Example B-9-5

Referring to the closed-loop system shown in Figure 9-63, design a lead compensator $G_c(s)$ such that the phase margin is 45^o, gain margin is not less than 8 dB, and the static velocity error constant K_v is 4.0 sec^{-1}. Plot unit-step and unit-ramp response curves of the compensated system with LabVIEW.

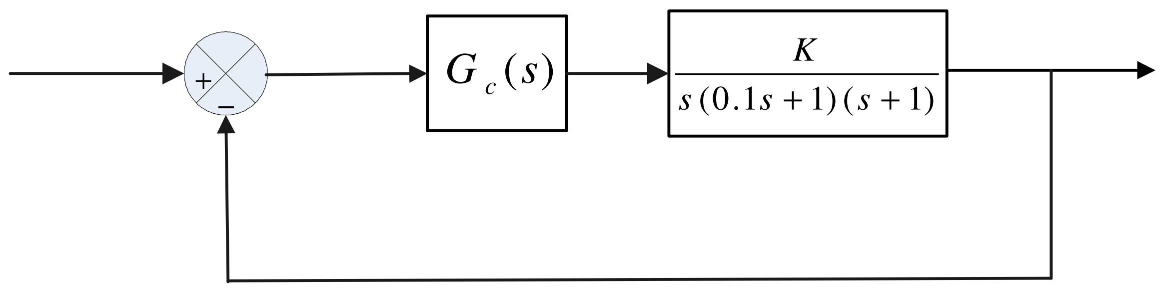


Figure 9-63. Closed-loop system

Solution. Let us use the lead compensator

$$G_c(s) = K_c\alpha\frac{Ts+1}{\alpha Ts+1} = K_c\frac{s+\frac{1}{T}}{s+\frac{1}{\alpha T}}$$

Since K_v is specified as 4.0 sec^{-1}, we have

$$K_v = \lim_{s\to 0} s\,K_c\,\alpha\frac{Ts+1}{\alpha Ts+1}\frac{K}{s(0.1s+1)(s+1)} = K_c\alpha K = 4$$

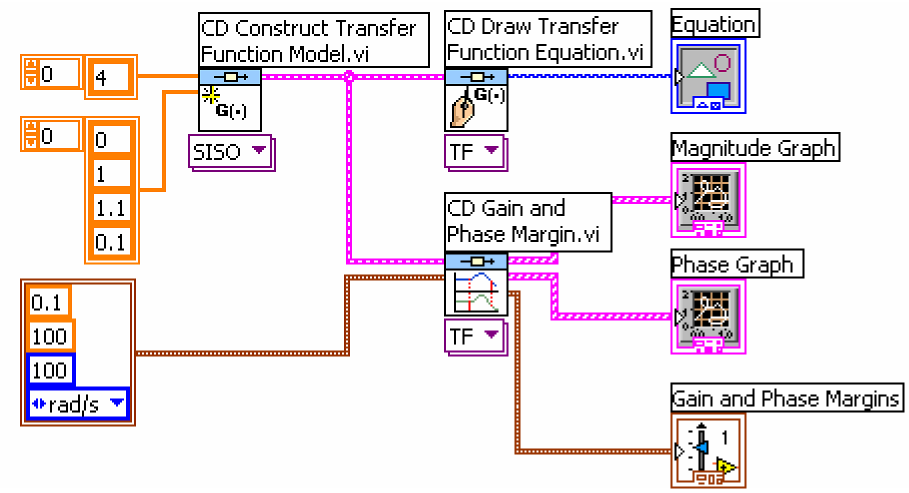
Let us set K = 1 and define $K_c\alpha = \hat{K}$. Then

$$\hat{K} = 4$$

Next, plot a Bode diagram of

$$\frac{4}{s(0.1s+1)(s+1)} = \frac{4}{0.1s^3+1.1s^2+s}$$

LabVIEW Program B-9-5(a) produces the Bode diagram shown in Figure 9-63(a).



