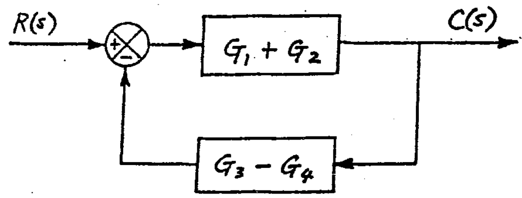
B-3-1.



$$\frac{C(s)}{R(s)} = \frac{G_1 + G_2}{1 + (G_1 + G_2)(G_3 - G_4)}$$

B-3-5. When D(s) is zero, the closed-loop transfer function $C_R(s)/R(s)$ is

$$\frac{C_R(s)}{R(s)} = \frac{G_c(s)\, G_p(s)}{1 + G_c(s)\, G_p(s)}$$

When R(s) = 0, the closed-loop transfer function $C_D(s)/D(s)$ is

$$\frac{C_D(s)}{D(s)} = \frac{1}{1 + G_c(s)\, G_p(s)}$$

When both the reference input and disturbance input are present, the output C(s) is the sum of $C_R(s)$ and $C_D(s)$. Hence

$$C(s) = C_R(s) + C_D(s) = \frac{1}{1 + G_c(s)\, G_p(s)} \left[G_c(s)\, G_p(s)\, R(s) + D(s) \right]$$

B-3-6. When only the reference input R(s) is present, the output $C_R(s)$ is given by

$$\frac{C_R(s)}{R(s)} = \frac{G_1(s)\, G_2(s)}{1 + G_1(s)\, G_2(s)}$$

For the reference input R(s), the desired output is R(s) for the unity-feedback system such as the present system. Thus, the error $E_R(s)$ is the difference between R(s) and the actual output $C_R(s)$. The error $E_R(s)$ is given by

$$E_R(s) = R(s) - C_R(s) = R(s)\left[1 - \frac{C_R(s)}{R(s)}\right]$$
$$= R(s)\left[1 - \frac{G_1(s)G_2(s)}{1+G_1(s)G_2(s)}\right] = \frac{1}{1+G_1(s)G_2(s)} R(s)$$

Assuming the system to be stable, the steady-state error $e_{ssR}(t)$ can be given by

$$e_{ssR}(t) = \lim_{t\to\infty} e_R(t) = \lim_{s\to 0} s\, E_R(s) = \lim_{s\to 0} \frac{s\, R(s)}{1+G_1(s)G_2(s)}$$

When only the disturbance input D(s) is present, the output $C_D(s)$ is given

$$\frac{C_D(s)}{D(s)} = \frac{G_2(s)}{1 + G_1(s)G_2(s)}$$

Since the desired output to the disturbance input D(s) is zero, the error can be given by

$$E_D(s) = 0 - C_D(s) = -C_D(s)$$

Hence

$$E_D(s) = -C_D(s) = -\frac{G_2(s)}{1+G_1(s)\, G_2(s)} D(s)$$

For the stable system, the steady-state error $e_{ssD}(t)$ is given by

$$e_{ssD}(t) = \lim_{t\to\infty} e_D(t) = \lim_{s\to 0} s\, E_D(s) = \lim_{s\to 0} \frac{-s\, G_2(s)\, D(s)}{1+G_1(s)G_2(s)}$$

The steady-state error when both the reference input R(s) and disturbance D(s) are present is the sum of $e_{ssR}(t)$ and $e_{ssD}(t)$ and is given by

$$e_{ss}(t) = e_{ssR}(t) + e_{ssD}(t)$$
$$= \lim_{s\to 0}\left[\frac{s\, R(s)}{1+G_1(s)\, G_2(s)} - \frac{s\, G_2(s)\, D(s)}{1+G_1(s)G_2(s)}\right]$$
$$= \lim_{s\to 0}\left\{\frac{s}{1+G_1(s)G_2(s)}\left[R(s) - G_2(s)\, D(s)\right]\right\}$$

B-3-9.

$$\dddot{y} + 3\ddot{y} + 2\dot{y} = u$$

Define

$$x_1 = y$$
$$x_2 = \dot{y}$$
$$x_3 = \ddot{y}$$

Then

$$\dot{x}_3 + 3x_3 + 2x_2 = u$$
$$\dot{x}_2 = x_3$$
$$\dot{x}_1 = x_2$$

or

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

B-3-11.

$$\mathbf{A} = \begin{bmatrix} -5 & -1 \\ 3 & -1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 2 \end{bmatrix}$$

The transfer function G(s) of the system is given by

$$G(s) = \mathbf{C}(s\mathbf{I}-\mathbf{A})^{-1}\mathbf{B} = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} s+5 & 1 \\ -3 & s+1 \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \end{bmatrix} \frac{1}{(s+5)(s+1)+3} \begin{bmatrix} s+1 & -1 \\ 3 & s+5 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

$$= \frac{1}{s^2+6s+8} \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 2s-3 \\ 5s+31 \end{bmatrix} = \frac{12s+59}{s^2+6s+8}$$

A MATLAB solution to this problem is given below.

```
A=[-5 -1;3 -1];
B=[2;5];
C=[1 2];
D=0;
[num,den] = ss2tf(A,B,C,D)

num =

   0   12   59


den =

   1   6   8
```