Example B-9-6

Consider the system shown in Figure 9-64. Design a compensator such that the static velocity error constant K_v is 50 sec^{-1}, phase margin is 50^{o}, and gain margin is not less than 8 dB. Plot unit-step and unit-ramp response curves of the compensated and uncompensated systems with LabVIEW.

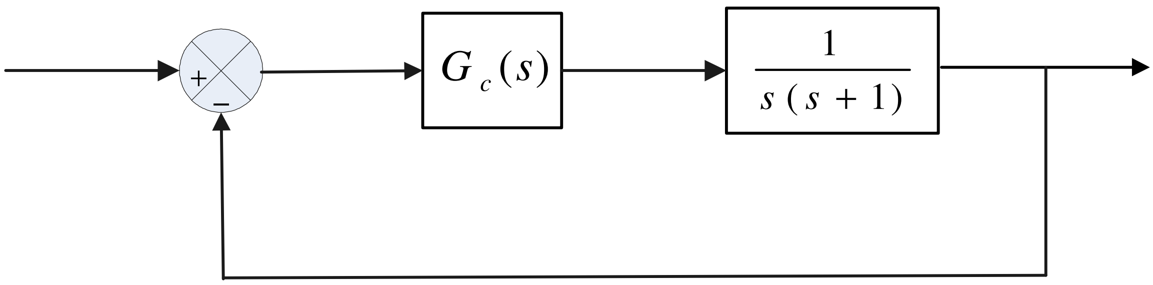


Figure 9-64. Control System

Solution. To satisfy the requirements, try a lead compensator $G_c(s)$ of the form

$$G_c(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

Define

$$G_1(s) = KG(s) = \frac{K}{s(s+1)}$$

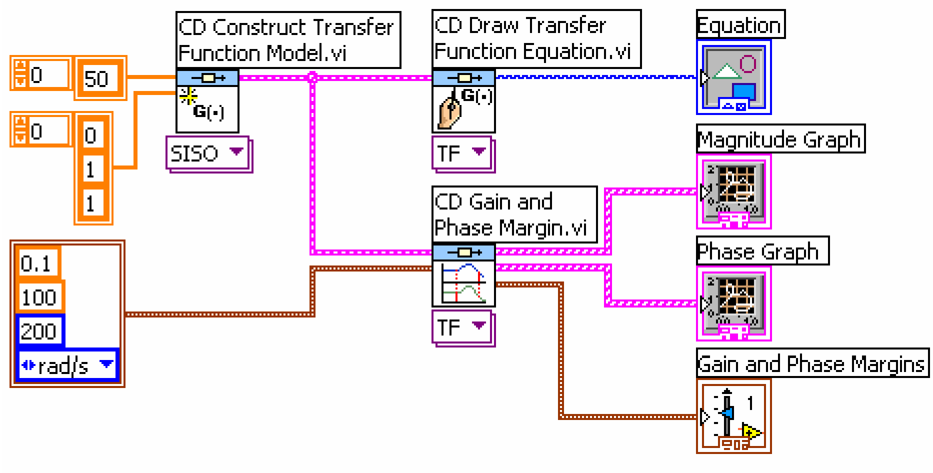
where $K = K_c\alpha$. Since the static velocity error constant K_v is given as 50 sec^{-1}, we have

$$K_v = \lim_{s\to 0} sG_c(s)G(s) = \lim_{s\to 0} s\frac{T\ s+1}{\alpha T\ s+1}\frac{K}{s(s+1)} = K = 50$$

We shall now plot the a Bode diagram of

$$G_1(s) = \frac{50}{s(s+1)}$$

LabVIEW Program B-9-6(a) produces the Bode diagram shown in Figure 9-64(a).



