen the double zero at s = -2 and double pole at s = -10. The value of a is to be determined later. Since the static velocity error constant is specified as 4 $\sec^{-1}$, we have

$$K_v = \lim_{s\to 0} s G_c(s) G(s) = \lim_{s\to 0} s K_c \frac{(s+2)^2}{(s+10)^2} \frac{s+a}{2s+0.1} \frac{2s+0.1}{s(s^2+0.1s+4)}$$

$$= K_c \frac{a}{100} = 4$$

Hence,

$$K_c\ a = 400$$

By several trials we find a = 4 will give satisfactory results. Therefore, we choose a = 4 and K_c = 100. Then, the transfer function of the compensator becomes

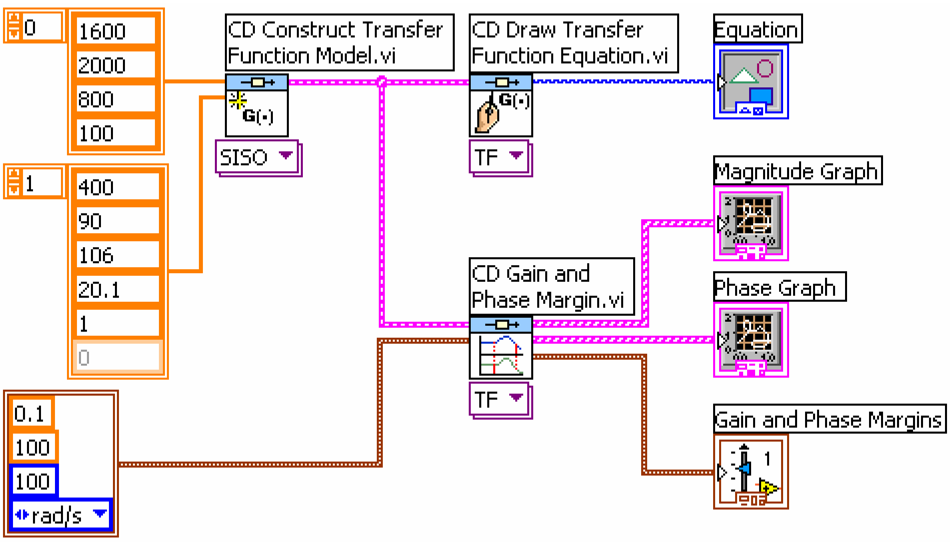
$$G_c(s) = 100 \frac{(s+2)^2}{(s+10)^2} \frac{s+4}{2s+0.1}$$

The open-loop transfer function becomes as follows:

$$G_c(s) G(s) = \frac{100(s+2)^2 (s+4)}{(s+10)^2\ s(s^2+0.1s+4)}$$

$$= \frac{100s^3 + 800s^2 + 2000s + 1600}{s^5 + 20.1s^4 + 106s^3 + 90s^2 + 400s}$$

LabVIEW Program B-9-8(c) produces the Bode diagram of $G_c(s)G(s)$. The resulting Bode diagram is shown in Figure 9-66(c).



LabVIEW Program B-9-8(c)