LabVIEW Program B-9-5(a)

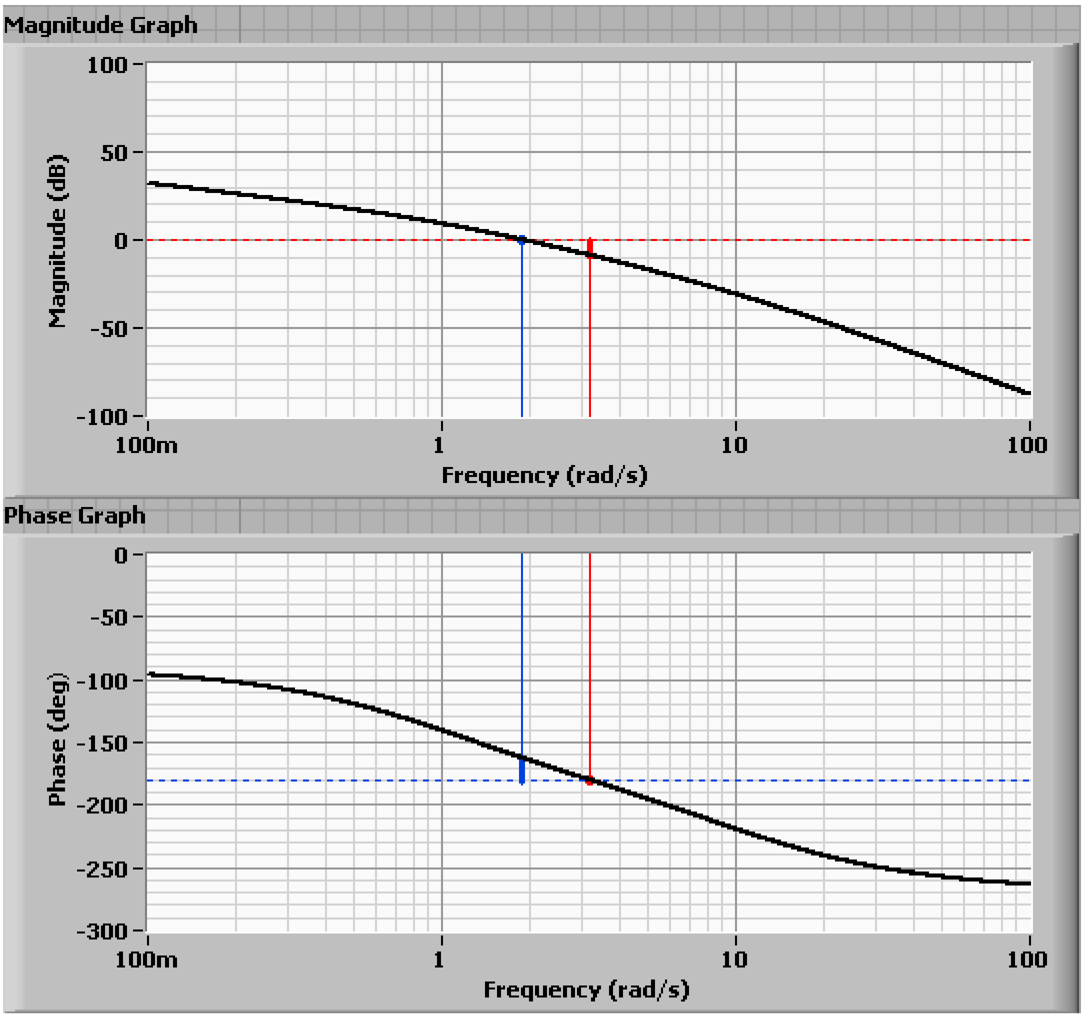


Figure 9-63(a). Bode plot of G(s) = [s(0.1s+1)(s+1)

From this plot, the phase and gain margins are 17° and 8.7 dB, respectively.

