Example B-9-7

Consider the system shown in Figure 9-65. Design a compensator such that the static velocity error constant is 4 sec^{-1}, phase margin is 50^{o}, and gain margin is 10 dB or more. Plot unit-step and unit-ramp response curves of the compensated system with LabVIEW. Also, draw a Nyquist plot of the compensated system with LabVIEW.

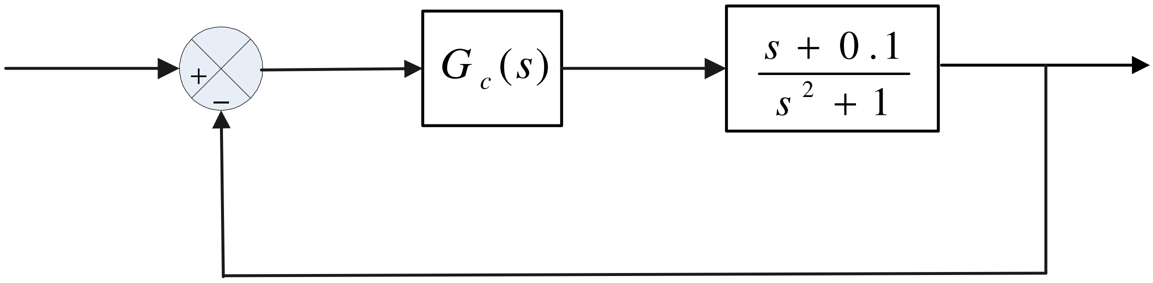


Figure 9-65. Control System

Solution.

Since the plant does not have an integrator, it is necessary to add an integrator in the compensator. Let us choose the compensator to be

$$G_c(s) = \frac{K}{s}\hat{G}_c(s), \qquad \lim_{s\to 0}\hat{G}_c(s) = 1$$

Where $\hat{G}_c(s)$ is to be determined later. Since the static velocity error constant is specified as 4 sec^{-1}, we have

$$K_v = \lim_{s\to 0} s\,G_c(s)\frac{s+0.1}{s^2+1} = \lim_{s\to 0} s\frac{K}{s}\hat{G}_c(s)\,\frac{s+0.1}{s^2+1} = 0.1K = 4$$

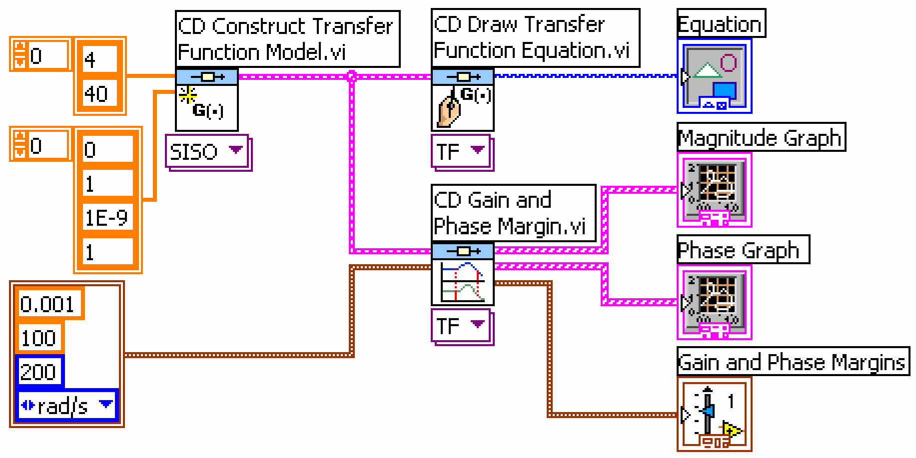
Thus, K = 40. Hence

$$G(s) = \frac{40}{s}\hat{G}_c(s)$$

Next, we plot a Bode diagram of

$$G(s) = \frac{40(s+0.1)}{s(s^2+1)}$$

LabVIEW Program B-9-7(a) produces a Bode diagram of G(s) as shown in Figure 9-65(a).



