Example B-9-9

Consider the system shown in Figure 9-67. Design a lag-lead compensator such that the static velocity error constant K_v is 20 sec^{-1}, phase margin is 60^o, and gain margin is not less than 8 dB. Plot the unit-step and unit-ramp response curves of the compensated system with LabVIEW.

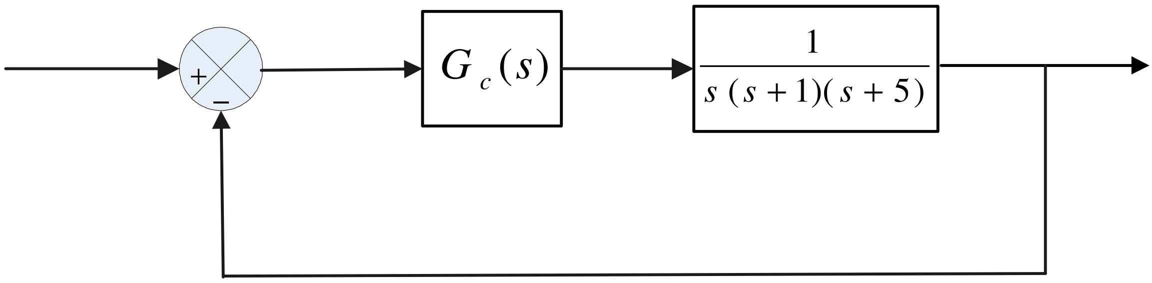


Figure 9-67. Control System

Solution.

Let us assume that the compensator $G_c(s)$ has the following form

$$G_c(s) = K_c \frac{(T_1 s + 1)(T_2 s + 1)}{(\frac{T_1}{\beta} s + 1)(\beta T_2 s + 1)} = K_c \frac{(s + \frac{1}{T_1})(s + \frac{1}{T_2})}{(s + \frac{\beta}{T_1})(s + \frac{1}{\beta T_2})}$$

Since K_v is specified as 20 sec^{-1}, we have

$$K_v = \lim_{s \to 0} s G_c(s) \frac{1}{s(s+1)(s+5)} = K_c \frac{1}{5} = 20$$

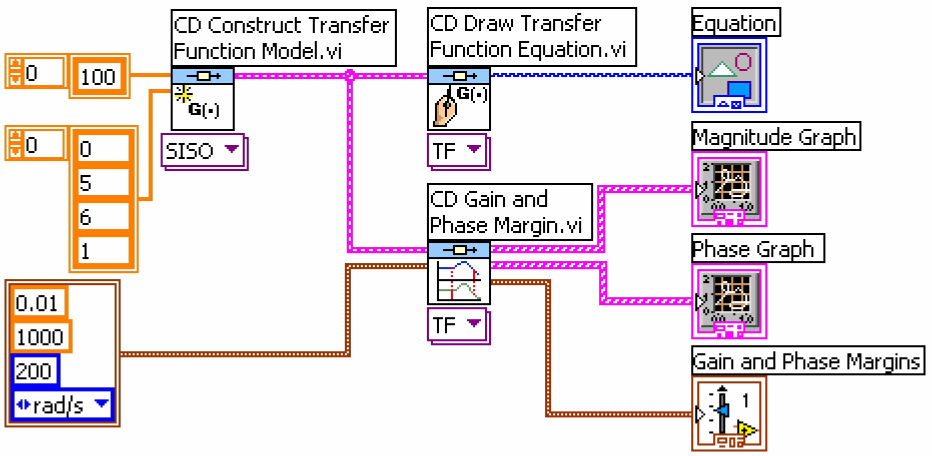
Hence

$K_c = 100$

Define

$$G_1(s) = 100 G(s) = \frac{100}{s(s+1)(s+5)}$$

LabVIEW Program B-9-9(a) produces the Bode plot of $G_1(s)$ as shown in Figure 9-67(a).



