QUIZ 2
Spring 2003-2004
(Thursday, May 13, 2004)
CIVE 311 – STRUCTURES I
CLOSED BOOK, 1 HOUR & 45 MN

Name: Solution **ID#:** 2

NOTES

- 2 PROBLEMS – 13 PAGES.
- ALL YOUR ANSWERS SHOULD BE PROVIDED ON THE QUESTION SHEETS.
- [redacted]
- [redacted]
- SOME ANSWERS MAY REQUIRE MUCH LESS THAN THE SPACE PROVIDED.
- *DO NOT* USE THE BACK OF THE SHEETS FOR ANSWERS.
- DRAFT BOOKLET WILL BE PROVIDED; BUT DO NOT USE FOR ANSWERS.
- BOTH QUESTION SHEETS AND DRAFT BOOKLET SHOULD BE RETURNED.
- CHECK BOXES ARE FOR YOU TO CONFIRM THAT HAVE SOLVED A QUESTION ☑

YOUR COMMENT(S)

Some tough calculations.

DO NOT WRITE IN THE SPACE BELOW

MY COMMENT(S)

Symmetry makes it easier

YOUR GRADE

Problem I: 65 /65
Problem II: 35 /35
Other: ---

TOTAL: 100 /100

Problem I: *(65 points)*