LabVIEW Program B-9-7(a)

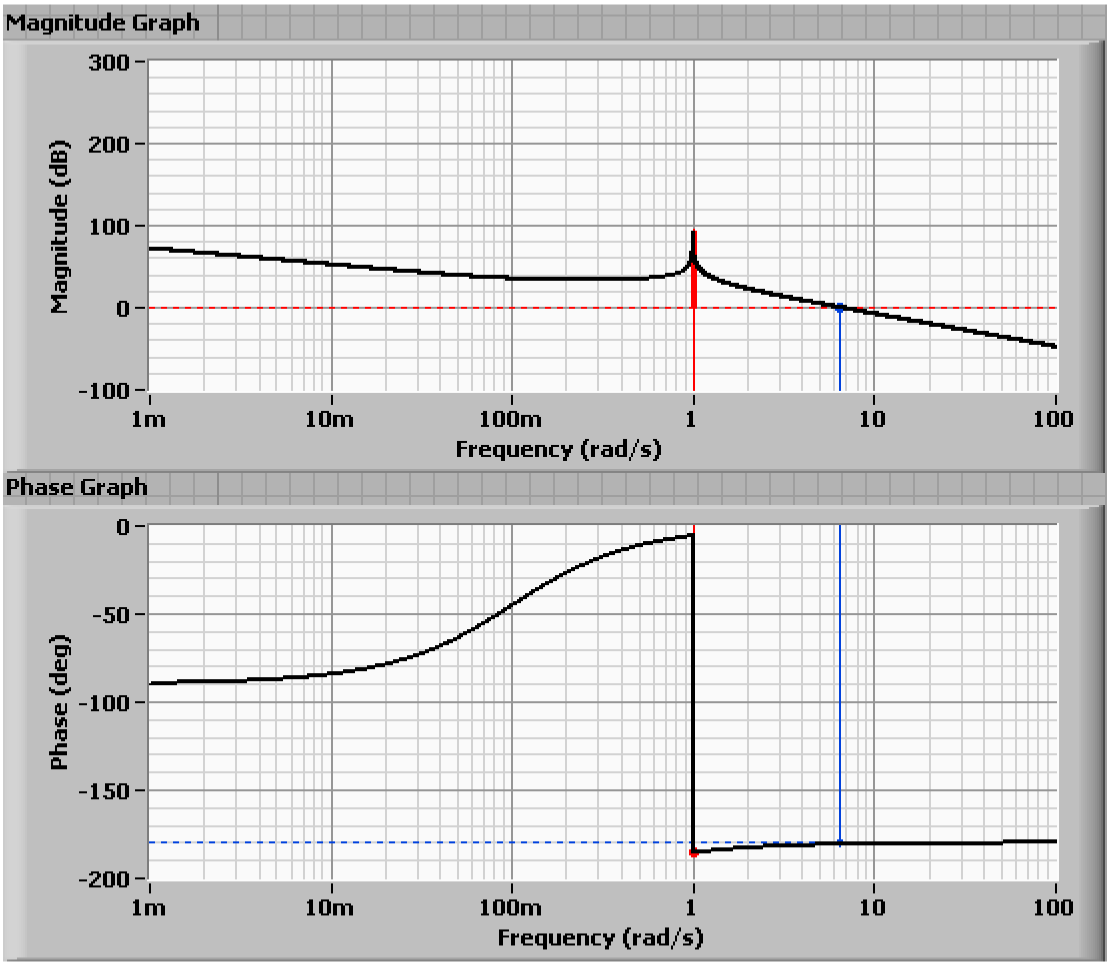
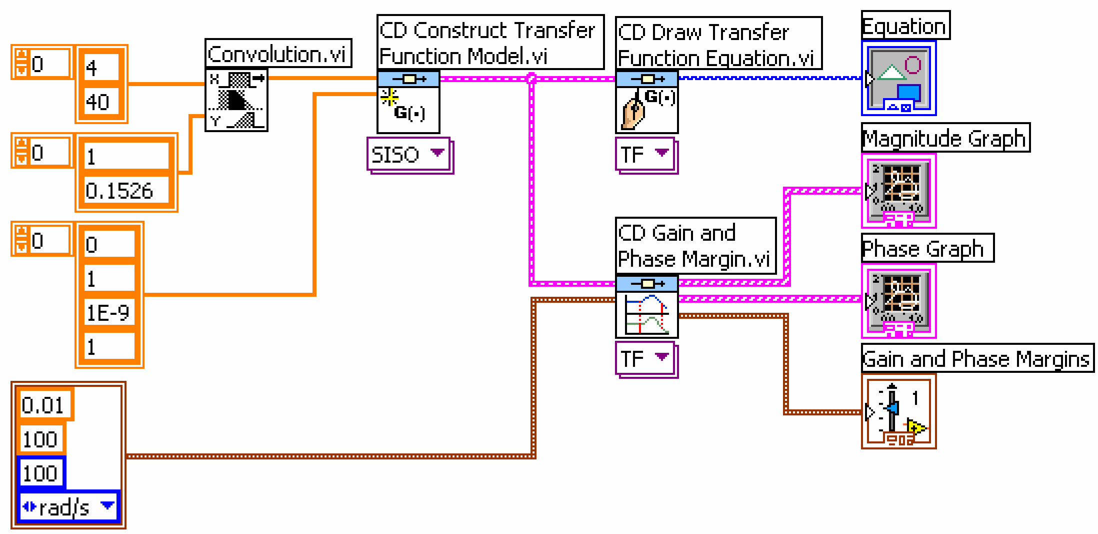