Since the specifications call for a phase margin of 45^o, let we choose

$$\phi_m = 45^o - 17^o + 12^o = 40^o$$

(This means that 12^o has been added to compensate for the shift in the gain crossover frequency.) The maximum phase lead is 40^o. Since

$$\sin \varphi_m = \frac{1-\alpha}{1+\alpha} \qquad (\varphi_m = 40^o)$$

α is determined as 0.2174. Let us choose, instead of 0.2174, α to be 0.21,
or

$$\alpha = 0.21$$

Next step is to determine the corner frequencies $\omega=1/T$ and $\omega=1/(\alpha T)$ of the lead compensator. Note that the maximum phase-lead angle φ_m occurs at the geometric mean of the two corner frequencies, or $\omega = 1/(\sqrt{\alpha} T)$. The amount of the modification in the magnitude curve at $\omega = 1/(\sqrt{\alpha} T)$ due to the inclusion of the term $(T s+1)/(\alpha T s+1)$ is

$$\left| \frac{1+j\omega T}{1+j\omega\alpha T} \right|_{\omega=1/(\sqrt{\alpha} T)} = \frac{1}{\sqrt{\alpha}}$$

Note that

$$\frac{1}{\sqrt{\alpha}} = \frac{1}{\sqrt{0.21}} = 6.7778\ dB.$$

We need to find the frequency point where, when the lead compensator is added, the total magnitude becomes 0 dB. The magnitude of $G(j\omega)$ is -6.7778 dB corresponds to $\omega = 2.81$ rad/sec. We select this frequency to be the new gain crossover frequency ω_c. Then we obtain

$$\frac{1}{T} = \sqrt{\alpha}\, \omega_c = \sqrt{0.21}\, x\, 2.81 = 1.2877$$

$$\frac{1}{\alpha T} = \frac{\omega_c}{\sqrt{\alpha}} = \frac{2.81}{\sqrt{0.21}} = 6.1319$$

Hence

$$G_c(s) = K_c \frac{s+1.2877}{s+6.1319}$$

and

$$K_c = \frac{\hat{K}}{\alpha} = \frac{4}{0.21}$$

Thus

$$G_c(s) = \frac{4}{0.21}\frac{s+1.2877}{s+6.1319} = 4\frac{0.7766s+1}{0.16308s+1}$$

The open-loop transfer function becomes as

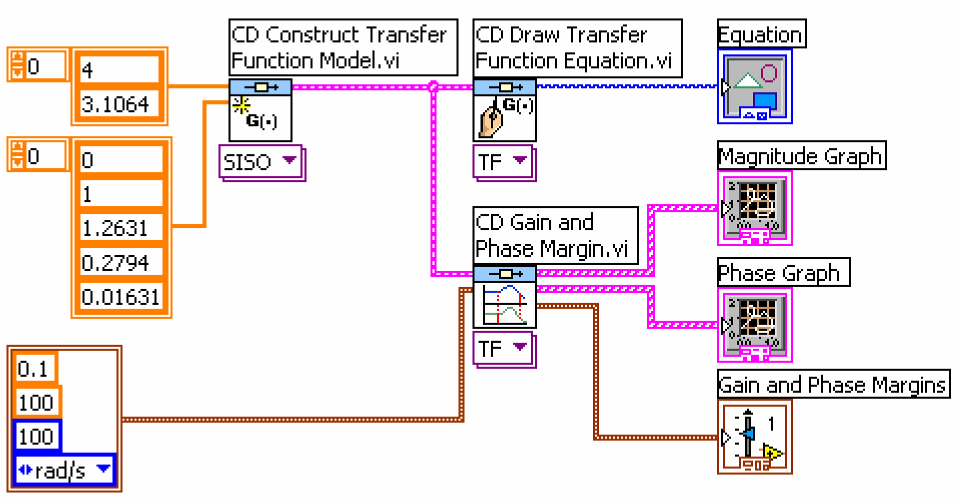
$$G_c(s)G(s) = 4\frac{0.7766s+1}{0.16308s+1}\frac{1}{s(0.1s+1)(s+1)}$$

$$= \frac{3.1064s+4}{0.01631s^4 + 0.2794s^3 + 1.2631s^2 + s}$$

the closed-loop transfer function is

$$\frac{C(s)}{R(s)} = \frac{3.1064s+4}{0.01631s^4 + 0.2794s^3 + 1.2631s^2 + 4.1064s + 4}$$

LabVIEW Program B-9-5(b) produces the Bode plots of the closed-loop transfer function as shown in Figure 9-63(b).



