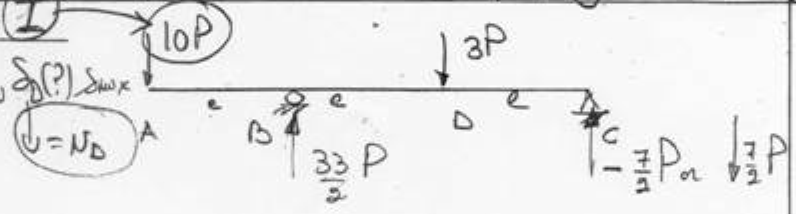
Problem (I)

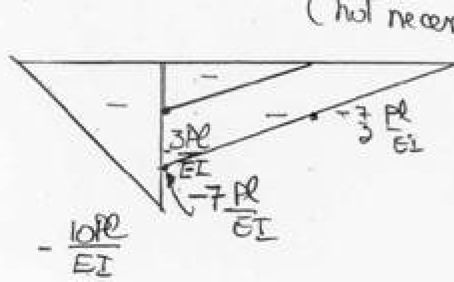
$\theta_B, \theta_C, \delta_A, \delta_D(?), \delta_{max}$

$\underline{10P}$ $\quad \downarrow = N_D$

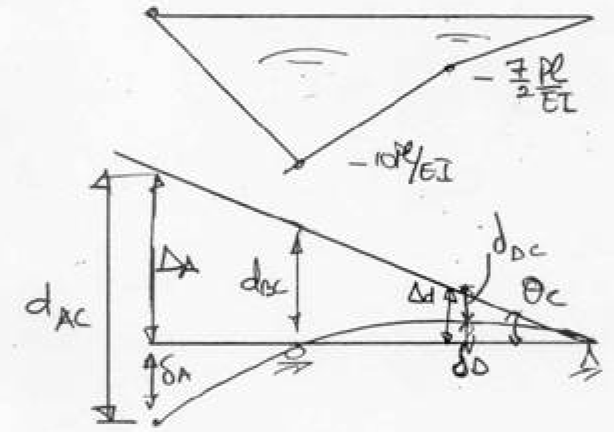


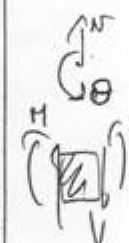
(not necessary by parts)

about (B): M/EI by parts



Total





$$d_{BC} = -\frac{1}{2}\left(\frac{3P\ell}{EI}\right)\times(\ell)\times\left(\frac{1}{3}\ell\right) - \frac{1}{2}\left(\frac{7P\ell}{EI}\right)(2\ell)\left(\frac{1}{3}\times 2\ell\right) = \ominus\frac{31}{6}\frac{P\ell^3}{EI}$$

ok

$$|\theta_C| = \frac{|d_{BC}|}{2\ell} = \frac{31}{12}\frac{P\ell^2}{EI}$$

$$|\Delta_A| = |\theta_C| \times 3\ell = \frac{31}{4}\frac{P\ell^3}{EI}$$

$$d_{AC} = -\frac{1}{2}\left(\frac{10P\ell}{EI}\right)\times(\ell)\times\left(\frac{2}{3}\ell\right) - \frac{1}{2}\left(\frac{3P\ell}{EI}\right)(\ell)\left(\frac{4}{3}\ell\right) - \frac{1}{2}\left(\frac{7P\ell}{EI}\right)(2\ell)\left(\ell + \frac{1}{3}\times 2\ell\right)$$

ok $= \ominus 17\frac{P\ell^3}{EI}$ (Point below tangent) $\quad |d_{AC}| > |\Delta_A|$