Figure 9-65(a). Bode diagram of G(s)

We need the phase margin of 50° and gain margin of 10 dB or more. Let us choose $\hat{G}_c(s)$ to be

$$\hat{G}_c(s) = a\,s + 1 \qquad\qquad (a > 0)$$

Then $G_c(s)$ will contribute up to 90° phase lead in the high frequency region. By simple trials, we find that a = 0.1526 gives the phase margin of 50° and gain margin of $+\infty$ dB. LabVIEW Program B-9-7(b) produces the Bode diagram of $G(s)\hat{G_c}(s)$ which is shown in Figure 9-65(b). From this Bode diagram we see that the static velocity error constant is 4 sec^{-1}, phase margin is 50°, and gain margin is $+\infty$ dB. Therefore the designed system satisfies all the requirements.



LabVIEW Program B-9-7(b)

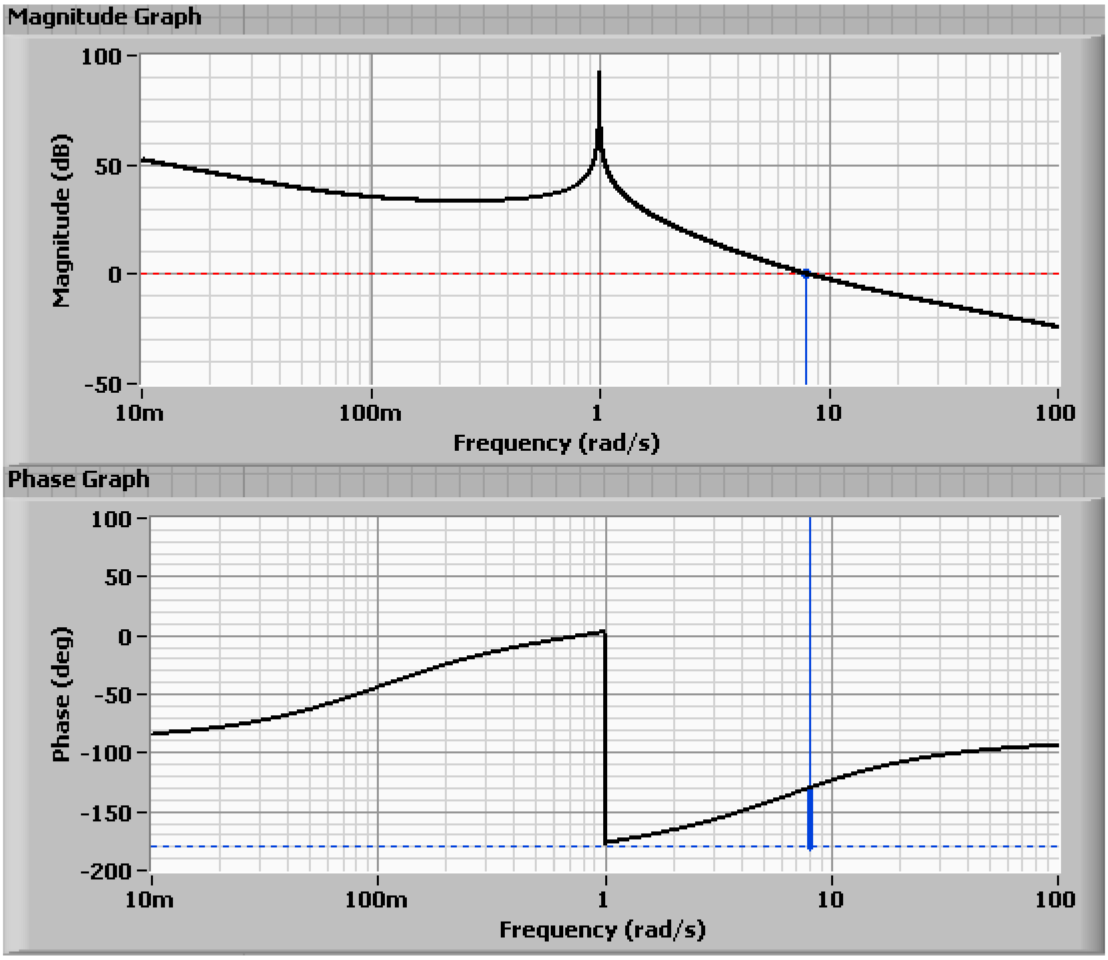


Figure 9-65(b). Bode diagram of $G(s)\hat{G}_c(s)$

The designed compensator has the following transfer function