LabVIEW Program B-9-5(b)

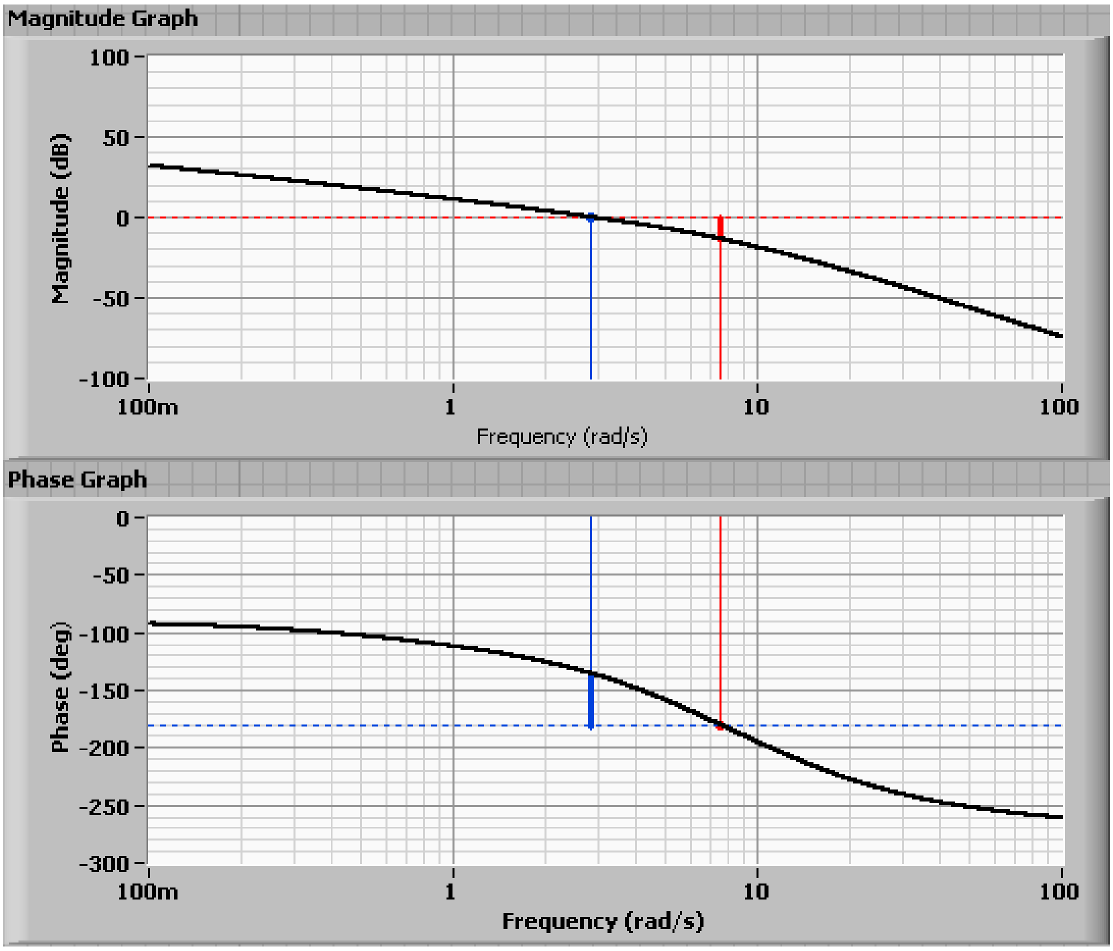
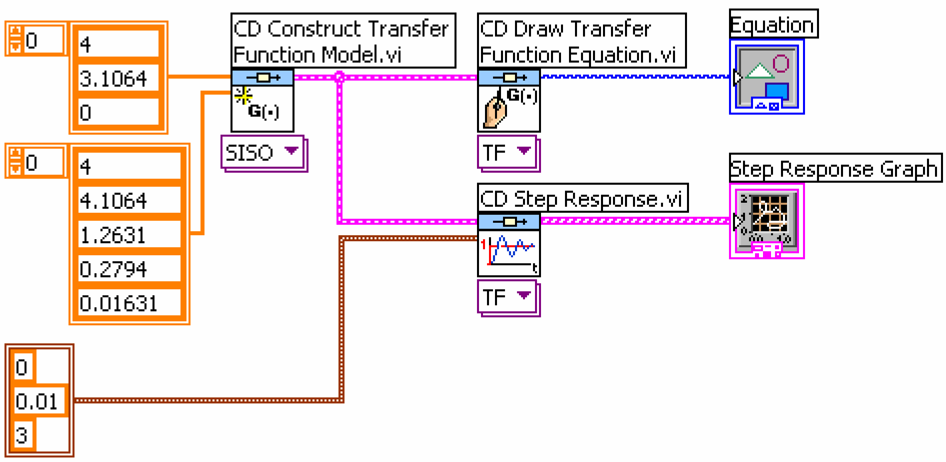


