DEFLECTION OF BEAMS (1)

CIVE311 – STRUCTURES I

(Wednesday, April 6, 2005)

Exercise I/I: *(partly from Quiz 1, Spring 1999-2000)*

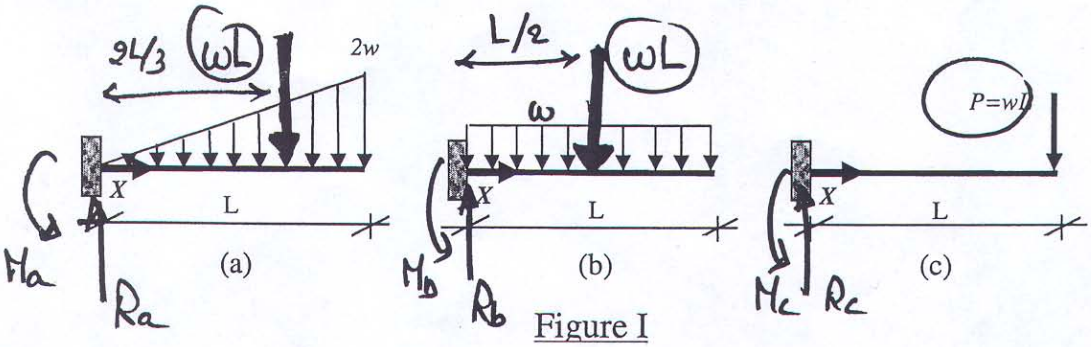


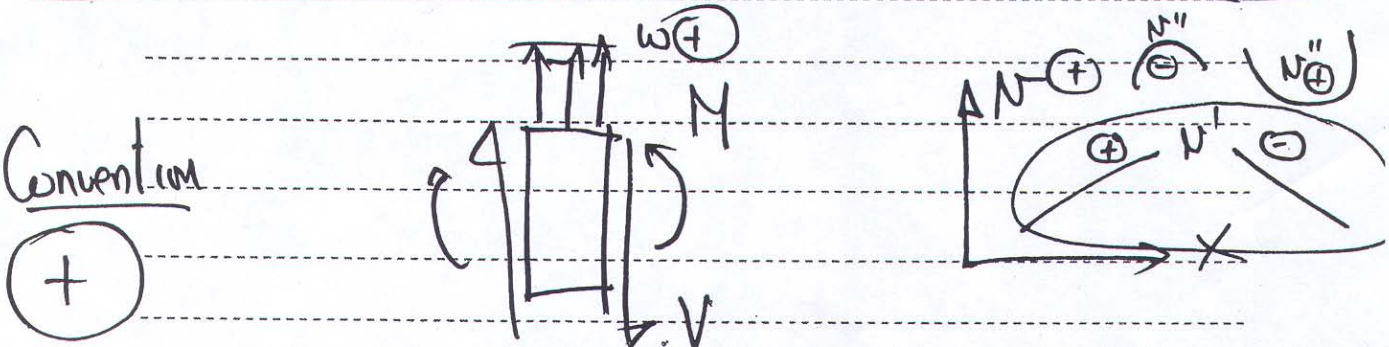
Figure I

1. Referring to Figure I, calculate the total load and reactions for the beams (a,b,c) & compare.

Calculations and Diagrams:

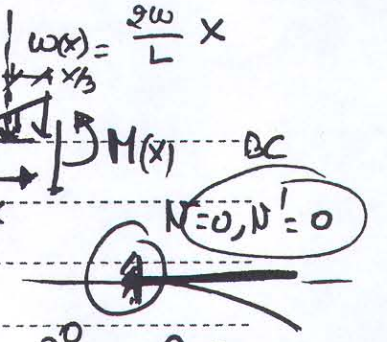
	(a)	(b)	(c)	Compare
Tot. W =	$\frac{1}{2} 2w \cdot L = wL$	wL	wL	Same
Reac R =	wL	wL	wL	Same
Reac M =	$\frac{2}{3} wL^2$	$\frac{wL^2}{2}$	wL^2	(c) > (a) > (b) Larger arm

Convention



2. Calculate the slopes and deflections at the tip of beams (a,b,c) and compare, using for
   (a) EIv"=M(x)
   (b) EIv""=w(x)
   (c) Moment-Area Theorems

Calculations and Diagrams:



(a) $M(x) = \omega L X - \frac{2\omega L^2}{3} - \frac{1}{2}\left(\frac{2\omega}{L}X\right)X\left(\frac{X}{3}\right)$ (3rd)

$M(x) = -\frac{1}{3}\frac{\omega}{L}X^3 + \omega L X - \frac{2}{3}\omega L^2$

$EI\,v'' = M(x)$

$EI\,v' = -\frac{1}{12}\frac{\omega}{L}X^4 + \frac{\omega L}{2}X^2 - \frac{2}{3}\omega L^2 X + C_1$ (BC: $X=0, v'=0 \Rightarrow C_1=0$)