$$G_c(s) = \frac{40}{s}\hat{G}_c(s) = \frac{40(0.1526s+1)}{s}$$

The open-loop transfer function of the designed system is

$$Open-loop\ transfer\ \ function = \frac{40(0.1526s+1)}{s}\ \frac{(s+0.1)}{(s^2+1)}$$

$$= \frac{6.104s^2 + 40.6104s + 4}{s(s^2+1)}$$

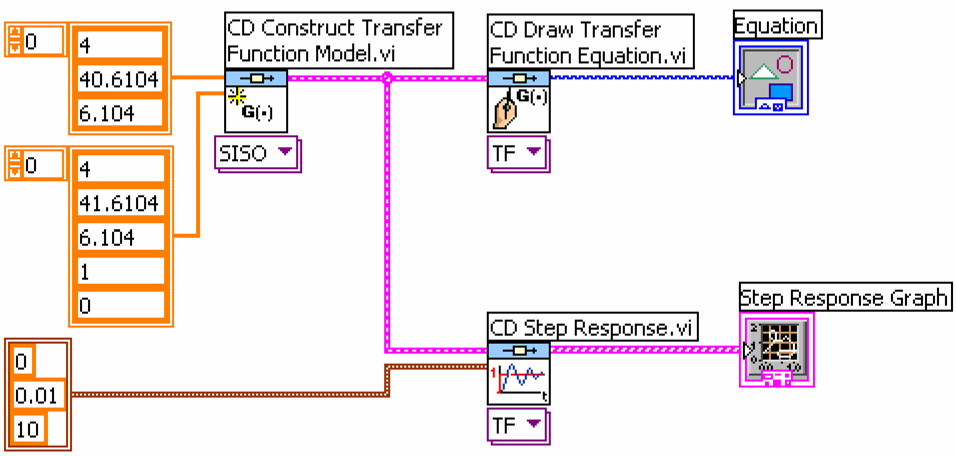
We shall next check the unit-step response and the unit-ramp response of the designed system. The closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{6.104s^2 + 40.6104s + 4}{s^3 + 6.104s^2 + 41.6104s + 4}$$

The closed-loop poles are located at

$$s = -3.0032 + j5.6573$$
$$s = -3.0032 - j5.6573$$
$$s = -0.0975$$

LabVIEW Program B-9-7(c) will produce the unit-step curve of the designed system. The resulting unit-step response curve is shown in Figure 9-65(c). Notice that the closed-loop pole at s = -0.0975 and the plant zero at s = -0.1 produce the long tail of small amplitude.



LabVIEW Program B-9-7(c)