Figure 9-63(b). Bode Diagram of the closed-loop system

LabVIEW Program B-9-5(c) produces the unit-step response curve as shown in Figure 9-63(c).



LabVIEW Program B-9-5(c)

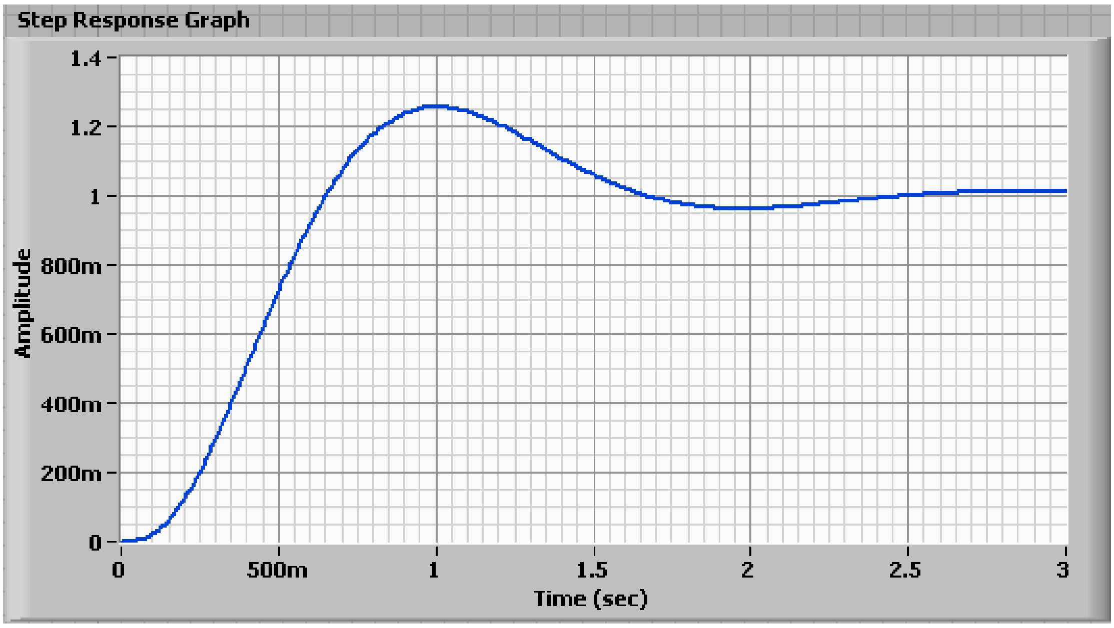
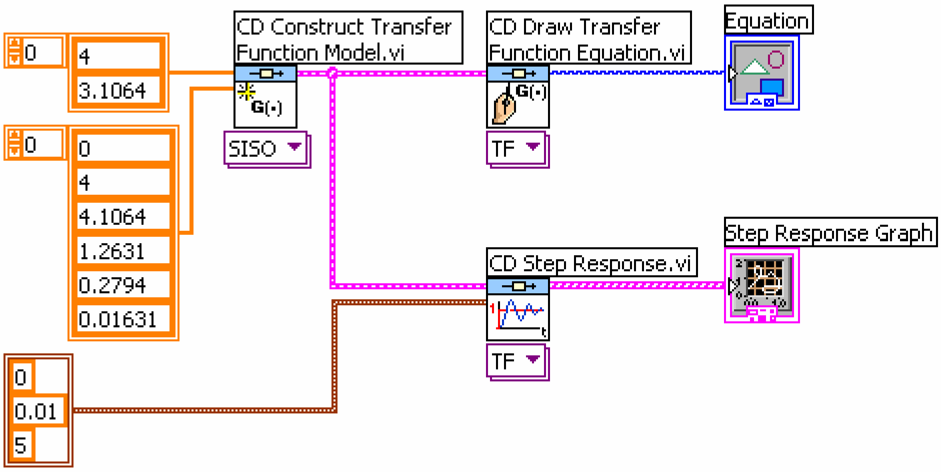