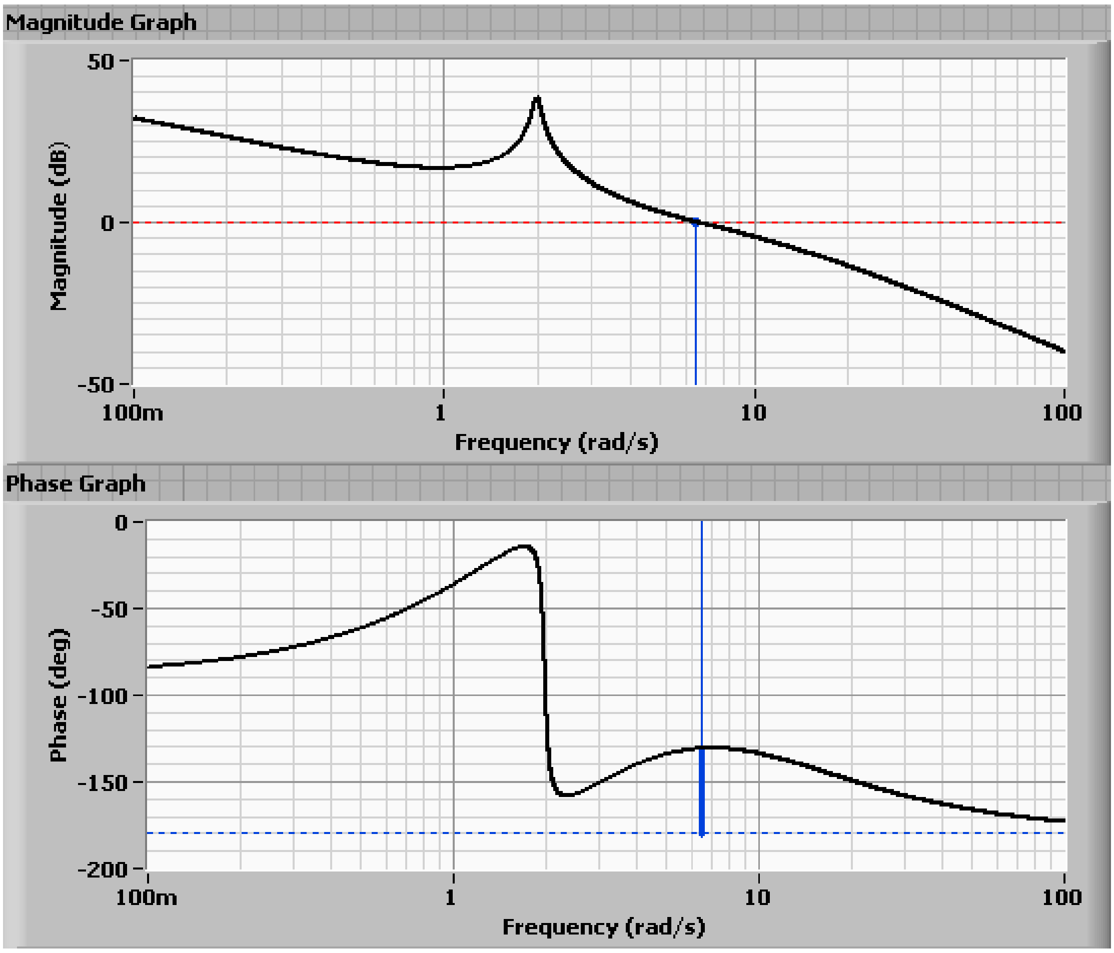


Figure 9-66(c). Bode diagram of $G_c(s)G(s)$.

From this Bode diagram, it is seen that $K_v = 4$ sec^{-1}, phase margin is $+\infty$ dB. So, all the requirements are met.

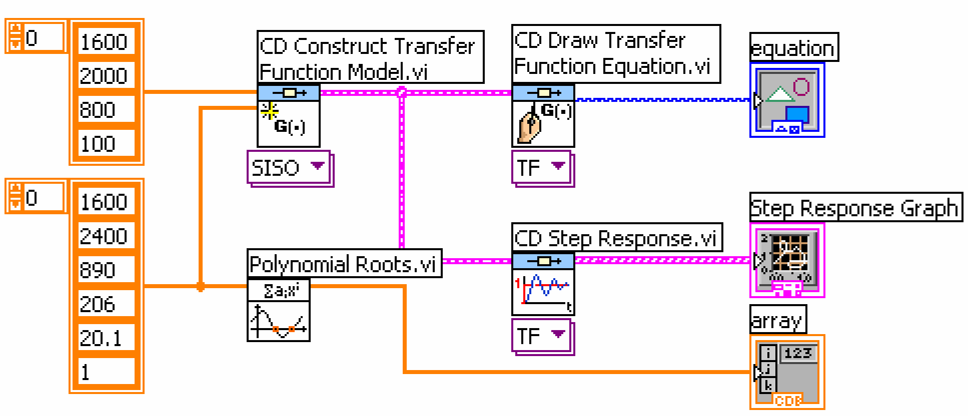
The closed-loop transfer function of the compensated system becomes as follows:

$$\frac{C(s)}{R(s)} = \frac{100s^3 + 800s^2 + 2000s + 1600}{s^5 + 20.1s^4 + 206s^3 + 890s^2 + 2400s + 1600}$$

The closed-loop poles of the compensated system can be found as follows.

