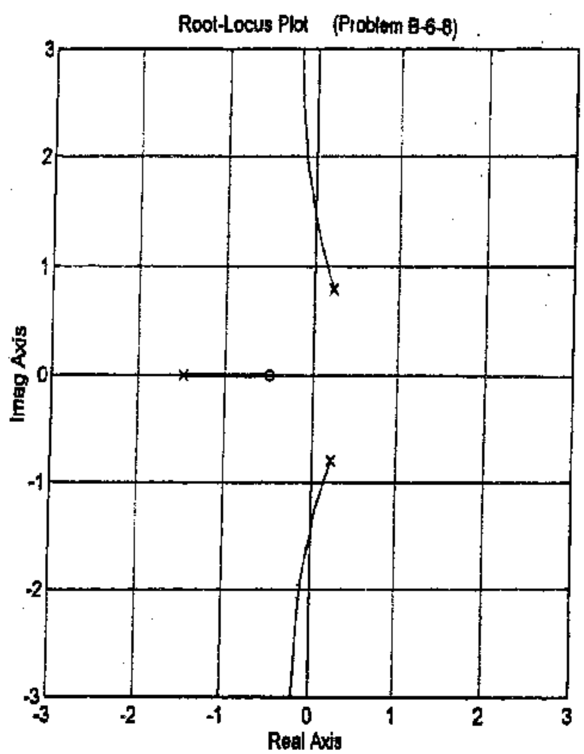
B-6-8. The open-loop transfer function

$$G(s)H(s) = \frac{K(s+0.5)}{s^3 + s^2 + 1}$$

has the poles at s = 0.2328 ± j 0.7926 and s = -1.4656. The zero is at s = -0.5. A MATLAB program to plot the root loci is shown below. The resulting root-locus plot is shown on the next page.

```
% ***** Root-locus plot *****

num = [0  0  1  0.5];
den = [1  1  0  1];
rlocus(num,den)
v = [-3  3  -3  3]; axis(v); axis('square')
grid
title('Root-Locus Plot     (Problem B-6-8)')
```



B-6-9. The open-loop transfer function

$$G(s)H(s) = \frac{K(s+9)}{s(s^2+4s+11)}$$

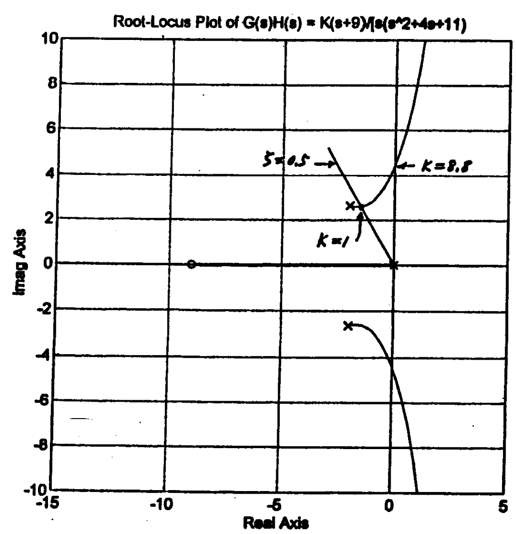
has the poles at s = 0, s = -2 ± j $\sqrt{7}$ and the zero at s = -9. The asymptotes have angles + 90° and meet the real axis at σ_a = 2.5. The complex branches cross the imaginary axis at s = ± j 4.45. The angle of departure from the complex pole in the upper half s plane is -16.5°.

The dominant closed-loop poles having the damping ratio ζ = 0.5 can be located as the intersection of the root loci and lines from the origin having angles ± 60°. The desired dominant closed-loop poles are found to be at

$$s = -1.5 \pm j\, 2.598$$

The third pole is at s = -1. The gain value corresponding to these dominant closed-loop poles is K = 1. A MATLAB program to plot the root-loci is shown below. The resulting root-locus plot is shown on the next page.

```
% ***** Root-locus plot *****

num = [0  0  1  9];
den = [1  4  11  0];
rlocus(num,den)
hold
Current plot held
x = [0,-3]; y = [0,5.196]; line(x,y);
v = [-15  5  -10  10]; axis(v); axis('square')
grid
title('Root-Locus Plot of G(s)H(s) = K(s+9)/[s(s^2+4s+11)]')
```