LabVIEW Program B-9-6(a)

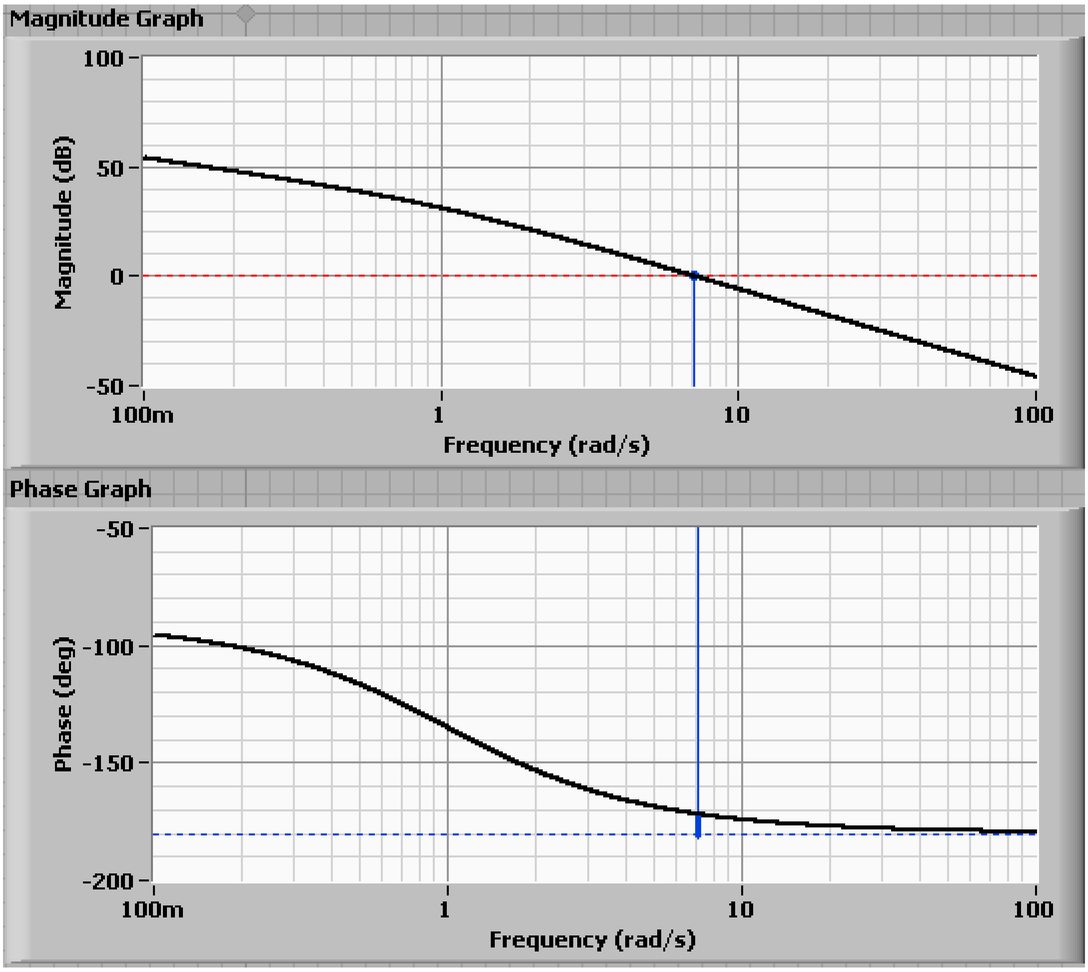


Figure 9-64(a). Bode diagram of $G_1(s)$

From this plot, the phase margin is found to be 7.8°. The gain margin is + ∞ dB. Since the specifications call for a phase margin of 50°, the additional phase lead angle necessary to satisfy the phase requirement is 42.2°. We may assume the maximum

phase lead required to be 48^o. This means that 5.8^o has been added to compensate for the shift in the gain crossover frequency.
Since

$$\sin \varphi_m = \frac{1-\alpha}{1+\alpha}$$

$\varphi_m = 48^o$ corresponds to α = 0.14735.(Note that α = 0.15 corresponds to $\varphi_m = 47.657^o$.) whether we choose $\varphi_m = 48^o$ or $\varphi_m = 47.657^o$ does not make much difference in the final solution. Hence, we choose α = 0.15.