LabVIEW Program B-9-9(a)

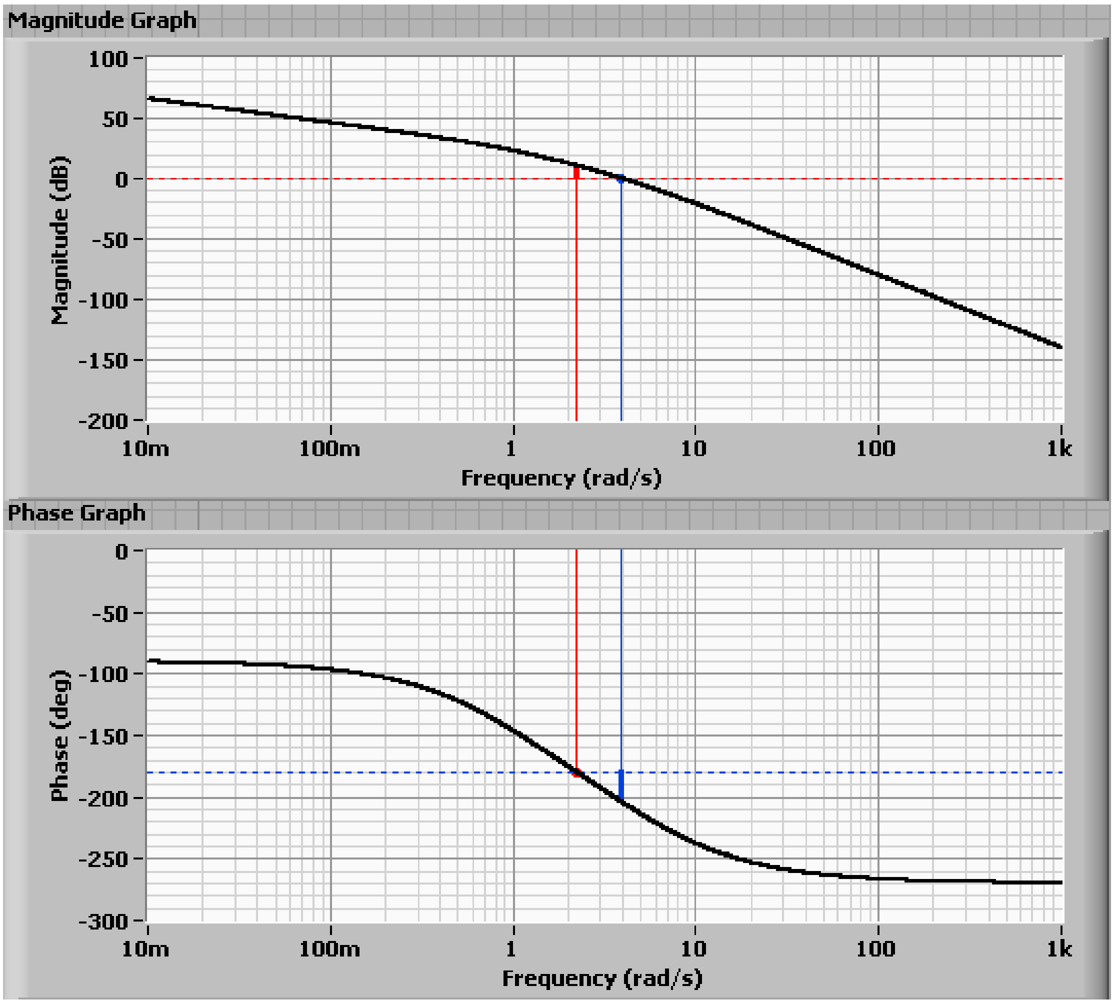


Figure 9-67(a). Bode plot of $G_1(s)$

From this Bode diagram we find the phase crossover frequency to be $\omega = 2.25$ rad/sec. Let us choose the gain crossover frequency of the designed system to be $\omega = 2.25$ rad/sec so that the phase lead angle required at $\omega = 2.25$ rad/sec is 60^o.