B-6-11. The characteristic equation for the system is

$$s^3 + 4s^2 + 8s + K = 0$$

If K is set equal to 2, then the characteristic equation becomes

$$s^3 + 4s^2 + 8s + 2 = 0$$

The closed-loop poles are located as follows:

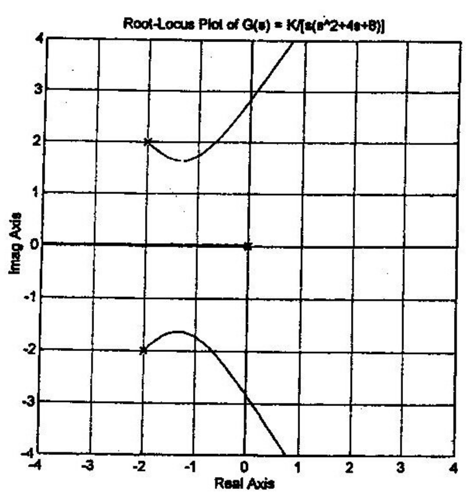
$$s = -1.8557 + j1.8669$$
$$s = -1.8557 - j1.8669$$
$$s = -0.2887$$

See the following MATLAB program for finding the closed-loop poles.

```
p = [1  4  8  2];
roots(p)

ans =

 -1.8557 + 1.8669i
 -1.8557 - 1.8669i
 -0.2887
```

A MATLAB program to plot the root loci is shown below. The resulting root-locus plot is also shown below.

```
% ***** Root-locus plot *****

num = [0  0  0  1];
den = [1  4  8  0];
rlocus(num,den)
axis('square')
grid
title('Root-Locus Plot of G(s) = K/[s(s^2+4s+8)]')
```

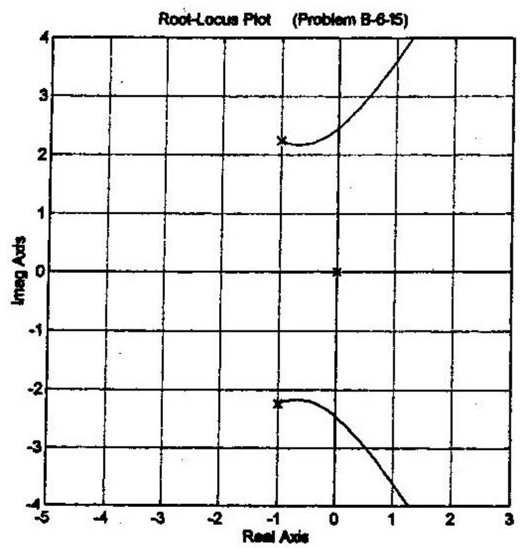


B-6-15. The term (s + 1) in the feedforward transfer function and the term (s + 1) in the feedback transfer function cancel each other. The reduced characteristic equation is

$$1 + G(s)H(s) = 1 + \frac{K(s+1)}{s(s^2+2s+6)} \frac{1}{s+1} = 1 + \frac{K}{s(s^2+2s+6)} = 0$$

The open-loop poles of G(s)H(s) is at s = 0 and $s = -1 \pm j\sqrt{5}$. The following MATLAB program produces the root-locus plot shown on the next page.

```
% ***** Root-locus plot *****

num = [0  0  0  1];
den = [1  2  6  0];
rlocus(num,den)

Warning: Divide by zero
v = [-5  3  -4  4]; axis(v); axis('square')
grid
title('Root-Locus Plot     (Problem B-6-15)')
```



To find the closed-loop poles when the gain K is set equal to 2, we may enter the following MATLAB program into the computer.

```
p = [1  2  6  2];
roots(p)

ans =

  -0.8147 + 2.1754i
  -0.8147 - 2.1754i
  -0.3706
```

Thus, the closed-loop poles are located at

$$s = -0.8147 \pm j\,2.1754, \qquad s = -0.3706$$