$\Rightarrow |\delta_A| = |d_{AC}| - |\Delta_A| = 9.25\, P\ell^3/EI$ down

$\theta_C = \frac{31}{12}\frac{P\ell^2}{EI}$ clockwise

$\delta_A = 9.25\frac{P\ell^3}{EI}$ down

$|\Delta d| = |\theta_d| \cdot \ell = \frac{31}{12}\frac{P\ell^2}{EI}$

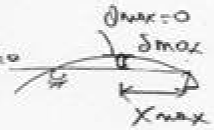
OK

$d_{DC} = -\frac{1}{2}\left(\frac{7P\ell}{2EI}\right)\cdot(\ell)\cdot\left(\frac{\ell}{3}\right) = -\frac{7}{12}\frac{P\ell^3}{EI}$ $\quad |d_{DC}| < |\Delta d|$

$|\delta_D| = |\Delta d| - |d_{DC}| = 2\frac{P\ell^3}{EI}$ (upward) $\qquad \delta_D = \frac{2P\ell^3}{EI}$ up

δ_{max} = ? locate x_{max} at $\theta_{max} = 0$

$B \to C$ $\quad \to \underline{N_{max}} = --$



θ_B = ?

$\theta_C - \theta_B = -\frac{1}{2}\left(\frac{7P\ell}{EI}\right)(2\ell) - \frac{1}{2}\left(\frac{3P\ell}{EI}\right)(\ell) = -8.5\frac{P\ell^2}{EI}$

where $\theta_C = -\frac{31}{12}\frac{P\ell^2}{EI}$

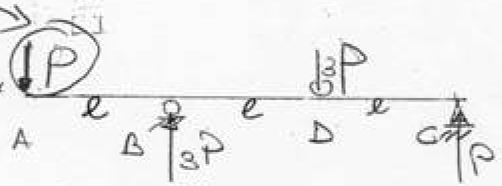
$\Rightarrow \theta_B = -\frac{31}{12}\frac{P\ell^2}{EI} + 8.5\frac{P\ell^2}{EI} = (+)\,5.92\frac{P\ell^2}{EI}$

counterclockwise

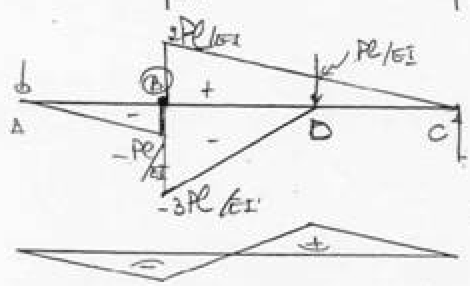
$\theta_B - \theta_A = --- \Rightarrow \underline{\theta_A} = ---$

Problem (I)

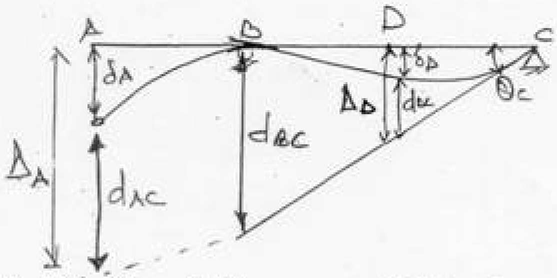
($\theta_B, \theta_C, \delta_A, \delta_D(?), \delta_{max}$) for $\underline{P}$



about (B): M/EI by parts



total



$d_{BC} = -\frac{1}{2}\left(\frac{3Pl}{EI}\right)\times(l)\times\left(\frac{1}{3}l\right) + \frac{1}{2}\left(\frac{2Pl}{EI}\right)(2l)\left(\frac{1}{3}\times 2l\right) = \frac{5Pl^3}{6EI}$

$\rightarrow \theta_C = \frac{d_{BC}}{2l} = \frac{5}{12}\frac{Pl^2}{EI}$ | $\theta_C = \frac{5}{12}\frac{Pl^2}{EI}$ counterclock

$\Delta_A = \theta_C\, 3l = \frac{5}{4}\frac{Pl^3}{EI}$

$d_{AC} = -\frac{1}{2}\left(\frac{Pl}{EI}\right)(l)\times\frac{2}{3}l - \frac{1}{2}\left(\frac{3Pl}{EI}\right)(l)\times\left(\frac{4}{3}l\right) + \frac{1}{2}\left(\frac{2Pl}{EI}\right)(2l)\left(l+\frac{1}{3}\times 2l\right)$