Once we choose the gain crossover frequency to be 2.25 rad/sec, we can determine the corner frequencies of the phase lag portion of the lag-lead compensator. Let us choose the corner frequency $1/T_2$ to be one decade below the new gain crossover frequency, or $1/T_2 = 0.255$. For the lead portion of the compensator, we first determine the value of β that provides $\varphi_m = 65^o$, (5^o added to 60^o.) Since

$$\sin\phi_m = \frac{1-\frac{1}{\beta}}{1+\frac{1}{\beta}} = \frac{\beta-1}{\beta+1}$$

We find β = 20 corresponds to 64.7912^o. Since we need 65^o phase margin, we may choose β = 20. Thus

$$\beta = 20$$

Then, the corner frequency $1/(\beta T_2)$ of the phase lag portion becomes as follows:

$$\frac{1}{\beta T_2} = \frac{1}{20x\frac{1}{0.225}} = \frac{0.225}{20} = 0.01125$$

Hence, the phase lag portion of the compensator becomes as

$$\frac{s+0.225}{s+0.01125} = 20\frac{4.4444s+1}{88.8889s+1}$$

For the phase lead portion, we first note that

$G_1(j\,2.25) = 10.35$ dB.

If the lag-lead compensator contributes -10.35 dB at ω = 2.25 rad/sec, then the new gain crossover frequency will be as desired. The intersections of the line with slope +20 dB/dec [passing through the point (2.25,-10.35 dB)] and the 0 dB line and -26.0206 dB line determine the corner frequencies. Such intersections are found as ω = 0.3704 and ω = 7.4077 rad/sec, respectively.

Thus, the phase lead portion becomes

$$\frac{s+0.3704}{s+7.4077} = \frac{1}{20}\left(\frac{2.6998s+1}{0.1350s+1}\right)$$

Hence the compensator can be written as

$$G_c(s) = 100\left(\frac{4.4444s+1}{88.8889s+1}\right)\left(\frac{2.6998s+1}{0.1350s+1}\right)$$

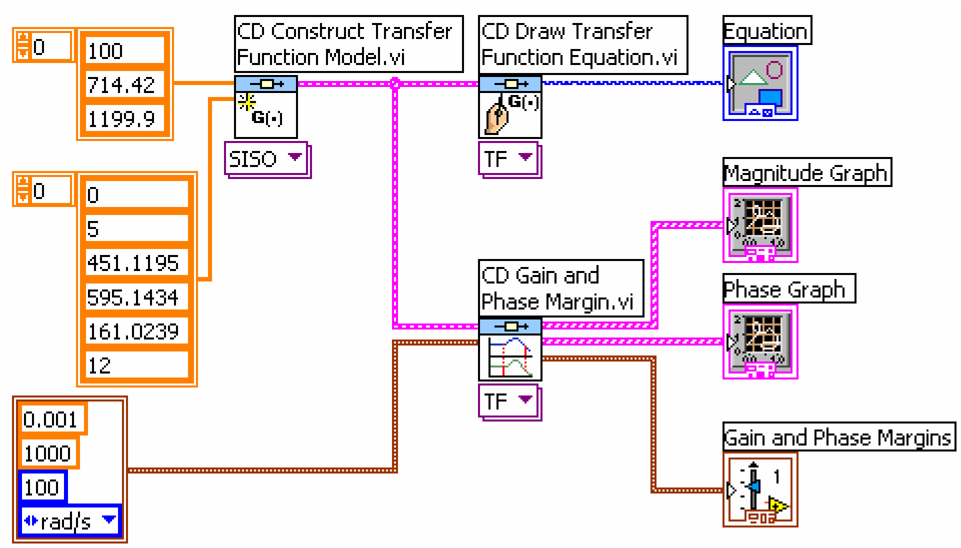
$$= 100\left(\frac{s+0.225}{s+0.01125}\right)\left(\frac{s+0.3704}{s+7.4.077}\right)$$

Then the open-loop transfer function $G_c(s)G(s)$ becomes as follows:

$$G_c(s) = 100\left(\frac{4.4444s+1}{88.8889s+1}\right)\left(\frac{2.6998s+1}{0.1350s+1}\right)\frac{1}{s(s+1)(s+5)}$$

$$= \frac{1199.90s^2 + 714.42s + 100}{12s^5 + 161.0239s^4 + 595.1434s^3 + 451.1195s^2 + 5s}$$

LabVIEW Program B-9-9(b) produces the Bode diagram of the open-loop transfer function. The resulting Bode diagram is shown in Figure 9-67(b).