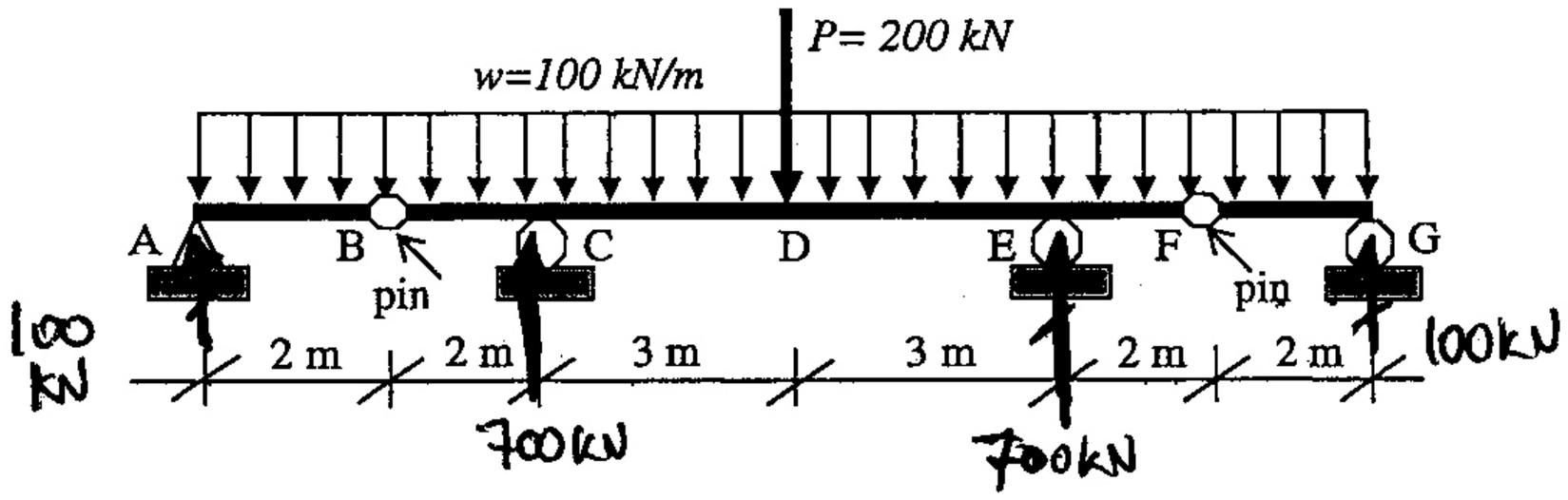


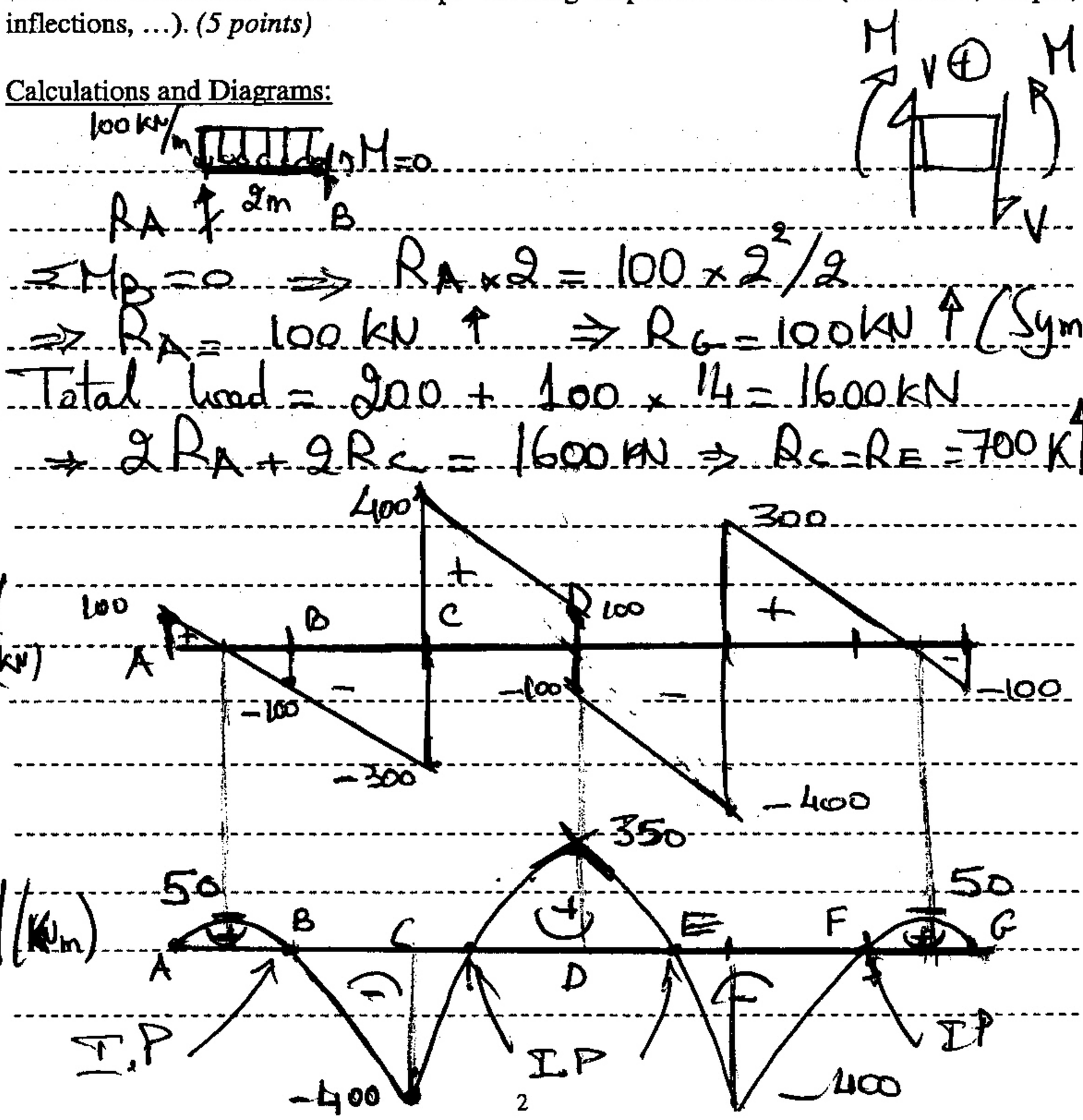
Figure I

Referring to Figure I, let EI=200,000 kN.m^2 throughout the beam. Neglect the own weight of the beam.

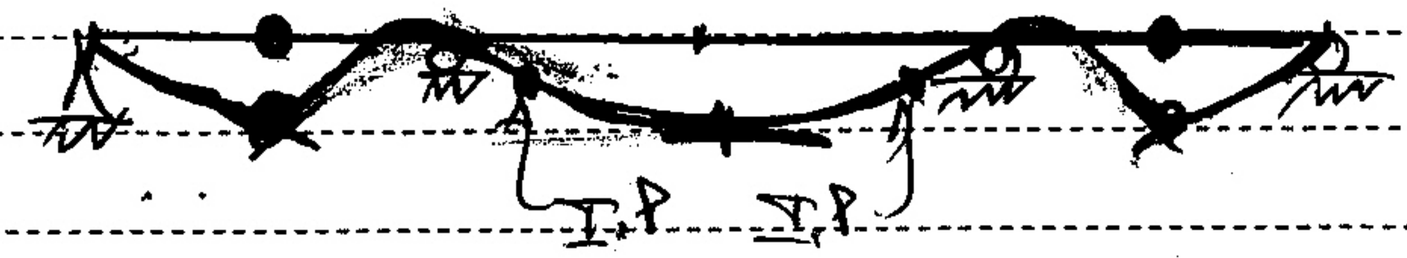
NOTE: THE SYSTEM IS SYMMETRICAL; YOU MAY TAKE ADVANTAGE OF THIS.

1. Calculate the reactions, and draw the shear and bending moment diagrams. *(15 points)* Sketch a reasonable deflected shape showing important features (deflections, slopes, inflections, …). *(5 points)*

Calculations and Diagrams:

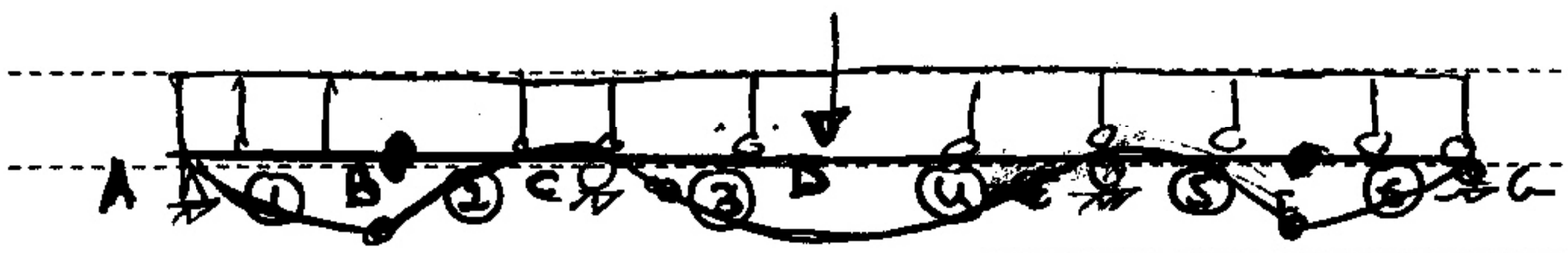


Calculations and/or Diagrams (cont'd):



2. Indicate how you would solve for the deflected shape using the method of INTEGRATION (Do not compute or write detailed equations; show an outline of the steps required). *(10 points)*