$= \frac{Pl^3}{EI}$ (Point A above tangent) $(< \Delta_A)$

$\rightarrow \delta_A = \Delta_A - d_{AC} = \frac{Pl^3}{4EI}$ down | $\delta_A = \frac{Pl^3}{4EI}$ down

$\Delta_D = \theta_C \times l = \frac{5}{12}\frac{Pl^3}{EI}$

$d_{DC} = +\frac{1}{2}\left(\frac{Pl}{EI}\right)\times(l)\times\left(\frac{1}{3}l\right) = \frac{Pl^3}{6EI}$ (Point D above tangent)
$< \Delta_D$

$\rightarrow \delta_D = \Delta_D - d_{DC} = \frac{Pl^3}{4EI}$ down | $\delta_D = \frac{Pl^3}{4EI}$ down

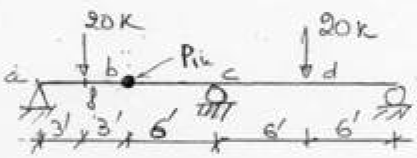
$\theta_B(?)$ $\theta_C - \theta_B = \frac{1}{2}\left(\frac{2Pl}{EI}\right)(2l) - \frac{1}{2}\left(\frac{3Pl}{EI}\right)l = \frac{Pl^2}{2EI}$

$\Rightarrow \theta_B = \frac{5}{12}\frac{Pl^2}{EI} - \frac{Pl^2}{2EI} = -\frac{Pl^2}{12EI} < 0$ clockwise)

δ_{max} ($A \to B$) : Locate X_{max}
@ $\theta_{max} = 0$
$\rightarrow \underline{\underline{N_{max}}} = —$

$\theta_B - \theta_A = - - - \Rightarrow \underline{\theta_A} = —$

Problem (II)



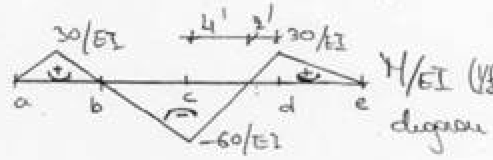
Same EI

$\delta = N$

(a) δ_b and $\delta_{20k} \left\langle \begin{matrix}\delta_d \\ \delta_f\end{matrix}\right.$

(b) δ_{max} and location (ce)

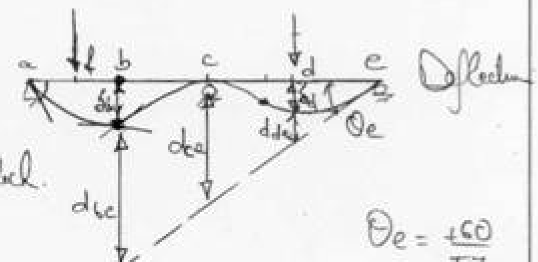
(c) Angles $\theta_a, \theta_e, \theta_b^{\oplus+\ominus}$



(a)+(c)

Consider $\underline{bcde}$

$d_{ce} = \cdots = \frac{720}{EI}$ (Point above tangent)



$\theta_e = \frac{d_{ce}}{12} = \boxed{\frac{60}{EI} = \theta_e}$ counterclock.

$\theta_e = \frac{+60}{EI}$

- $\delta_d = \Delta_d - d_{de}$

$\Delta_d = \theta_e \times 6 = \frac{360}{EI}$

$d_{de} = \cdots = \frac{180}{EI}$

$\boxed{\delta_d = \frac{180}{EI}}$ down

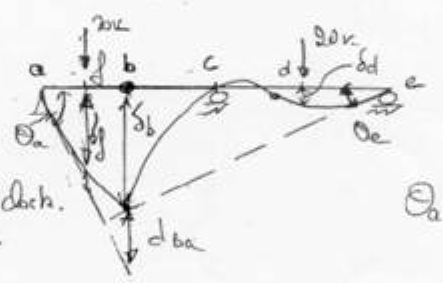
- $\delta_b = \Delta_b - d_{be}$

$\Delta_b = \theta_e \times 18 = \frac{1080}{EI}$

$d_{be} = \cdots = 0$ ← Note (Point coincides with tangent)

$\boxed{\delta_b = \frac{1080}{EI}}$ down

Consider $\underline{afb}$



- $d_{ba} = \cdots = \frac{270}{EI}$ (point above tangent)

$\theta_a = \frac{\delta_b + d_{ba}}{6} = \boxed{\frac{225}{EI} = \theta_a}$ clock.