LabVIEW Program B-9-9(b)

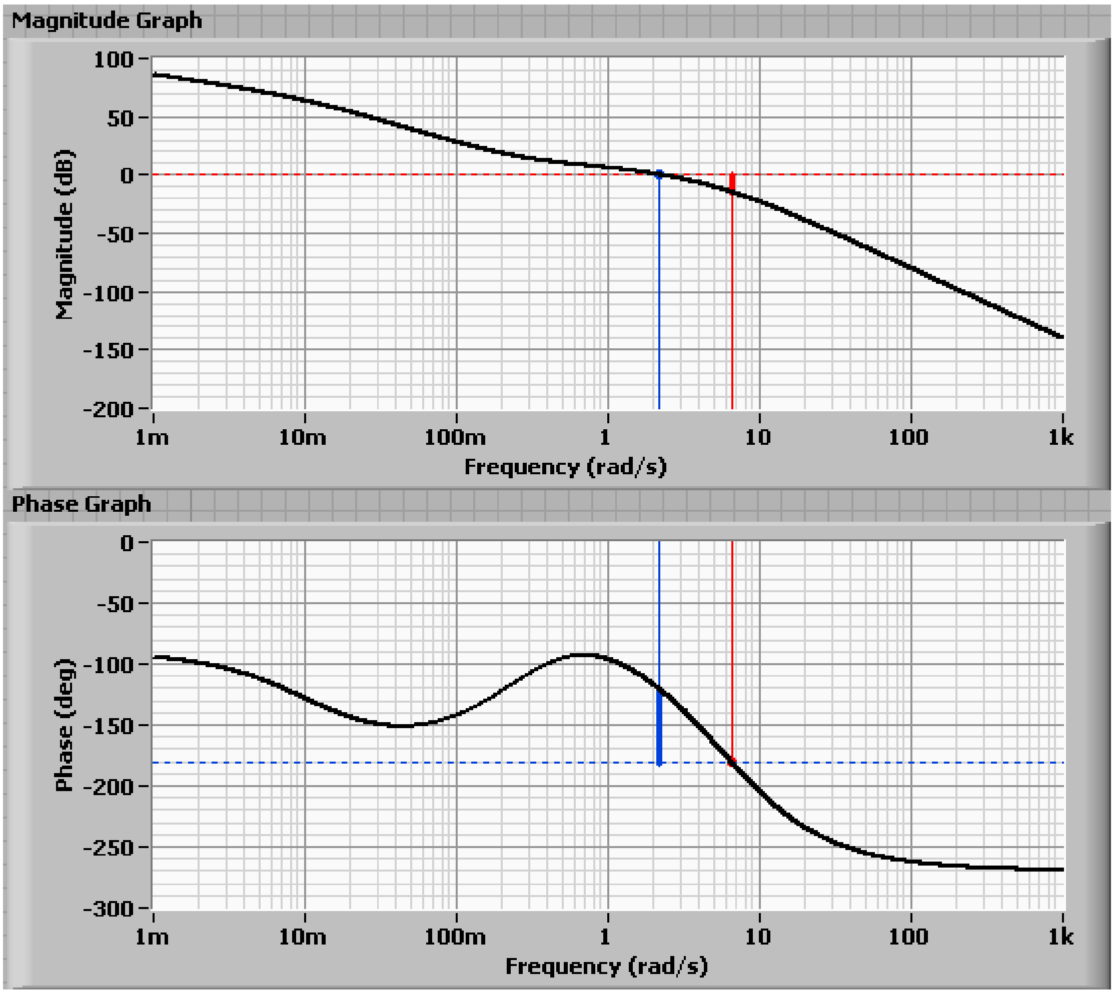


Figure 9-67(b). Bode diagram of the open-loop transfer function.

To read the phase margin and gain margin precisely, we need to expand the diagram between $\omega = 1$ and $\omega = 10$ rad/sec. This can be easily done by clicking on the first value and final of ω in the graph and changing them to 1 and 10 respectively. The resulting Bode diagram is shown in Figure 9-67(b1).