Calculations and Diagrams:



	A	B	C	D	E	F	G
	$M_1(x)$	$M_2(x)$	$M_3(x)$	$M_4(x)$	$M_5(x)$	$M_6(x)$	
Curv. $N'' = \frac{M}{EI}$ →	$N''_1(x)$	$N''_2(x)$	—	—	—	$N''_6(x)$	
Slope $N'(x) = \int N''(x)$ →	Cst$_1$	A_1	B_1	C_1	D_1	E_1	F_1
Def $N(x) = \int N'(x)$ →	Cst$_2$	A_2	B_2	C_2	D_2	E_2	F_2

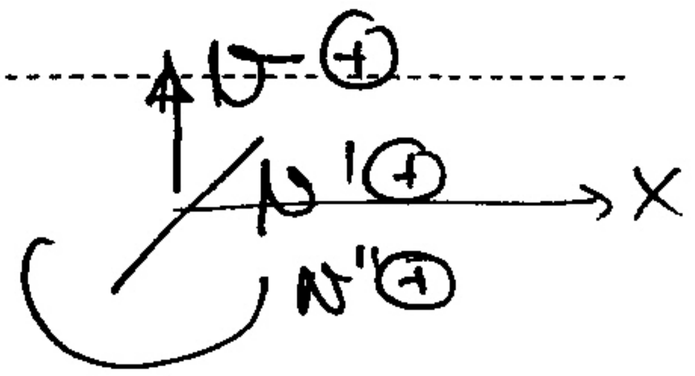
12 cst → 12 Boundary Conditions → 12 eqns?

6 eqs ($N_{①A} = 0$ $N_{③C} = 0$ $N_{④E} = 0$ $N_{⑥G} = 0$ $N_{③C} = 0$ $N_{⑤G} = 0$

3 eqs ($N'_{①C} = N'_{③C}$ $N'_{④E} = N'_{⑤E}$ $N'_{③D} = N'_{④D}$

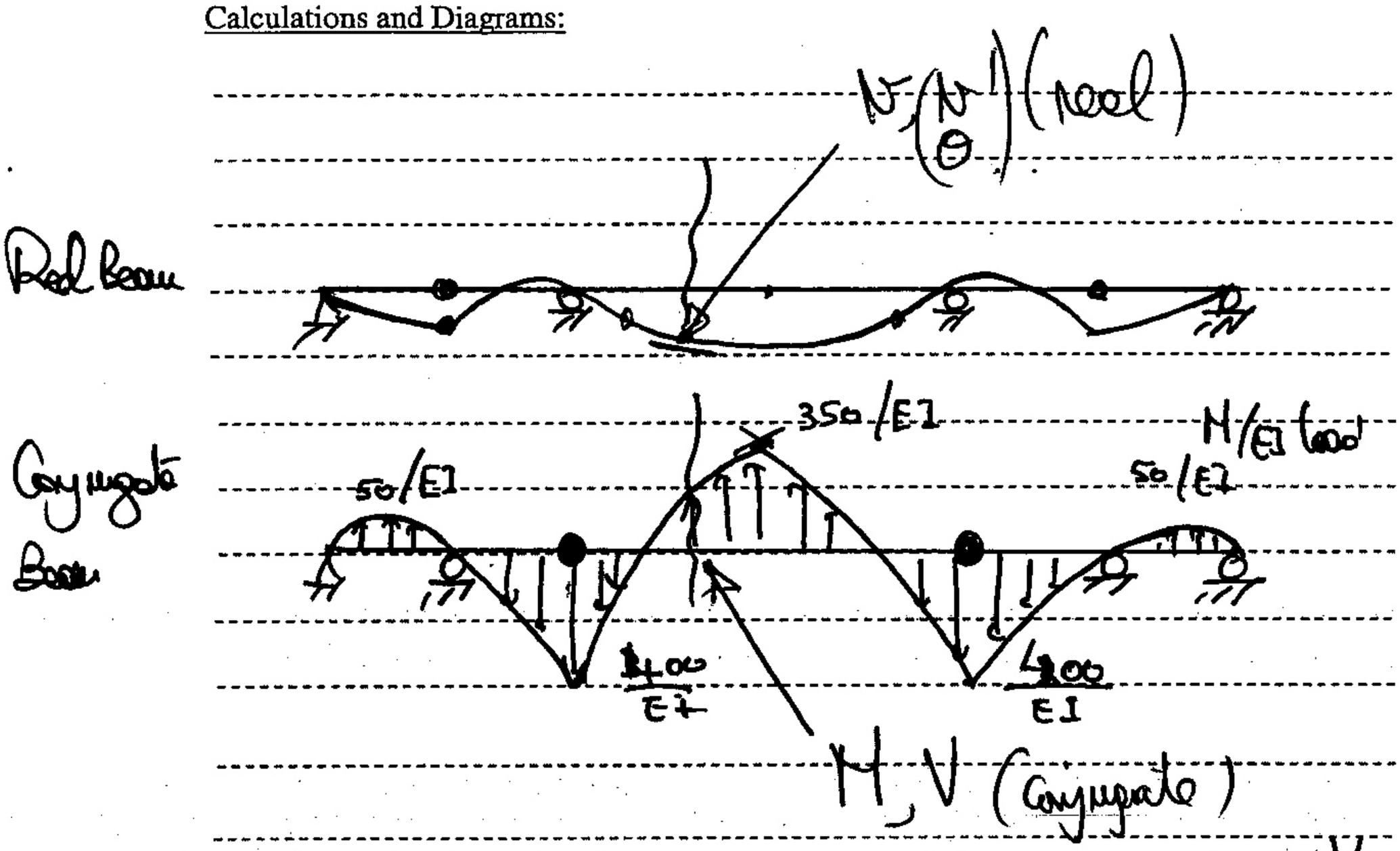
3 eqs ($N_{①B} = N_{②B}$ $N_{③D} = N_{④D}$ $N_{⑤F} = N_{⑥F}$

