AMERICAN UINVERSITY OF BEIRUT
FACULTY OF ENGINEERING AND ARCHITECTURE
EECE 460 Control Systems
Spring 2006-2007

Quiz II Solution

Problem 1 (60 points):

a) There is a pole at ω = 0 and at ω = 0.3 → G(s) is of the form $\dfrac{K}{s(1+\frac{s}{0.3})}$.

At ω = 10^{-2} G(s) should be 48 dB (from figure) → K contributes to 8 dB or K = 2.52.

Thus G(s) = $\dfrac{2.52}{s(1+\frac{s}{0.3})}$ → G(s) = $\dfrac{0.756}{s(s+0.3)}$

b) Pole at ω = 0 → marginally stable

c) $K_v = \lim_{s\to 0} s\,G(s) = 2.52$

d) $G_M = \infty$, $P_M = 19.5°$

e) $G_M > 0$, $P_M > 0$ and min. phase OLTF → stable CLTF

f) Choose the lead compensator: The maximum phase margin that a lead compensator can contribute is 65°. The phase margin of the uncompensated system is 19.5° and thus the lead compensator should contribute around 35° (=25 + 10) which is beyond the allowable range.

g) The lead compensator is of the form: $K\dfrac{Ts+1}{\alpha Ts+1}$ where $K = K_C\alpha$

$\Phi_m = 35°$ → $\alpha = 0.27$

The static error velocity should remain the same: $K_v = 2.52$

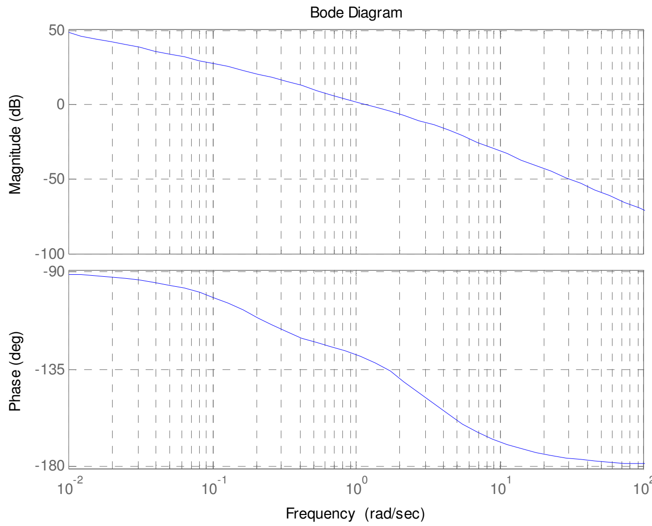
→ $\lim_{s\to 0} s\,\dfrac{0.756}{s(s+0.3)}\,K\dfrac{Ts+1}{\alpha Ts+1} = 2.52$ → K = 1 → $K_c = 1/\alpha = 3.7$

$-20\log\left(\dfrac{1}{\sqrt{\alpha}}\right) = -5.6$ → The gain cross over frequency, $\omega_c \approx 1.2$

→ $\omega_1 = \dfrac{1}{T} = \sqrt{\alpha}\,\omega_C = 0.62$ and $\omega_2 = \dfrac{1}{\alpha T} = \dfrac{\omega_C}{\sqrt{\alpha}} = 2.31$ rd/s

→ $G_c(s) = \dfrac{1.6s+1}{0.43s+1}$ → O.L.T.F = G(s) $G_c(s)$ = $\dfrac{1.6s+1}{0.43s+1}\,\dfrac{0.756}{s(s+0.3)}$

The bode diagram of the compensated system is shown below. The new G_M and P_M are ∞ and 49.5°, respectively.



<u>Problem 2 (60 points):</u>

a) S-plane poles: 0, -4
   S-plane zeros: +2i, -2i
   → The system is marginally stable

b) K → 0, Therefore C.L.T.F poles = O.L.T.F poles = 0, -4

c) Range of pure real poles of C.L.T.F = [-4, 0]

d) System critically damped → $\varsigma = 1$. The C.L.T.F = $\dfrac{KG}{1+KG} =$

$$\frac{K(s^2+4)}{(1+K)s^2+4s+4K} = \frac{\left(\dfrac{K}{1+K}\right)(s^2+4)}{s^2+\left(\dfrac{4}{1+K}\right)s+\left(\dfrac{4K}{1+K}\right)}$$

→ $2\omega_n = \left(\dfrac{4}{1+K}\right)$ and $\omega_n^2 = \left(\dfrac{4K}{1+K}\right)$ → K = 0.618

e) The C.L.T.F poles converge to the O.L.T.F zeros = +2i and -2i

f) 0.618 < K < ∞

g)