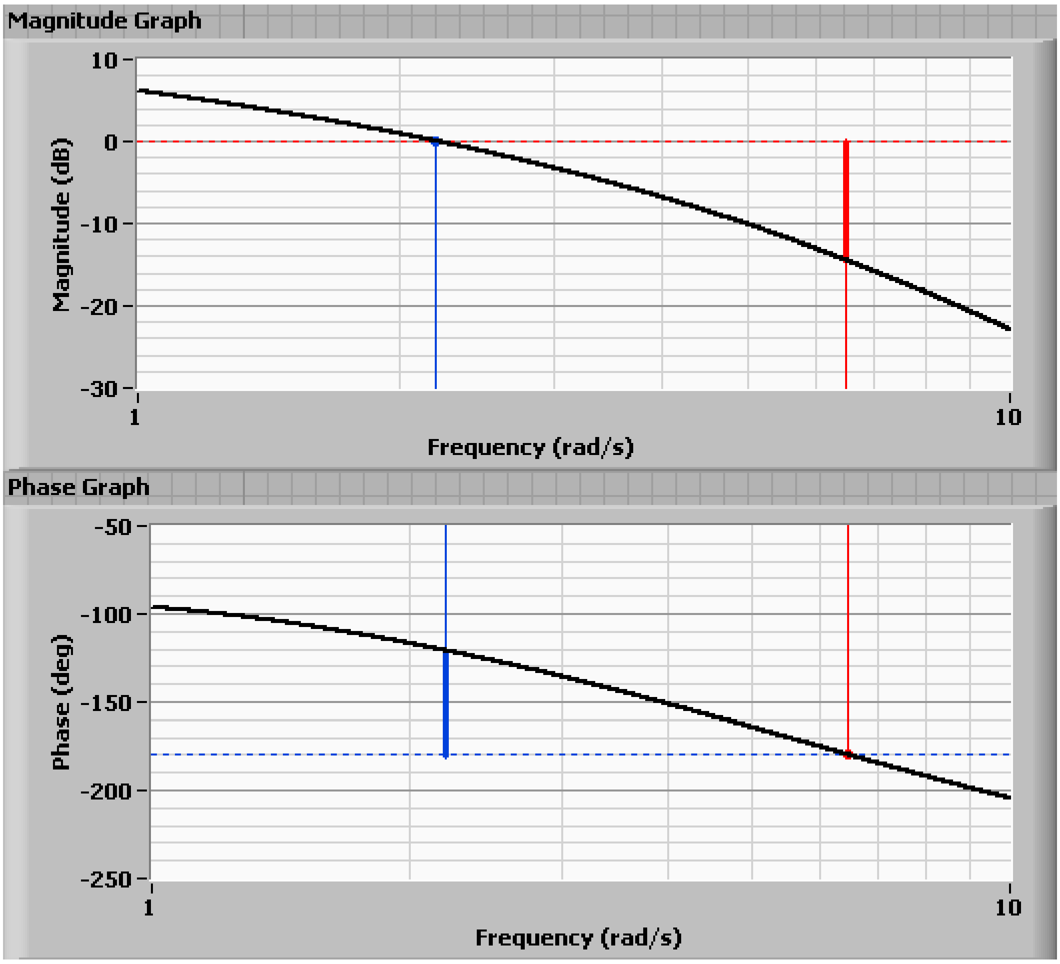


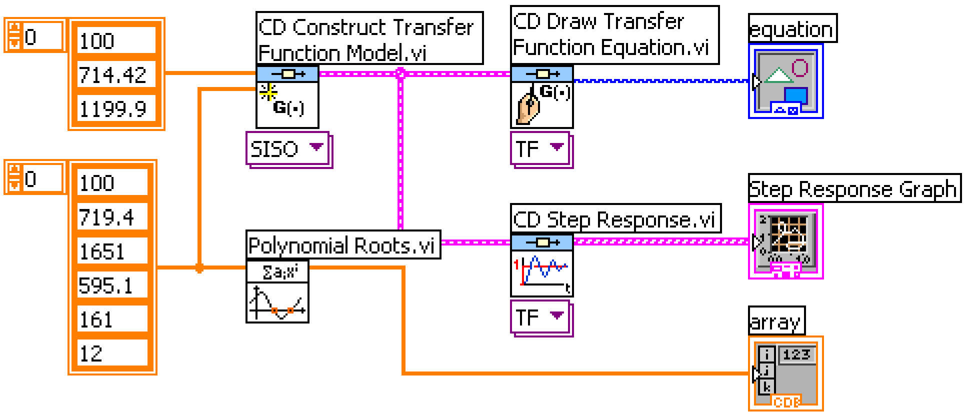
Figure 9-67(b1). Expanded diagram of open-loop transfer function

From this diagram we find that the phase margin is approximately 60° and gain margin is 14.35 dB. The static velocity error constant is 20 $\sec^{-1}$.