$EI\,v = -\frac{1}{60}\frac{\omega}{L}X^5 + \frac{\omega L}{6}X^3 - \frac{2}{6}\omega L^2 X^2 + C_2$ (BC: $X=0, v=0 \Rightarrow C_2=0$)

Tip $v'_a(L) = -\frac{1}{4}\frac{\omega L^3}{EI} = -0.25\frac{\omega L^3}{EI}$

Tip $v_a(L) = -\frac{11}{60}\frac{\omega L^4}{EI} = -0.1833\frac{\omega L^4}{EI}$ (a)

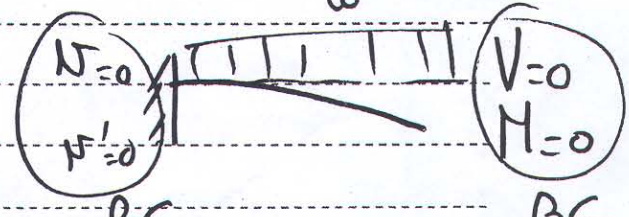
(b) $EI\,v'''' = -\omega$

$EI\,v''' = -\omega X + C_1 = V$

$X = L \Rightarrow V = 0 \Rightarrow C_1 = \omega L$

$EI\,v'' = -\frac{\omega X^2}{2} + \omega L X + C_2 = M$

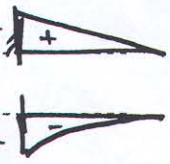
$X = L \Rightarrow M = 0 \Rightarrow C_2 = \frac{\omega L^2}{2} - \omega L^2 = -\frac{\omega L^2}{2}$



So

$(V=)\ EI\,v''' = -\omega X + \omega L$

$(M=)\ EI\,v'' = -\frac{\omega X^2}{2} + \omega L X - \frac{\omega L^2}{2}$



$EI\,v' = -\frac{\omega X^3}{6} + \frac{\omega L X^2}{2} - \frac{\omega L^2}{2}X + C_3$

$X = 0 \quad v' = 0 \quad C_3 = 0$

$EI\,v = -\frac{\omega X^4}{24} + \frac{\omega L X^3}{6} - \frac{\omega L^2}{4}X^2 + C_4$

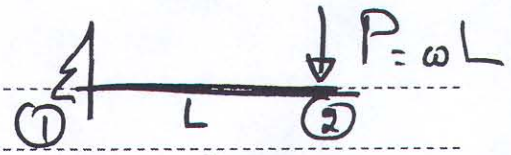
$X = 0 \quad v = 0 \Rightarrow C_4 = 0$

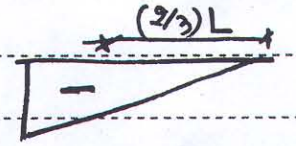
Tip $v'_b(L) = -\frac{\omega L^3}{6EI} = -0.167\frac{\omega L^3}{EI}$

Tip $v_b(L) = -\frac{\omega L^4}{8EI} = -0.125\frac{\omega L^4}{EI}$ (b)

Calculations and Diagrams (cont'd):

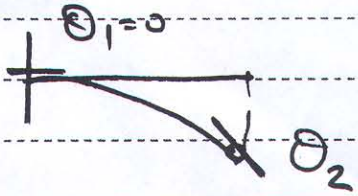
(c) Moment area



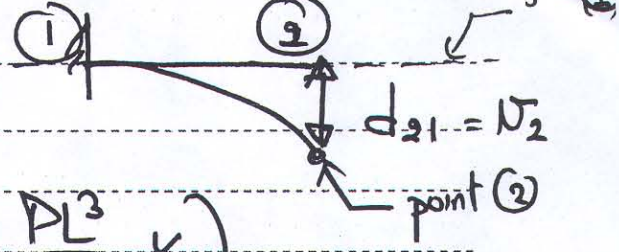


$$\frac{M}{EI} = \frac{-PL}{EI} = -\frac{\omega L^2}{EI}$$

$$\theta_2 - \theta_1 = \frac{1}{2}\left(-\frac{\omega L^2}{EI}\right)\times(L)$$



$$\theta_2 = -\frac{\omega L^3}{2EI}$$



$$d_{2/1} = \frac{1}{2}\left(-\frac{\omega L^2}{EI}\right)\times(L)\left(\frac{2}{3}L\right)$$

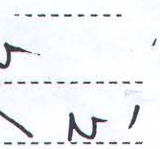
point ↵ ↳ tangent

$$= \ominus\ \frac{\omega L^4}{3EI} \qquad \left(\frac{PL^3}{3EI}\ \downarrow\right)$$

↳ Point (2) below tangent from (1) ↙

Typ $N'_c(L) = -\frac{\omega L^3}{2EI} = \ominus\ 0.5\frac{\omega L^3}{EI}$

Typ $|N_c(L)| = \frac{\omega L^4}{2EI} = 0.333\frac{\omega L^4}{EI}$ ©



Comparaison Absolute

$$N'_c > N'_a > N'_b$$

0.5 0.25 0.167

$$N_c > N_a > N_b$$

0.333 0.1833 0.125

Expected ↙