The next step is to determine the corner frequencies ω=1/*T* and ω=1/ (α*T*) of the lead compensator. note that the maximum phase-lead angle φ_m occurs at the geometric mean of the two corner frequencies, or $\omega = 1/(\sqrt{\alpha}T)$. The amount of the modification in the magnitude curve at $\omega = 1/(\sqrt{\alpha}T)$ due to the inclusion of the term (*T* s+1)/(α*T*s+1) is

$$\left|\frac{1+j\omega T}{1+j\omega\alpha T}\right|_{\omega=1/(\sqrt{\alpha}T)} = \left|\frac{1+j\frac{1}{\sqrt{\alpha}}}{1+j\alpha\frac{1}{\sqrt{\alpha}}}\right| = \frac{1}{\sqrt{\alpha}}$$

Note that

$$\frac{1}{\sqrt{\alpha}} = \frac{1}{\sqrt{0.15}} = 2.5820 = 8.239\ dB.$$

We need to find the frequency point where, when the lead compensator is added, the total magnitude becomes 0 dB. The frequency at which the magnitude of $G_1(j\omega)$ is equal to -8.239 dB occurs between ω = 10 and 100 rad/sec. From the Bode diagram we find the frequency point where $|G_1(j\omega)| = -8.239\ dB$ occurs at ω = 11.4 rad/sec. Noting that this frequency corresponds to $1/(\sqrt{\alpha}T)$, or

$$\omega_c = \frac{1}{\sqrt{\alpha}\,T}$$

We obtain

$$\frac{1}{T} = \omega_c\sqrt{\alpha} = 11.4\sqrt{0.15} = 4.4152$$

$$\frac{1}{\alpha T} = \frac{\omega_c}{\sqrt{\alpha}} = \frac{11.4}{\sqrt{0.15}} = 29.4347$$

The lead compensator thus determined is

$$G_c(s) = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} = K_c \frac{s + 4.4152}{s + 29.4347}$$

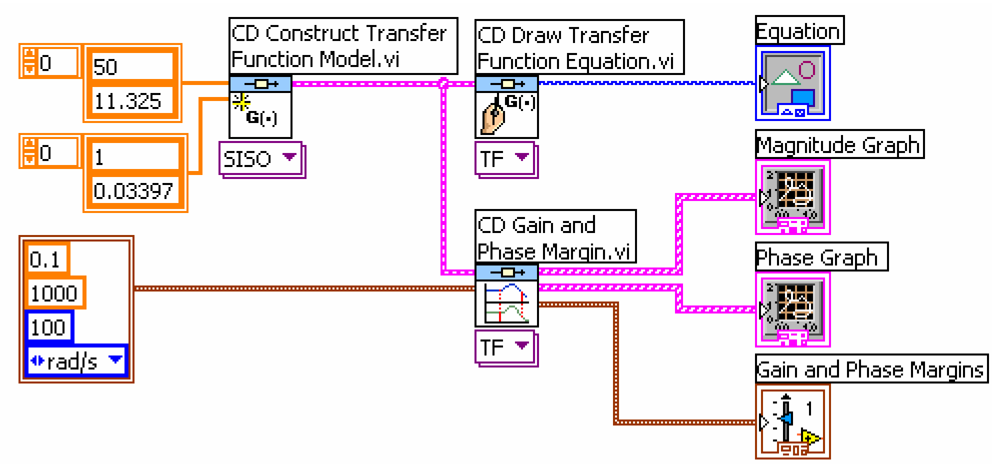
Where K_c is determined as

$$K_c = \frac{K}{\alpha} = \frac{50}{0.15} = \frac{1000}{3}$$

Thus,

$$G_c(s) = \frac{1000}{3} \frac{s + 4.4152}{s + 29.4347} = 50 \frac{0.2265s + 1}{0.03397s + 1}$$

LabVIEW Program B-9-6(b) produces the Bode diagram of the lead compensator just designed, as shown in Figure 9-64(b).