$\theta_a = -\frac{225}{EI}$

- $\Delta_f = d_{fa} + \delta_f$

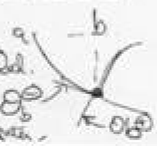
$\Delta_f = \theta_a \times 3 = \frac{675}{EI}$

$d_{fa} = \cdots = \frac{45}{EI}$

$\delta_f = \frac{675-45}{EI} = \boxed{\frac{630}{EI} = \delta_f}$ down

- $\Delta\theta_a^{b\ominus} = \theta_{b\ominus} - \theta_a = \frac{-1}{2}\frac{30}{EI} \times 6 = \frac{90}{EI} \Rightarrow \boxed{\theta_{b\ominus} = -\frac{135}{EI}}$ clock.

- $\Delta\theta_b^{e\oplus} = \theta_e - \theta_b^{\oplus} = -\frac{180}{EI}$ $= \boxed{\theta_b^{\oplus} = \frac{+240}{EI}}$ counter.

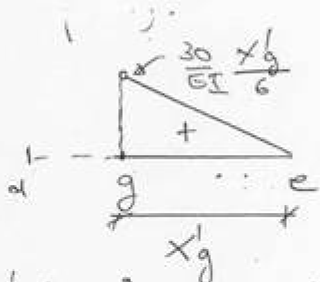


(b) δ_{max} and location (ce)

Check between de : (g)

$\delta_{max} \Rightarrow \theta_g$ (slope) $= 0$

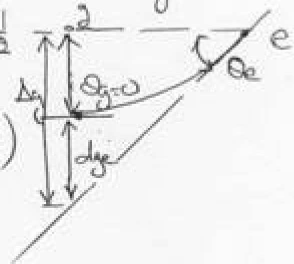
M/EI



$\Delta\theta_g^e = \theta_e - \theta_g = \triangle = \frac{30}{6EI} x'_g \times x'_g \times \frac{1}{2}$

$\Rightarrow \frac{60}{EI} = \frac{5}{2EI} x'^2_g$

$\Rightarrow x'_g = \sqrt{24} \simeq 4.89\,ft < 6\,ft$ (OK)

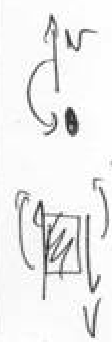


$\delta_g = \Delta g - d_{ge}$

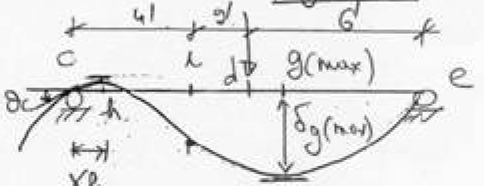
$\Delta g = \theta_e \times x'_g = \frac{293.94}{EI}$

$d_{ge} = \frac{5}{2EI} \times (24) \times \frac{\sqrt{24}}{3} = \frac{97.98}{EI}$

$\Rightarrow \boxed{\delta g \simeq \frac{196}{EI}}$ max down between d + e : @ $\boxed{x'_g = 4.89\,ft}$ from e



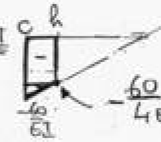
Check between (c and i)
(h)
inflection



$\delta_{max} \Rightarrow \theta_h = 0$

or $\begin{cases} \Delta\theta_{he} = 0 \\ \Delta\theta_{bh} = 0 \\ \Delta\theta_{ch} = 0 \text{ (if } \theta_c \text{ is calculated)} \\ \Delta\theta_{hg} = 0 \end{cases}$

use



$-\frac{60}{4EI}(4 - X_h) = -\frac{15}{EI}(4 - X_h)$
$(-60 + 15X_h)/EI$

$\Delta\theta_b^c = \theta_c - \theta_b = -\frac{60}{EI} \times \frac{6}{2} = -\frac{180}{EI}$