Closed Loop Denominator Roots
-0.918881 +0 i
-2.24243 +3.37514 i
-2.24243 -3.37514 i
-7.34813 -7.21451 i
-7.34813 +7.21451 i

LabVIEW Program B-9-8(d) produces the unit-step response of the designed system. The unit-step response curve is shown in Figure 9-66(d).



LabVIEW Program B-9-8(d)

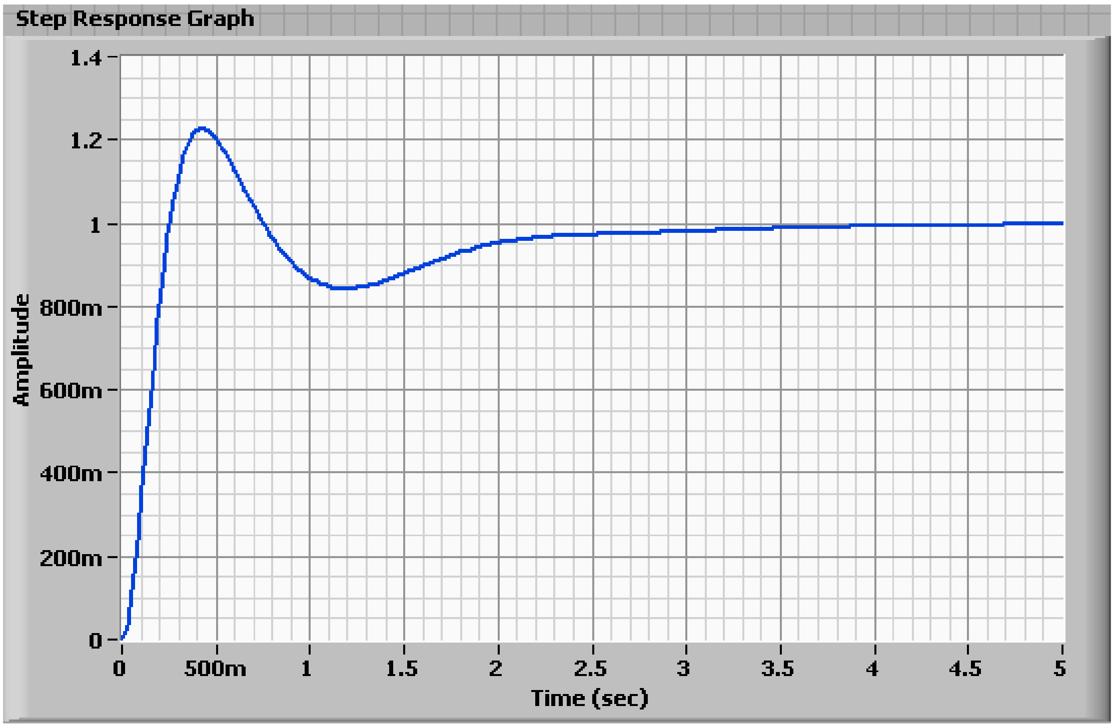
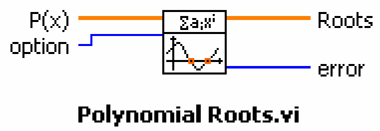


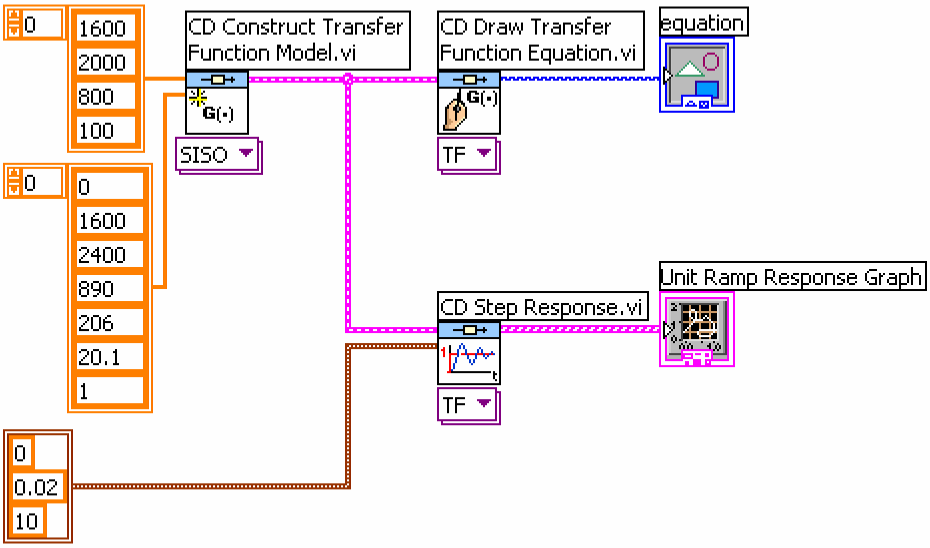
Figure 9-66(d). Unit-step response of the designed system.