LabVIEW Program B-9-6(b)

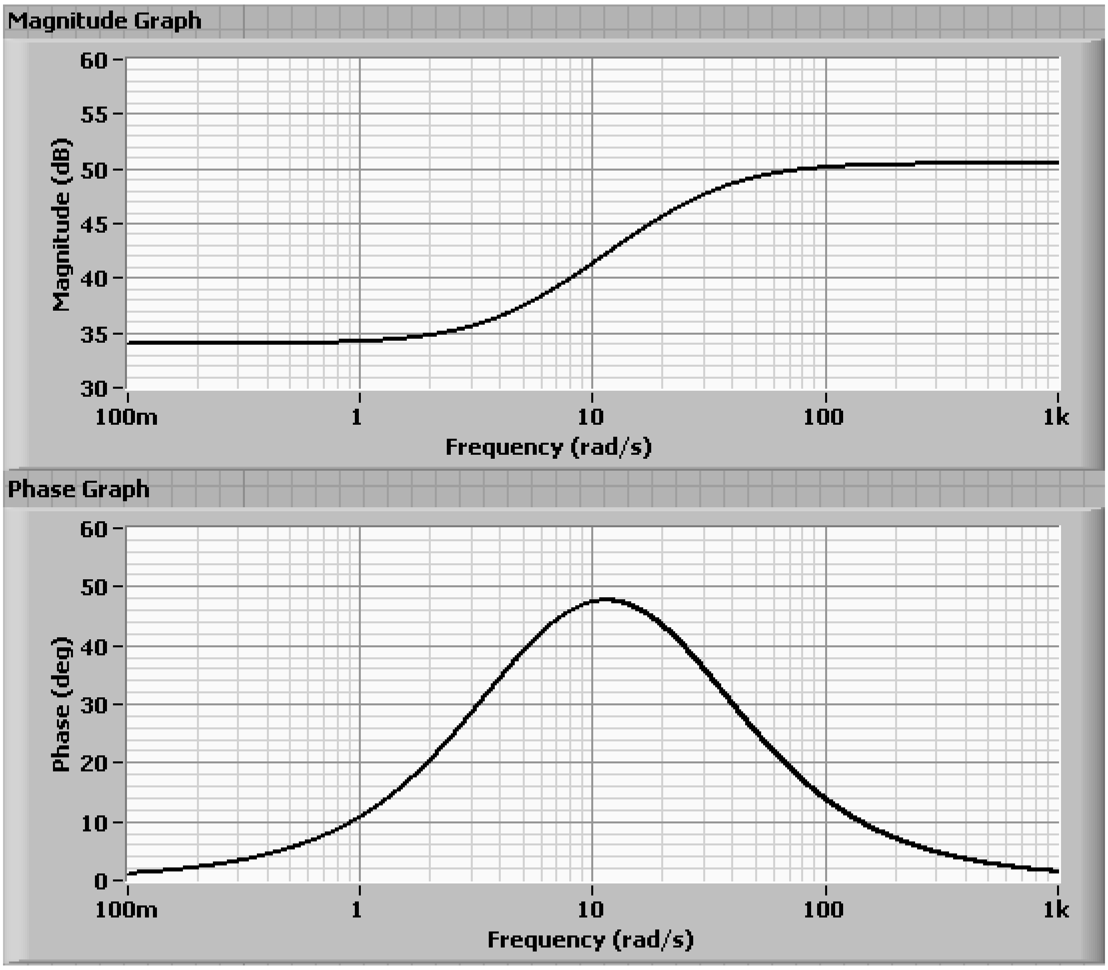
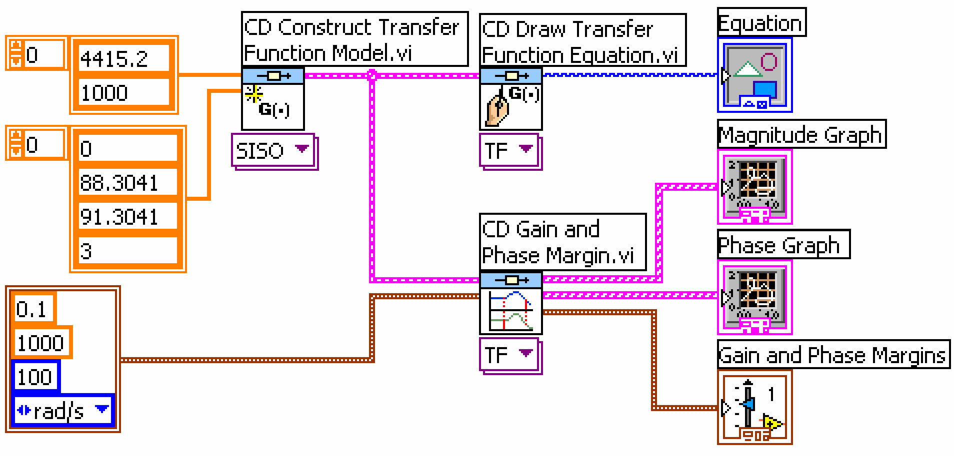