$\Rightarrow \theta_c = \frac{-180 + 240}{EI} = \frac{60}{EI}$ counter clockwise

Up. Move

between c. $\delta_h = \frac{48.8}{EI}$

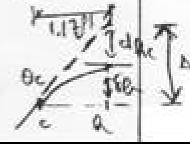
$X_h = 1.17\,ft$ from C

$\Delta\theta_c^h = \cancel{\theta_h} - \theta_c = -\frac{1}{EI}(60 + 60 - 15X_h) \times \frac{1}{2} \times X_h = -\frac{60}{EI}$

$\Rightarrow 7.5 X_h^2 - 60 X_h + 60 = 0 \qquad X_h^2 - 8X_h + 8 = 0$

$X_h = +\frac{8 \pm \sqrt{32}}{2} = 1.17\,ft < 4\,ft$ (OK)

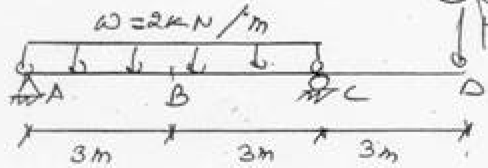
$\delta_h = \Delta h - d_{hc} = \theta_c \times 1.17 - \left[\frac{-15 \times 1.17}{EI} \times \frac{1}{2} \times \frac{1.17}{3} + (-60 + 15 \times 1.17)\frac{1.17}{2EI}\right] = \frac{48.8}{EI}$



Problem III

$\theta_C, \delta_B, \delta_D$

$\delta_{max(AC)}$



$E = 20 \times 10^6\ kN/m^2$

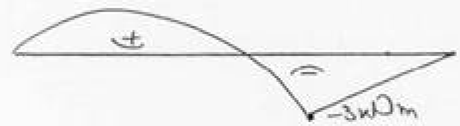
$I = \frac{0.20 \times 0.5^3}{12}\ (m^4)$

$EI = 41666.7\ kNm^2$

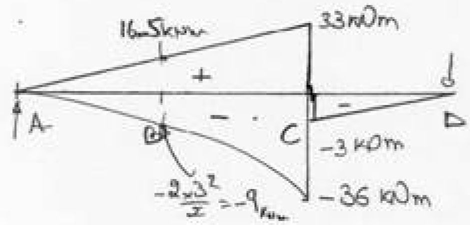
Reactions: $\Sigma M_C = 0 \Rightarrow R_A = 5.5kN \uparrow$

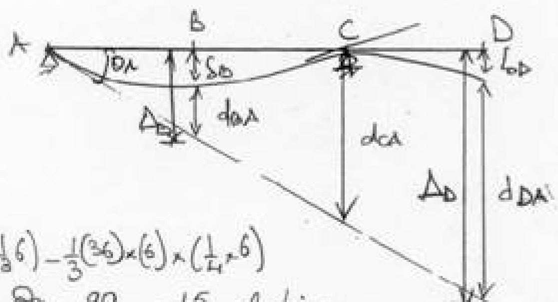
$\Sigma F_y = 0 \Rightarrow R_C = 7.5kN \uparrow$

Total: M



By Parts: M





$EI d_{CA} = \frac{1}{2}(33)(6) \times (\frac{1}{3}6) - \frac{1}{3}(36)(6) \times (\frac{1}{4} \times 6)$

$\Rightarrow d_{CA} = \frac{90}{EI} \Rightarrow \theta_A = \frac{90}{6EI} = \frac{15}{EI}$ clockwise