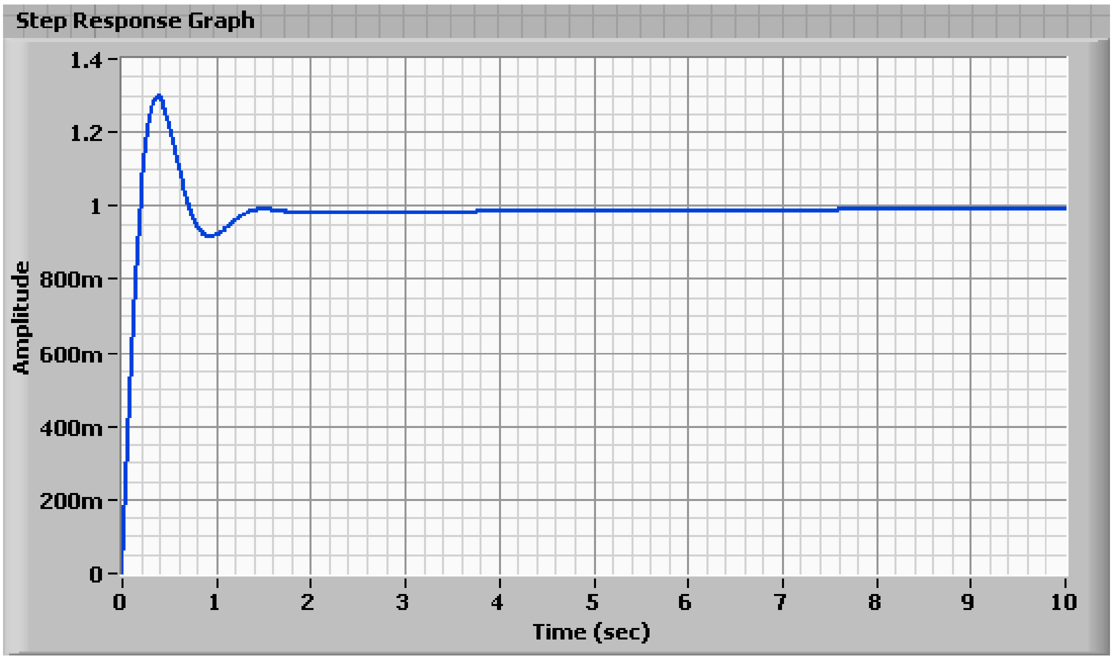
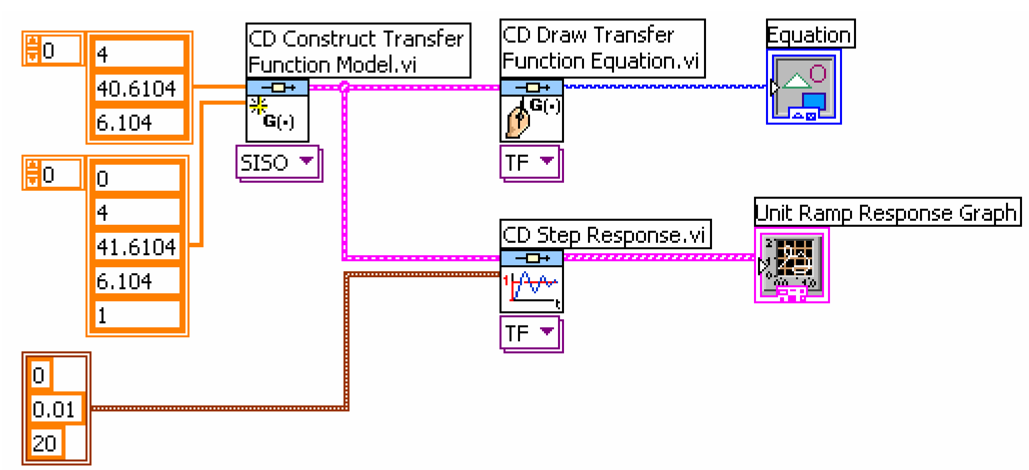


Figure 9-65(c). Unit-step curve of the designed system

LabVIEW Program B-9-7(d) produces the unit-ramp response curve of the designed system. The resulting response curve is shown in Figure 9-65(d).



LabVIEW Program B-9-7(d)