Figure 9-64(b). Bode Diagram of $G_c(s)$

The open-loop transfer function of the designed system is

$$G_c(s)\,G(s) = \frac{1000}{3}\left(\frac{s+4.4152}{s+29.4347}\right)\frac{1}{s(s+1)}$$

LabVIEW Program B-9-6(c) produces the Bode diagram of $G_c(s)G(s)$,which is shown in Figure 9-64(c).



LabVIEW Program B-9-6(c)

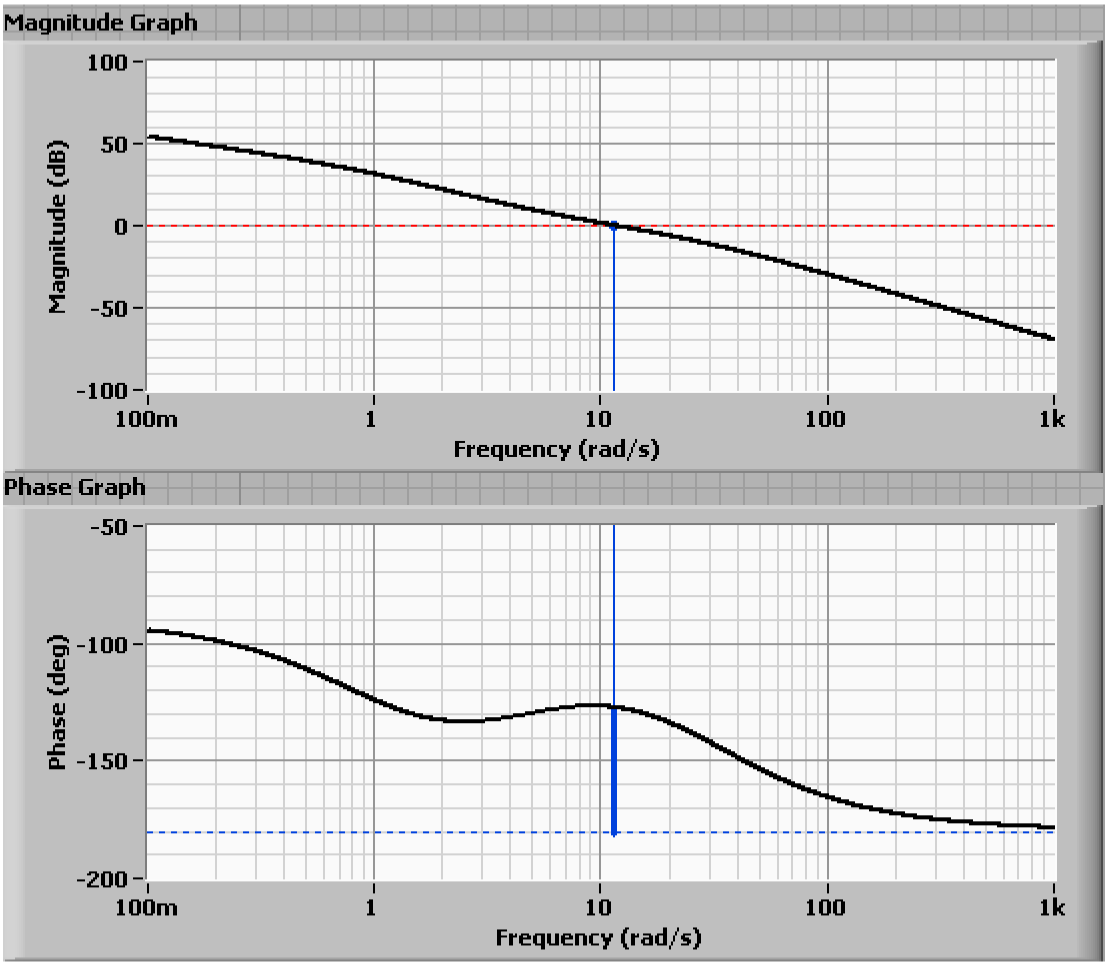


Figure 9-64(c). Bode diagram of $G_c(s)G(s)$,

From this diagram, it is clearly seen that the phase margin is approximately 52°, the gain margin is $+\infty$ dB, and $K_v = 50$ sec^{-1} ; all specifications are met.

Next we shall obtain the unit-step and unit-ramp responses of the original uncompensated system and the compensated system. The original uncompensated system has the following closed-loop transfer function:

$$\frac{C(s)}{R(s)} = \frac{1}{s^2 + s + 1}$$

The closed-loop transfer function of the compensated system is

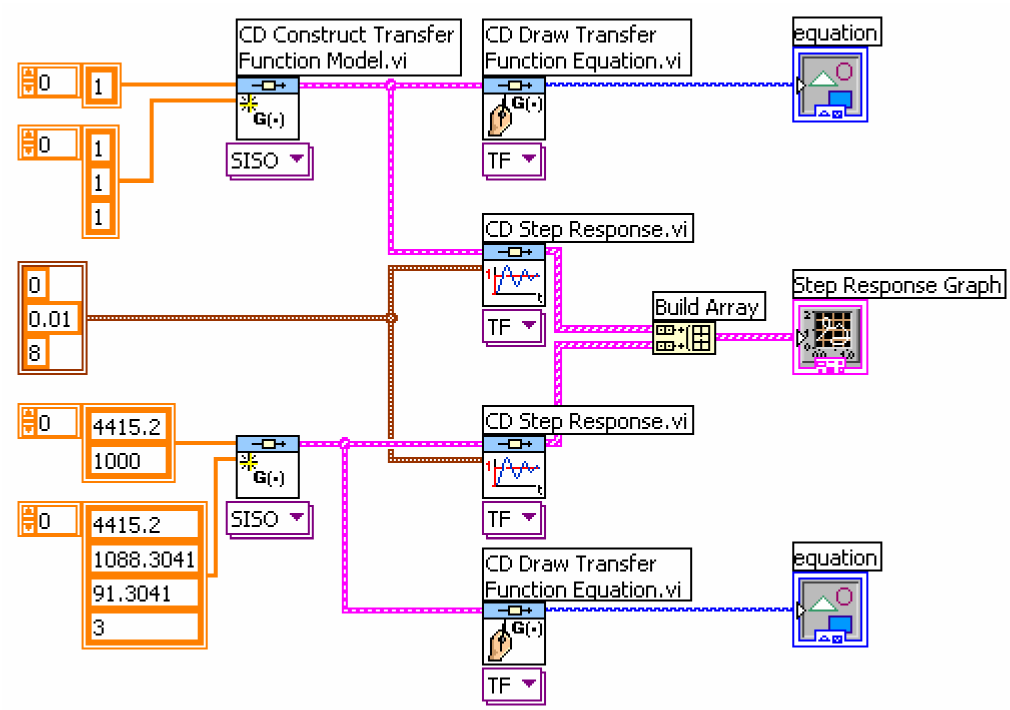
$$\frac{C(s)}{R(s)} = \frac{1000(s + 4.4152)}{3(s + 29.4347)s(s + 1) + 1000(s + 4.4152)}$$

$$= \frac{1000s + 4415.2}{3s^3 + 91.3041s^2 + 1088.3041s + 4415.2}$$

The closed-loop poles of the compensated system are as follows:

$$s = -11.1772 + j7.5636$$
$$s = -11.1772 - j7.536$$
$$s = -8.0804$$

LabVIEW Program B-9-6(d) produces the unit-step responses of the uncompensated and compensated systems. The resulting response curves are shown in Figure 9-64(d).