Solve for $A_1, \ldots F_2$



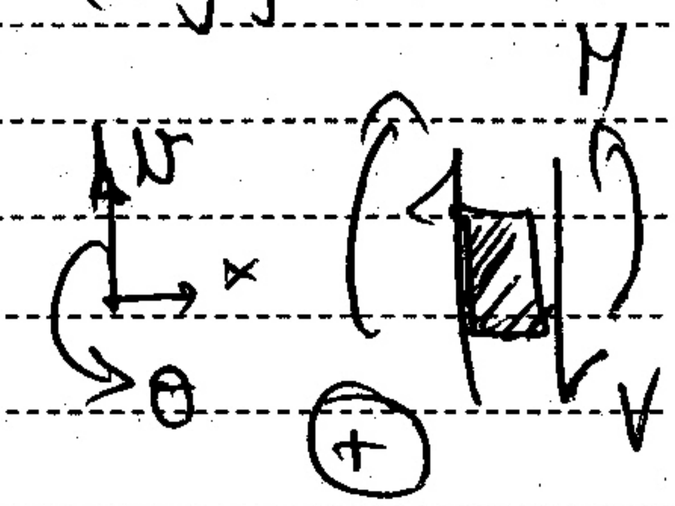
3. Draw the CONJUGATE BEAM with the corresponding load. Explain in ***two lines*** how you would solve for a deflection and slope at a point (Do not solve). *(5 points)* ✓

Calculations and Diagrams:



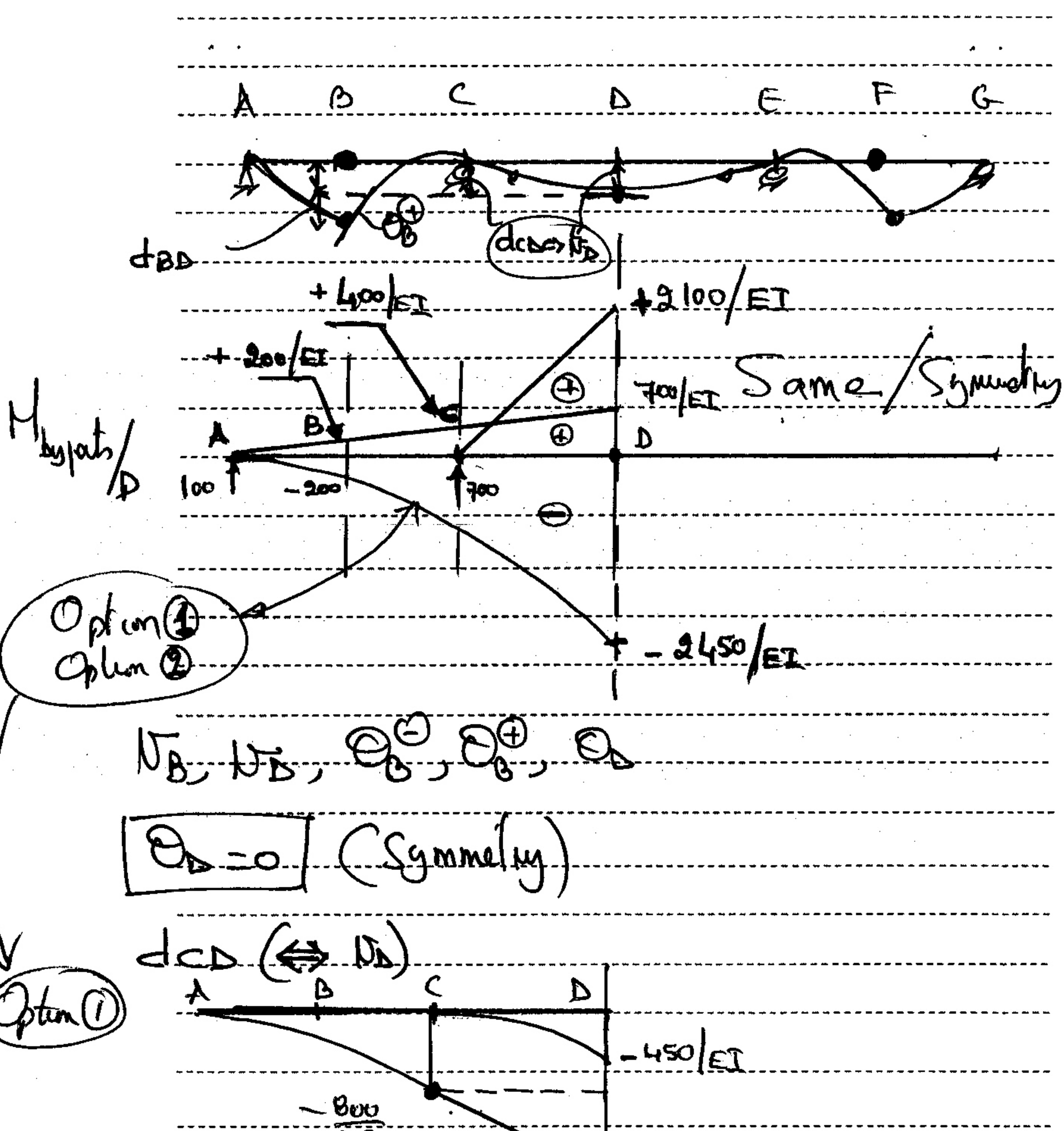
$M_{real} = M_{conjugate}$

$\theta_{real} \text{ or } M'_{real} = V_{conjugate}$



4. Using the MOMENT-AREA METHOD, compute the vertical deflections and slopes at points B and D (Again, symmetry can help here). Is the vertical deflection at B maximum between A and C; why or why not? *(30 points)*