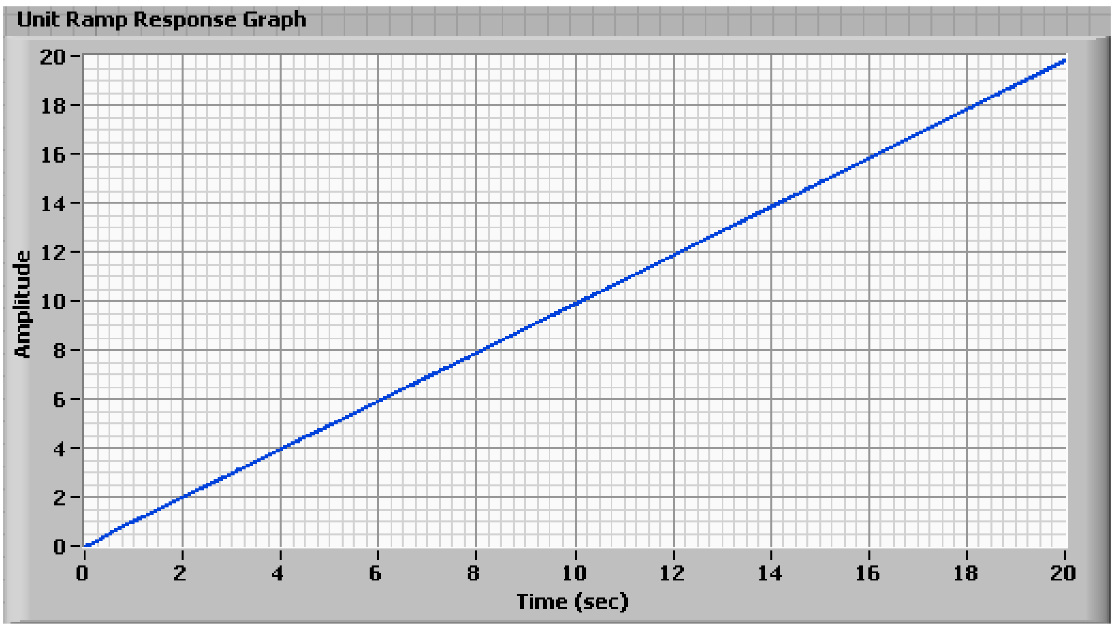


Figure 9-65(d). Unit-ramp curve of the designed system.

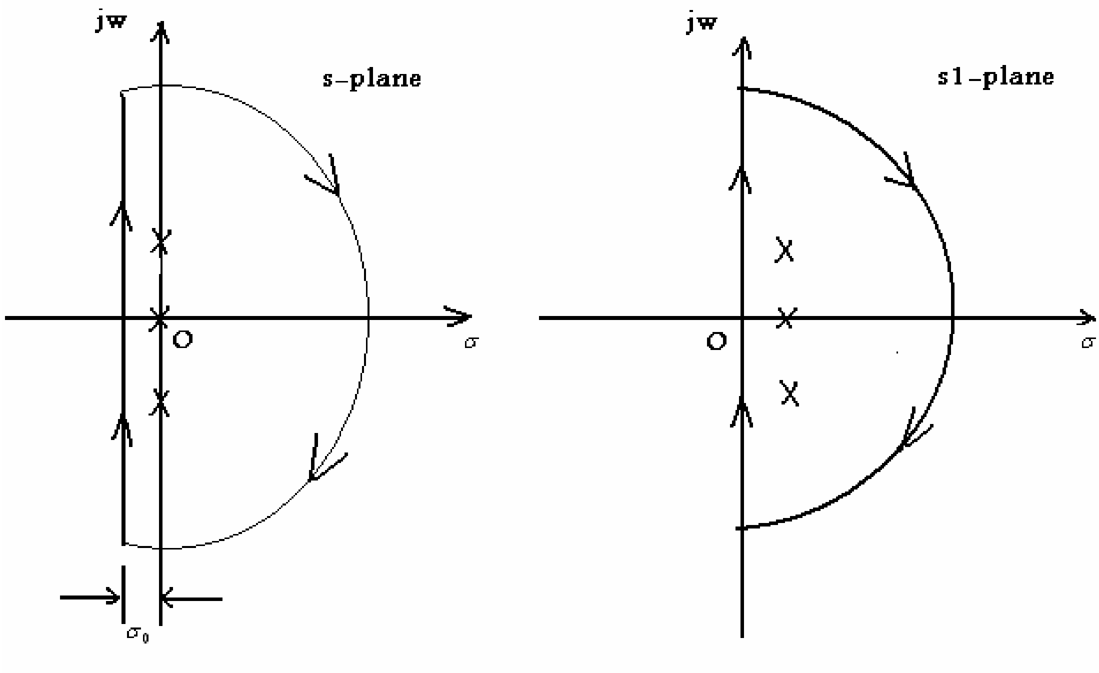
<u>Nyquist plot</u>: Define the open-loop transfer function as G(s). Then

$$G(s) = G_c(s)\,\frac{s+0.1}{s^2+1} = \frac{6.104s^2 + 40.6104s + 4}{s(s^2+1)}$$

Let us choose a modified Nyquist path in the s-plane as shown in Figure (a) below. The modified path encloses three open-loop poles (s = 0, s = j1, s = -j1). Then the

Nyquist path becomes as shown in Figure (b) below. In the s_1 plane, the open-loop transfer function has three poles in the right-half s_1 plane.