LabVIEW Program B-9-6(d)

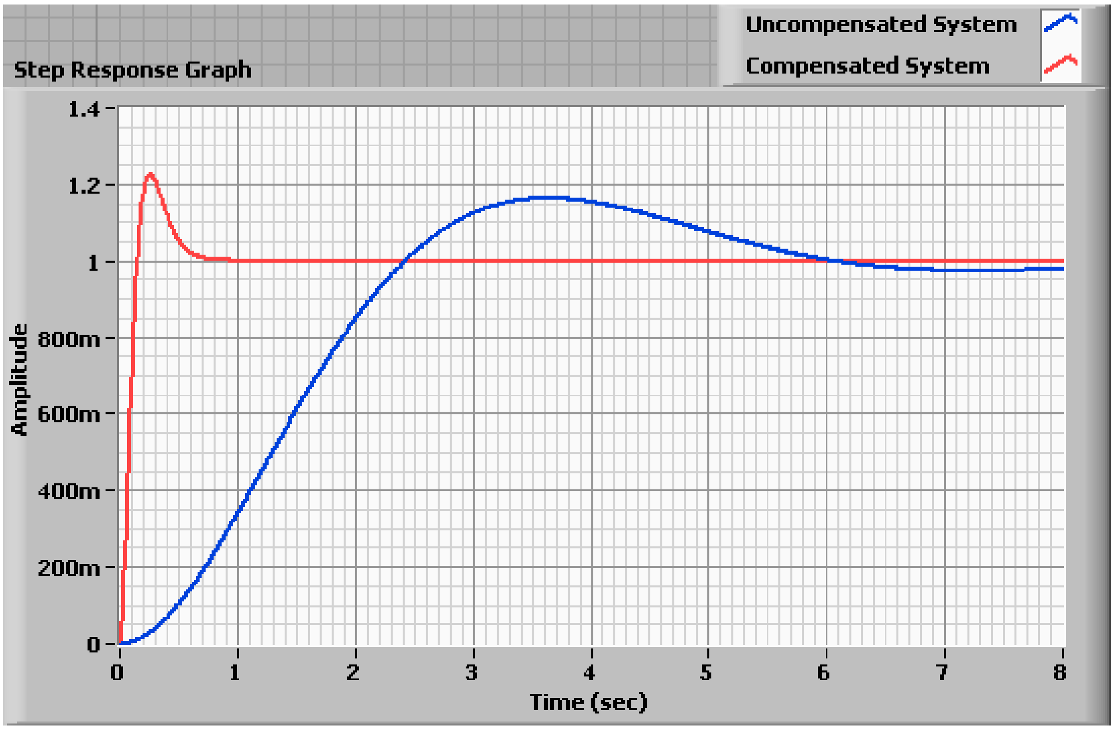
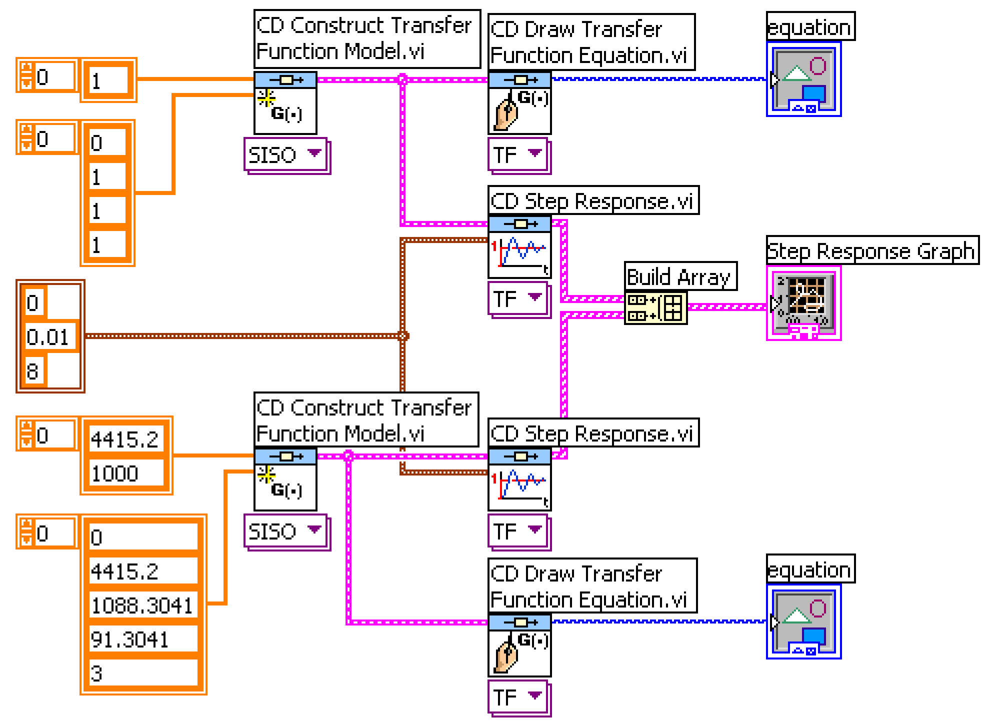