Calculations and Diagrams:



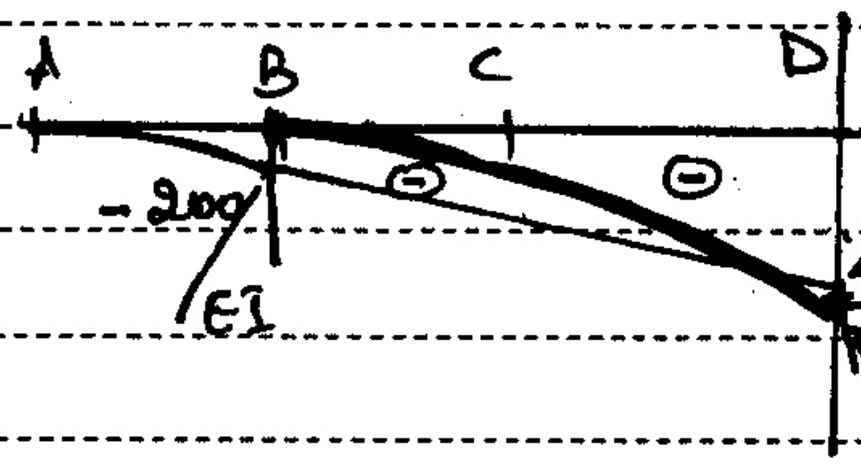
Calculations and/or Diagrams (cont'd):

$d_{CD} = \frac{\oplus 787.5}{EI} = 3.94 \times 10^{-3}$ m ⊕ Point C Above ✓

$\Delta_D = 3.94 \times 10^{-3}$ m ↓

$\Theta_B^{\oplus} : \cancel{\Theta_B^{0}} - \Theta_B^{\oplus} = \text{Area } M_{D \to B}$

Qtun 2



$\cancel{\Theta_B^{0}} - \Theta_B^{\oplus} = \frac{1}{EI}\left[\left(\frac{1}{2}\right)(-1200-200)(5) + \left(\frac{1}{3}\right)(-1250)(5)\right.$

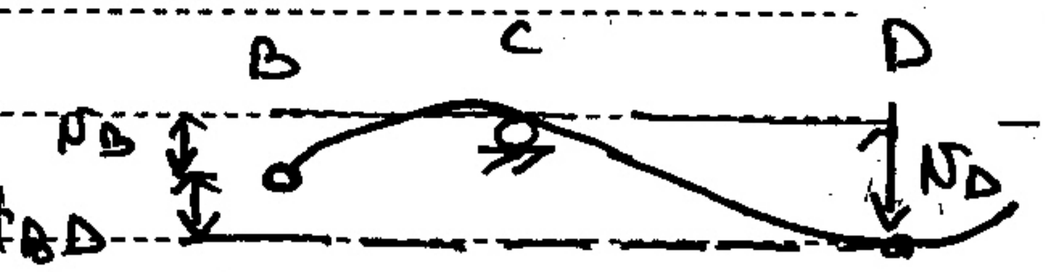
$\left. + \left(\frac{1}{2}\right)(2100)(3) + \left(\frac{1}{2}\right)(700+200)(5)\right] = -\frac{183.33}{EI}$

$\Theta_B^{\oplus} = +0.917 \times 10^{-3}$ rad ⇒ C.C.W ✓

$d_{BD} = \frac{1}{EI}\left[\left(\frac{1}{2}\right)(-1200+200)(5)\left(\frac{2}{3}\times 5\right) + (1)(-200)(5)\left(\frac{1}{2}\times 5\right) + \left(\frac{1}{3}\right)(-1250)(5)\left(\frac{3}{4}\times 5\right)\right.$

$\left. + \left(\frac{1}{2}\right)(2100)(3)\left(2+\frac{2}{3}\times 3\right) + \left(\frac{1}{2}\right)(700-200)(5)\left(\frac{2}{3}\times 5\right) + (1)(200)(5)\left(\frac{1}{2}\times 5\right)\right]$

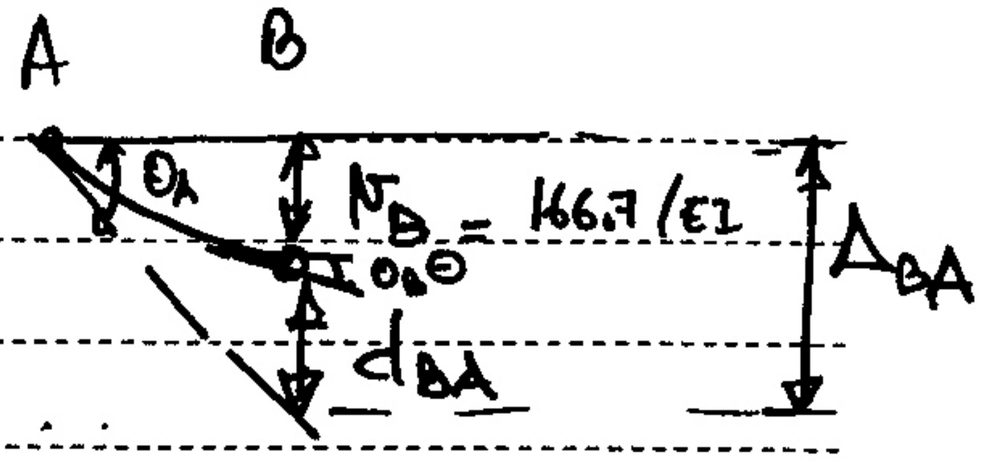
$= \frac{\oplus 620.8}{EI}$ Point B above