LabVIEW Program B-9-8(e) produces the unit-ramp response of the compensated system. The unit-ramp response curve is shown in Figure 9-66(e).

Note the use of the Polynomial Roots VI



which finds the roots of the polynomial P(x).



LabVIEW Program B-9-8(e)

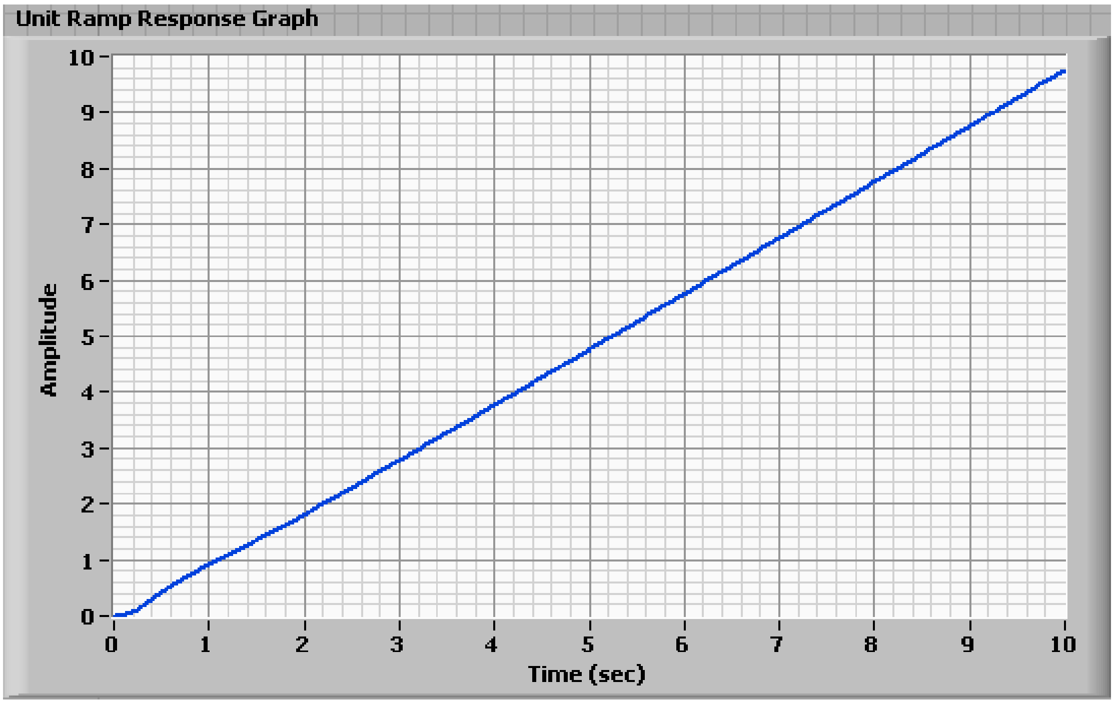


Figure 9-66(e). Unit-ramp response of the compensated system.

It is noted that there are infinitely many possible compensators for this system. A few possible compensators are shown below.

$$G_c(s) = 400\frac{(s+1)^2}{(s+25)(2s+0.1)}$$

$$G_c(s) = 320\frac{(s+1)^2}{(s+20)(2s+0.1)}$$

$$G_c(s) = 160\frac{s+4}{s+30}\,\frac{s+1}{s+0.1333}$$

$$G_c(s) = 1212.12\,\frac{s+9.81}{s+74.32}$$