Figure 9-64(d). Unit-step responses of the uncompensated and compensated systems.

LabVIEW Program B-9-6(e) produces the unit-ramp responses of the uncompensated system and compensated system. The resulting response curves are shown in Figure 9-64(e).



LabVIEW Program B-9-6(e)

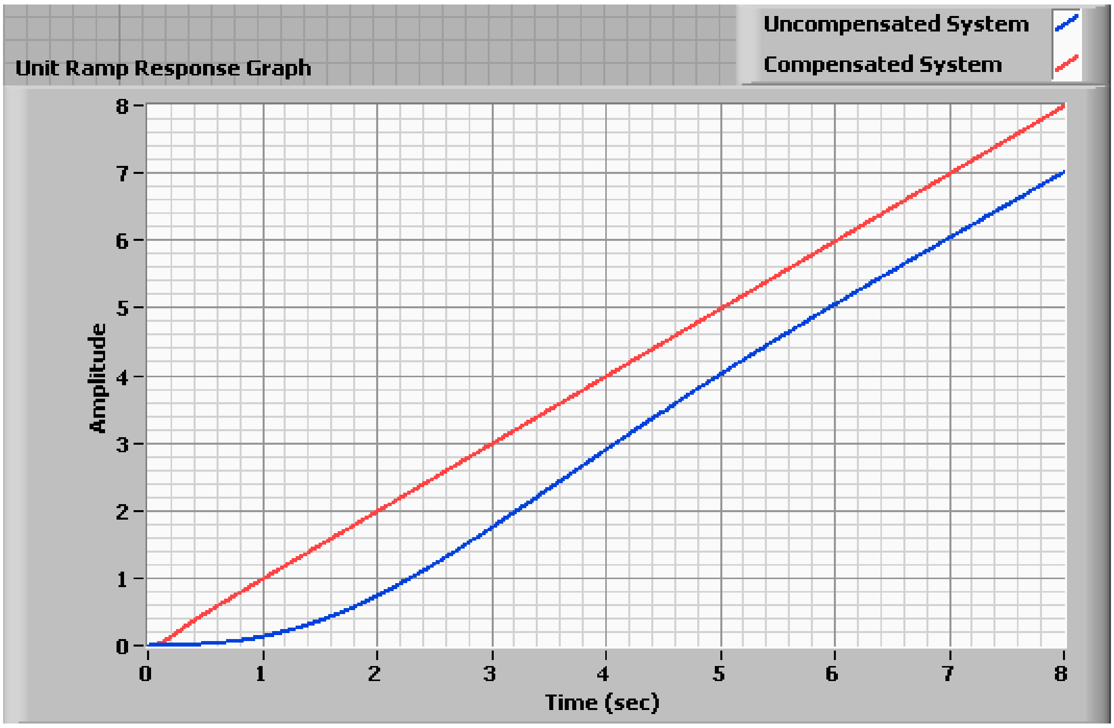


Figure 9-64(e). Unit-ramp responses of the Uncompensated and Compensated systems.