$|\Delta_B| = |\Delta_D| - |d_{BD}| = \left|\frac{787.5}{EI}\right| - \left|\frac{620.8}{EI}\right| = \frac{166.7}{EI}$ ↓

$\Delta_B = 0.833 \times 10^{-3}$ m ↓

Calculations and/or Diagrams (cont'd):



Point

$$d_{BA} = \frac{1}{EI}\left[\left(\frac{1}{2}\right)(200)(2)\left(\frac{1}{3}\times 2\right) + \left(\frac{1}{3}\right)(200)(2)\left(\frac{1}{4}\times 2\right)\right]$$

$$= \oplus \frac{66.67}{EI} \quad \oplus \text{ Point Above}$$

$$\Delta_{BA} = \frac{233.3}{EI} \Rightarrow |\Theta_A| = \left|\frac{\Delta_{BA}}{2}\right| = \frac{116.67}{EI} \quad \text{C.W.} \ (-)$$

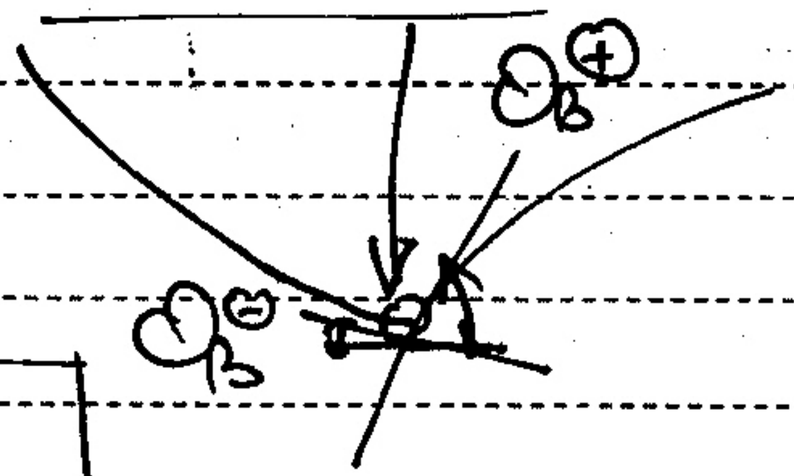
$$\Theta_B^{(-)} - \Theta_A = \frac{1}{EI}\left[\left(\frac{1}{2}\right)(200)(2) + \left(\frac{1}{3}\right)(-200)(2)\right] = +\frac{66.67}{EI}$$

$$\Rightarrow \Theta_B^{(-)} = \frac{1}{EI}\left(66.67 + \Theta_A\right) = \left(66.67 - 116.67\right)\frac{1}{EI} = -\frac{50}{EI} \text{ CW}$$