Figure 9-63(c). Unit-step response curve

Similarly, LabVIEW Program B-9-5(d) produces the unit-ramp response curve as shown in Figure 9-63(d).



LabVIEW Program B-9-5(d)

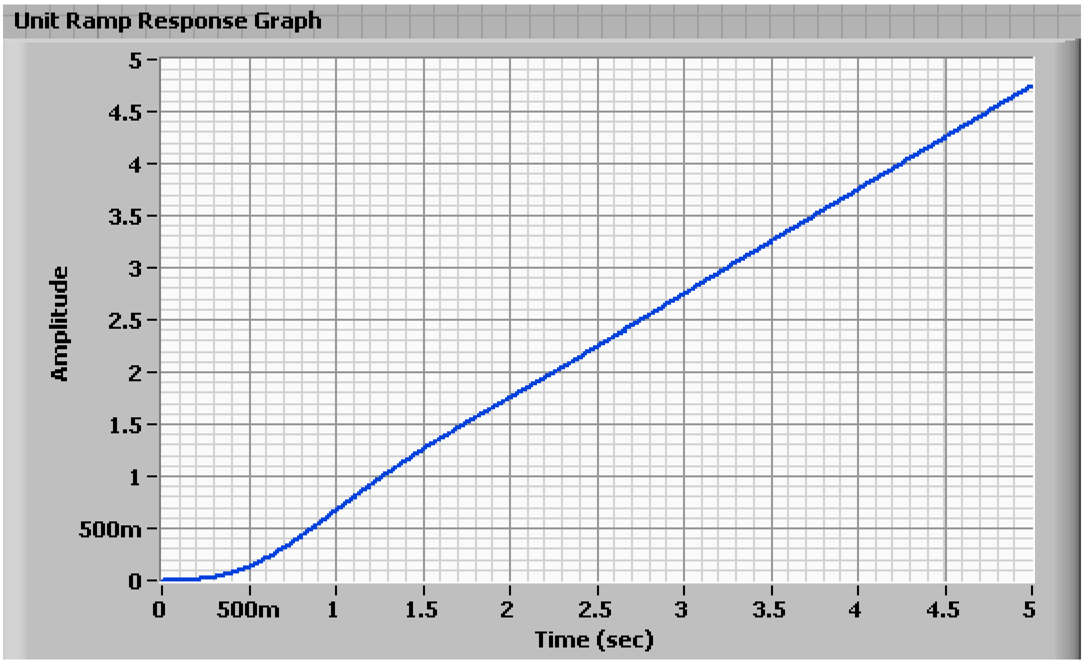


Figure 9-63(d). Unit-ramp response curve