The closed-loop transfer function of the designed system is

$$\frac{C(s)}{R(s)} = \frac{1199.90s^2 + 714.42s + 100}{12s^5 + 161s^4 + 595.1s^3 + 1651s^2 + 719.4s + 100}$$

LabVIEW Program B-9-9(c) produces the unit-step response. The resulting unit-step response curve is shown in Figure 9-67(c).



LabVIEW Program B-9-9(c)

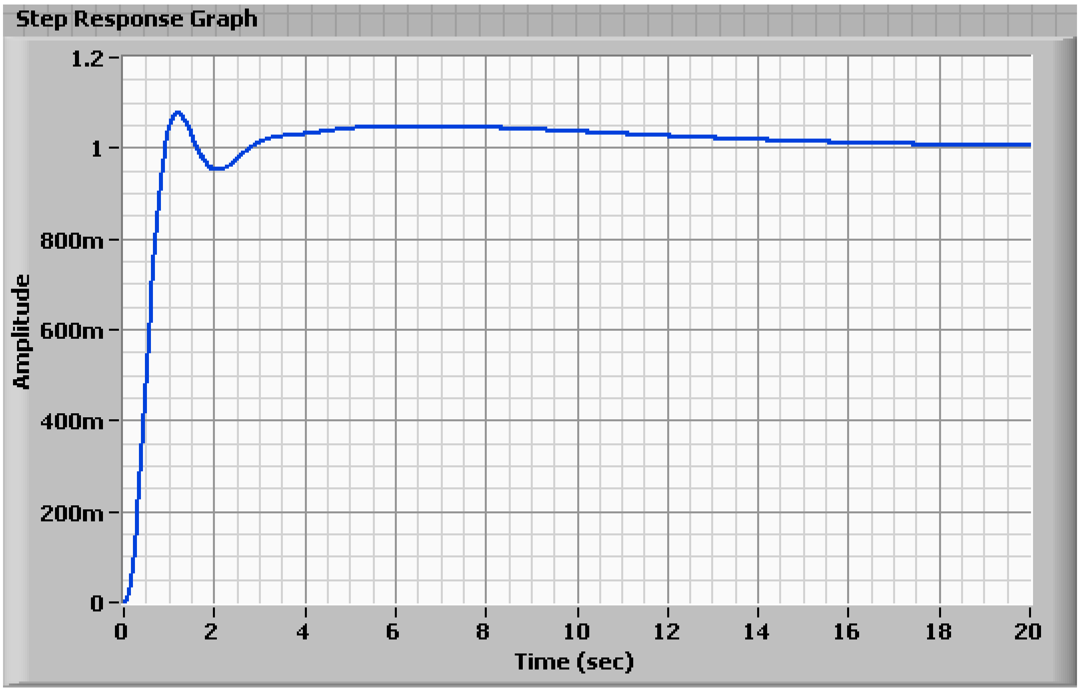


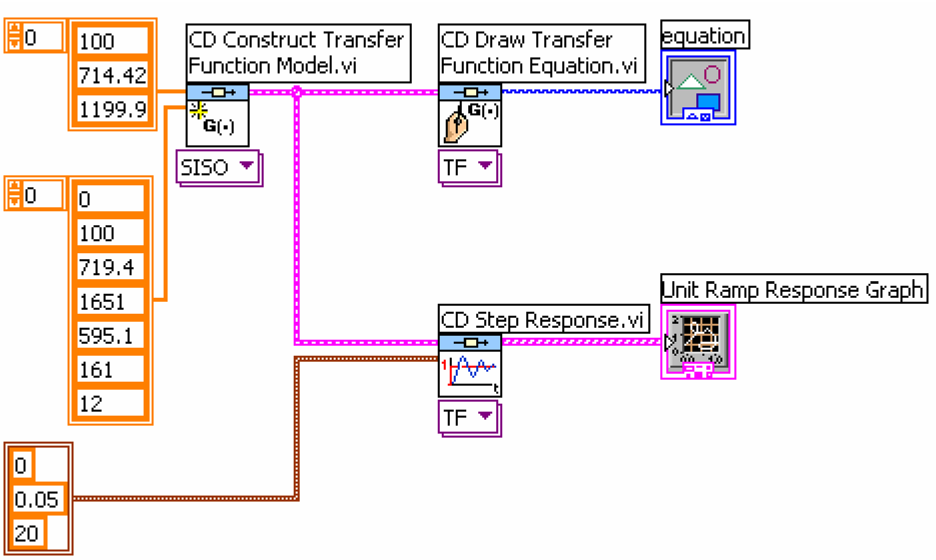
Figure 9-67(c). Unit-step response curve.

The closed-loop poles are:

Closed Loop Denominator Roots