$$\Theta_B^{(-)} = -0.00025 \text{ rad} = -0.25\times 10^{-3} \text{ rad CW}$$

$\Theta_B^{(-)} \rightarrow \ominus$ CW

$\Theta_B^{(+)} \rightarrow \oplus$ CCW



$\Rightarrow$ N_B max between A → C

Problem II: *(35 points)*

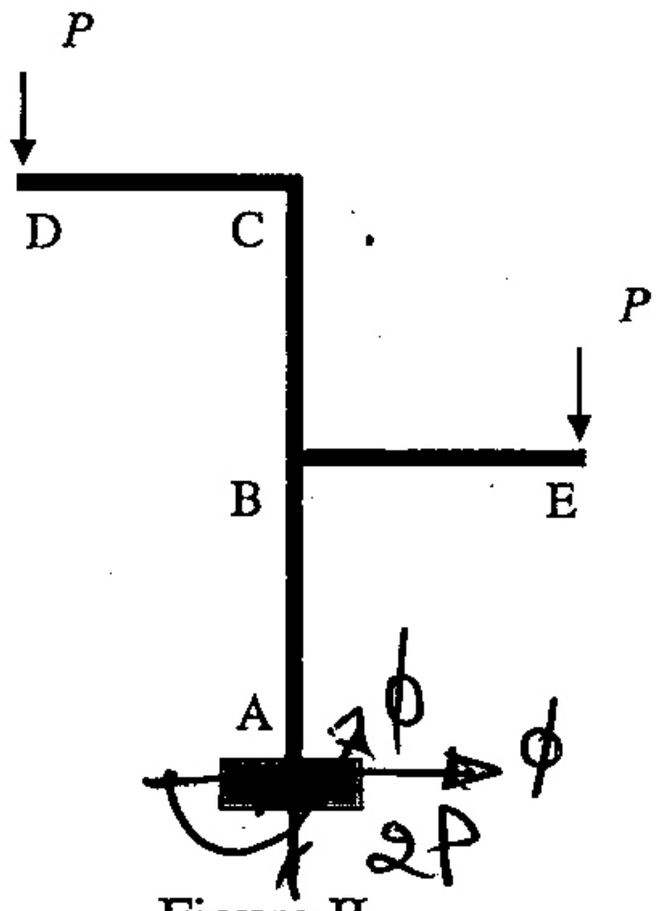
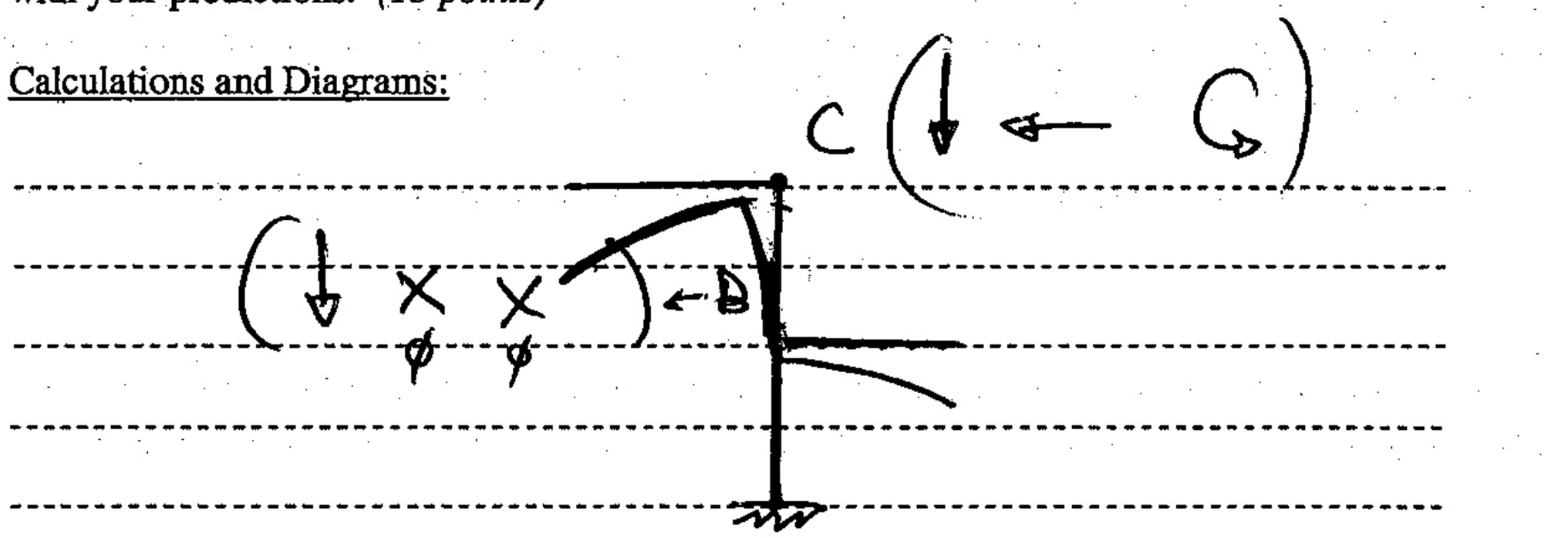


Figure II

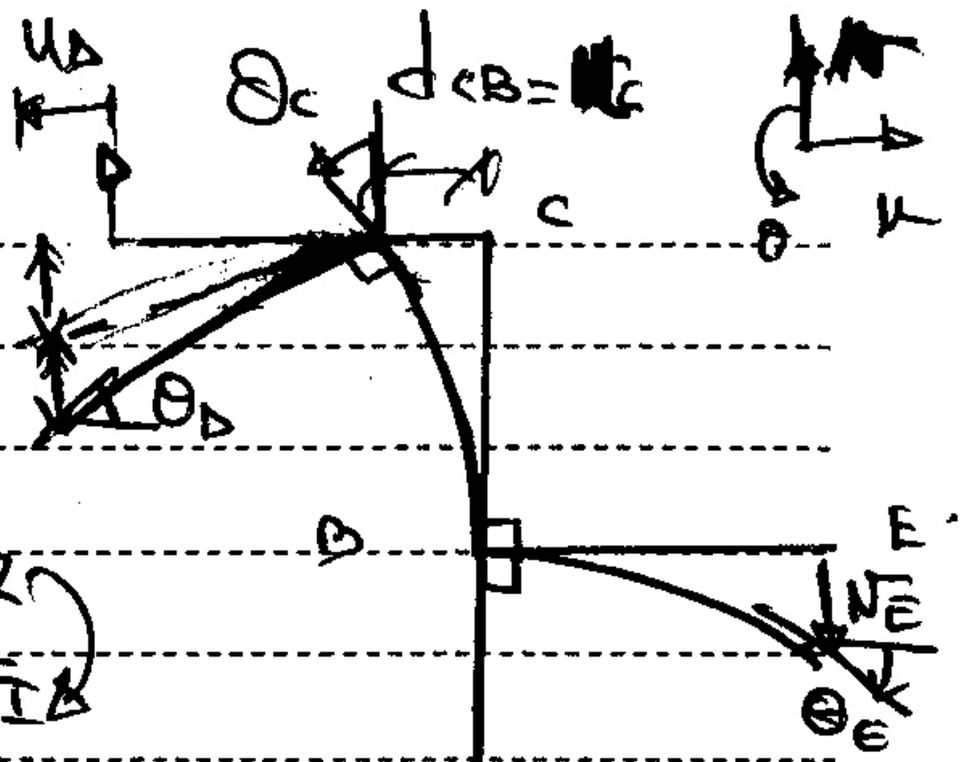
Referring to Figure II, the frame members have AB, BC, CD, and BE have the same length L. Neglect the own weight of the frame.