$\theta_C - \theta_A = \frac{1}{2}\frac{33 \times 6}{EI} - \frac{1}{3}\frac{36 \times 6}{EI} = \frac{27}{EI} \Rightarrow \theta_C = \frac{27}{EI} + \theta_A = \frac{27}{EI} - \frac{15}{EI}$ | $\theta_C = \frac{12}{EI}$ counterclock. $= 2.88 \times 10^{...}$

$\Delta_B = \theta_A \times 3 = \frac{45}{EI}$

$EI d_{BA} = \frac{1}{2} \times (16.5)(3)(\frac{3}{3}) - \frac{1}{3}(9)(3)(\frac{1}{4} \times 3) = 18 \Rightarrow d_{BA} = \frac{18}{EI}$ (Point above) $< \Delta_B$

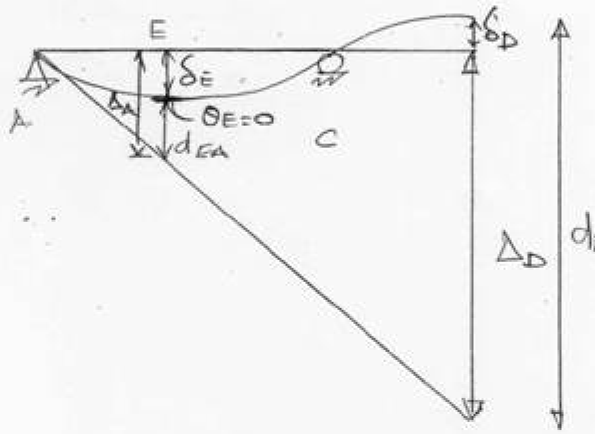
$\delta_B = \Delta_B - d_{BA} = \frac{45}{EI} - \frac{18}{EI} = \frac{27}{EI}$ down. | $\delta_B = \frac{27}{EI}$ down

6.48

$\Delta_D = \theta_A \times 9 = \frac{135}{EI}$

$EI d_{DA} = \frac{1}{2}(33)(6)(\frac{6}{3} + 3) - \frac{1}{3}(36)(6) \times (\frac{6}{4} + 3) - \frac{1}{2}(3)(3) \times (\frac{2}{3}3) = 162$ (Point above)

NOTE: $d_{DA} > \Delta_D \Rightarrow$ adjust deflected shape $\rightarrow$

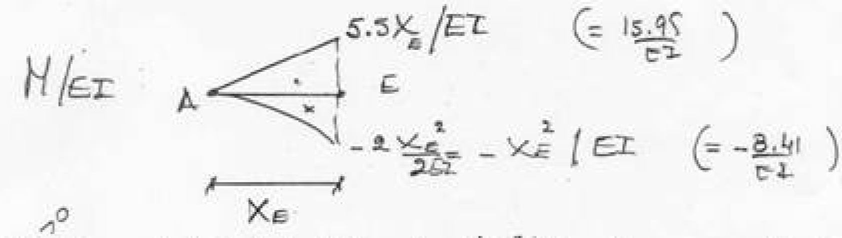


$\delta_D = d_{DA} - \Delta_D = \frac{27}{EI}$ upward.

$\delta_D = \frac{27}{EI}$ up

6.48×10^{-4}

Between AC: δ_{MAX} @ E $\rightarrow \delta_E \Rightarrow \theta_E = 0$ @ X_E



$\cancel{\theta_E}^{0} - \theta_A = \frac{1}{2}\left(\frac{5.5X_E}{EI}\right)(X_E) - \frac{1}{3}\left(\frac{X_E^2}{EI}\right)(X_E) = \frac{2.75X_E^2}{EI} - \frac{X_E^3}{3EI}$

$= -\left(-\frac{15}{EI}\right)$

$\Rightarrow X_E^3 - 8.25X_E^2 + 45 = 0$

trial + error $X_E = 2.9m$ (just to left of B)

$\delta_E = \Delta_E - d_{EA}$

$\Delta_E = \theta_E \times X_E = \frac{43.5}{EI}$

$EI\, d_{EA} = \frac{1}{2}(15.95)(2.9)\left(\frac{2.9}{3}\right) - \frac{1}{3}(8.41)(2.9)\left(\frac{2.9}{4}\right) = 16.46$