-9.70216 +0 i
-0.246254 +0.107582 i
-0.246254 -0.107582 i
-1.611 +3.04935 i
-1.611 -3.04935 i

Notice that there are two zeros (s = -0.225 and s = -0.4939) near the closed-loop poles at s = -0.246254 ± j0.107582. Such a pole-zero combination generates a long tail with small amplitude in the unit-step response.

LabVIEW Program B-9-9(d) will produce the unit-ramp response as shown in Figure 9-67(d).



LabVIEW Program B-9-9(d)

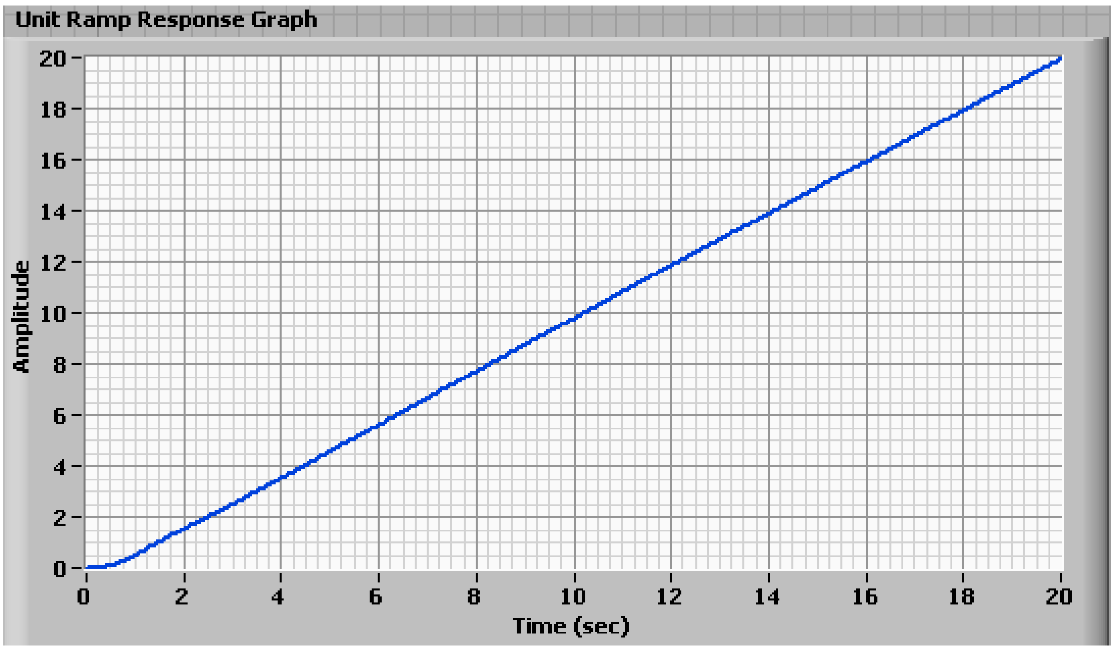


Figure 9-67(d). Unit-ramp response curve.