1. Let all members have the same EI. Predict, without calculations, the direction of deflections/rotations of joints B and C (up or down, left or right, cw or ccw), and sketch your predicted deflected shape of the frame. *(5 points)*
   Neglecting axial deformations, compute the deflections/rotations in all joints, and compare with your predictions. *(15 points)*

Calculations and Diagrams:



Calculations and/or Diagrams (cont'd):



- $u_A = v_A = \theta_A = 0$ (fixed)
- $u_B = v_B = \theta_B = 0$
- $u_E = 0 \quad v_E = \frac{Pl^3}{3EI} \downarrow \quad \theta_E = \frac{Pl^2}{2EI} \circlearrowright$
- $v_C = 0 \quad |v_C| = |d_{CB}| = \frac{Pl^3}{2EI} \leftarrow$

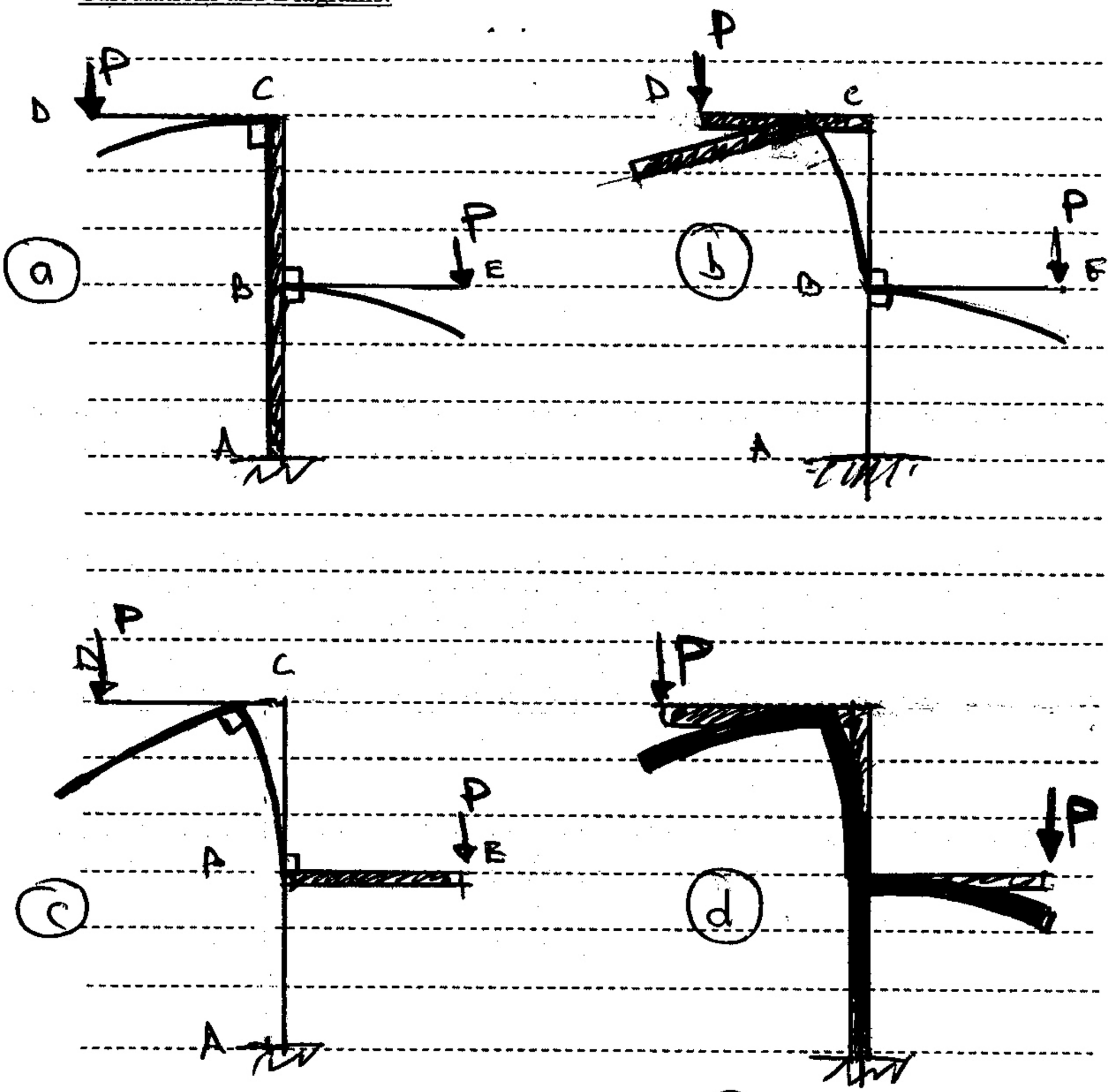
$$\theta_C = \frac{Pl^2}{EI} \circlearrowright \qquad (\text{Axial Def} \rightarrow 0)$$

- $u_D = v_C = \frac{Pl^3}{2EI} \leftarrow$

$$|v_D| = |\Delta_{AC}| + |d_{AC}| = \theta_C \times l + \frac{Pl^3}{3EI} = \frac{4}{3}\frac{Pl^3}{EI} \downarrow$$

2. Sketch the deflected shapes for the frames under the following conditions: *(15 points)*
   a. Member ABC is very stiff (all other members are normal).
   b. Member CD is very stiff (all other members are normal).
   c. Member BE is very stiff (all other members are normal).
   d. All members are very stiff.

Calculations and Diagrams:



Same as Question 1 (with smaller values of deflections)