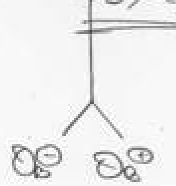
$\Rightarrow \delta_E = 27.04/EI$ $(\simeq \delta_D)$

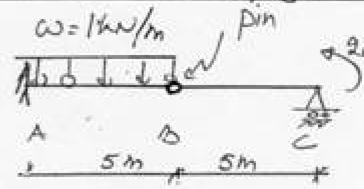
$\delta_E = \frac{27.04}{EI}$

6.4896×10^{-4}

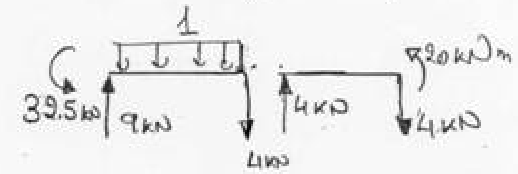
Problem (IV)

$\theta_B, \theta_C, \delta_B$ (?)

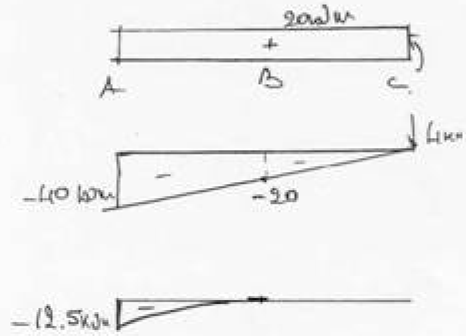




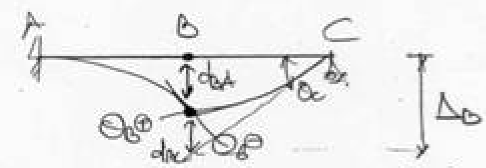
EI = 46666.7 kN m²



Moment: by parts about (A)



Pin at B
⇒ Consider
AB
& BC



$$\theta_B^{\ominus} - \cancel{\theta_A}^{\,0} = \left(\frac{20}{EI}\right)\times(5) - \left(\frac{20}{EI}\right)(5) - \frac{1}{2}\left(\frac{20}{EI}\right)(5) - \frac{1}{3}(12.5)(5)$$
$$= -70.83/EI$$

$\Rightarrow \theta_B^{\ominus} = -70.83/EI$ (clockwise)

$\theta_B^{\ominus} = 70.83/EI$ clockwise

1.7×10^{-3}

$$EI\,d_{BA} = 20\times5\times\frac{5}{2} - 20\times5\times\frac{5}{2} - \frac{1}{2}(20)(5)\left(\frac{2}{3}5\right) - \frac{1}{3}(12.5)(5)\left(\frac{3}{4}5\right)$$

$EI\,d_{BA} = -244.8 \Rightarrow d_{BA} = \dfrac{-244.8}{EI}$ (Point B below)

$\delta_B = \dfrac{244.8}{EI}$ (down)

$\delta_B = \dfrac{244.8}{EI}$

5.87×10^{-3} m

$$EI\,d_{BC} = 20\times5\times2.5 - \frac{1}{2}(20)(5)\left(\frac{1}{3}\times5\right) = 166.7 \text{ (above tangent)}$$

1.975×10^{-3}

$\Delta_B = |d_{BA}| + |d_{BC}| = \dfrac{411.5}{EI} \Rightarrow \theta_C = \dfrac{\Delta_B}{5} = \dfrac{82.3}{EI}$ counterclockwise

$\theta_C = 82.3/EI$ counterclockwise

$\theta_C - \theta_B^{\oplus} = \dfrac{20\times5}{EI} - \dfrac{1}{2}\dfrac{20\times5}{EI} = \dfrac{50}{EI} \Rightarrow \theta_B^{\oplus} = \dfrac{-50+82.3}{EI} = \dfrac{32.3}{EI}$

$\theta_B^{\oplus} = 32.3$ counterclockwise

7.75×10^{-4}