CHAPTER 11

B-11-1.

(a) Controllable canonical form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 6 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

(b) Observable canonical form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -6 \\ 1 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 6 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

11-10:

```
num = [0   10.4   47   160];
den = [1   14   56   160];
[A,B,C,D] = tf2ss(num,den)

A =

  -14   -56  -160
    1     0     0
    0     1     0

B =

    1
    0
    0

C =

   10.4000   47.0000   160.0000

D =

    0
```

The state-space representation is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -14 & -56 & -160 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = \begin{bmatrix} 10.4 & 47 & 160 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0\,u$$

B-11-13. The controllability and observability of the system can be determined by examining the rank conditions of

$$[\mathbf{B} \quad \mathbf{A}\mathbf{B} \quad \mathbf{A}^2\mathbf{B}]$$

and

$$[\mathbf{C}^* \quad \mathbf{A}^*\mathbf{C}^* \quad (\mathbf{A}^*)^2\mathbf{C}^*]$$

respectively.


```
A = [-1  -2  -2;0  -1  1;1  0  -1];
B = [2;0;1];
C = [1  1  0];
D = [0];
rank([B  A*B  A^2*B])

ans =

   3

rank([C'  A'*C'  A'^2*C'])

ans =

   3
```

Since the rank of $[\mathbf{B} \quad \mathbf{A}\mathbf{B} \quad \mathbf{A}^2\mathbf{B}]$ is 3 and the rank of $[\mathbf{C}' \quad \mathbf{A}'\mathbf{C}' \quad \mathbf{A}'^2\mathbf{C}']$ is also 3, the system is completely state controllable and observable.

B-11-15.

```
A = [0  1  0;0  0  1;-6  -11  -6];
B = [0;0;1];
C = [20  9  1];
D = [0];
rank([B  A*B  A^2*B])

ans =

    3

rank([C'  A'*C'  A'^2*C'])

ans =

    3
```

Since the rank of [$\underset{\sim}{B}$ $\underset{\sim}{AB}$ $\underset{\sim}{A}^2\underset{\sim}{B}$] is 3 and that of [$\underset{\sim}{C}'$ $\underset{\sim}{A}'\underset{\sim}{C}'$ $\underset{\sim}{A}'^2\underset{\sim}{C}'$] is also 3, the system is completely state controllable and completely observable.