Let us choose $\sigma_o = 0.01$. Since s = $s_1 - \sigma_o$, we have



(a) (b)

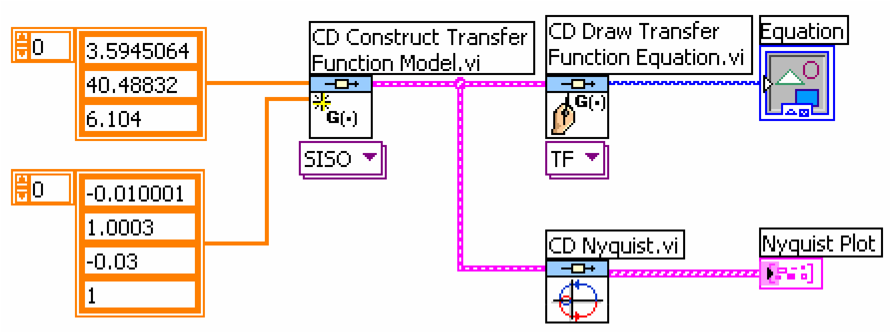
$$G(s) = g(s_1 - 0.01)$$

Open-loop transfer function in the s_1 plane

$$= \frac{6.104(s_1^2 - 0.02s_1 + 0.0001) + 40.6104(s_1 - 0.01) + 4}{(s_1 - 0.01)(s_1^2 - 0.02s_1 + 1.0001}$$

$$= \frac{6.104s_1^2 + 40.48832s_1 + 3.5945064}{s_1^3 - 0.03s_1^2 + 1.0003s_1 - 0.010001}$$

LabVIEW Program B-9-7(e) is used to obtain the Nyquist plot shown in Figure 9-65(e).



LabVIEW Program B-9-7(e)

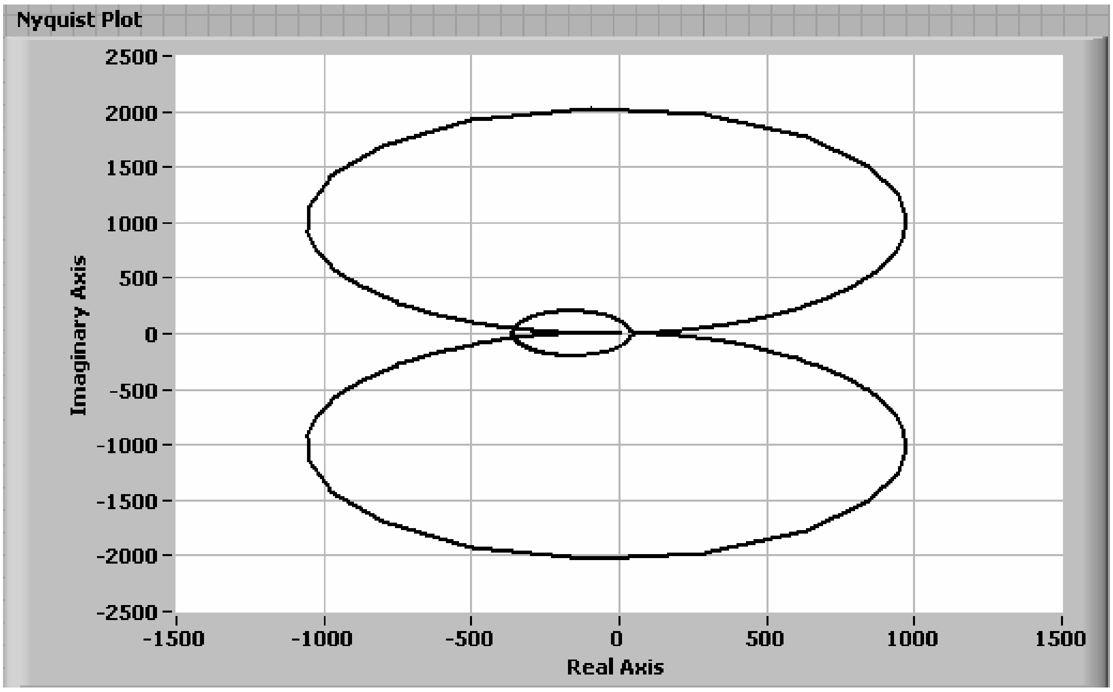


Figure 9-65(e). Nyquist plot