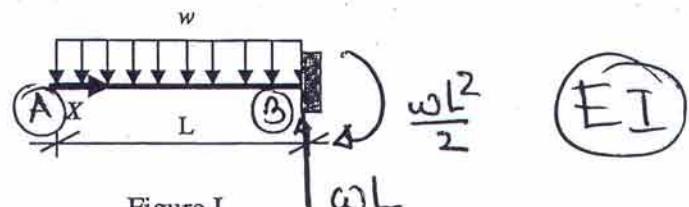


DEFLECTION OF BEAMS (2)CIVE311 - STRUCTURES I

(Wednesday, April 13, 2005)

Exercise I

- Referring to Figure I, calculate the slope and deflection at the tip, using the Moment-Area theorems.

Calculations and Diagrams:

V: Free body diagram of the beam showing a downward point load w at the left end and a fixed support at the right end.

M: Bending moment diagram showing a parabolic shape. The maximum bending moment at the tip is labeled $\frac{\omega L^2}{2}$.

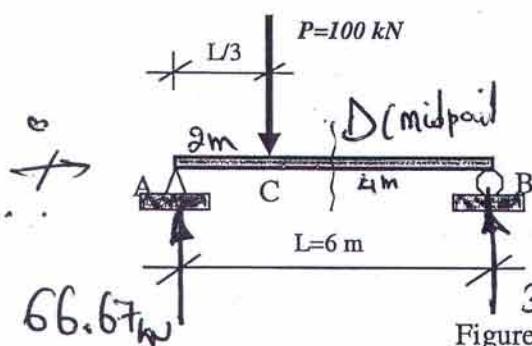
$\theta_B - \theta_A = \frac{1}{3} \left(\frac{-\omega L^2}{2EI} \right) L = -\frac{\omega L^3}{6EI}$

$\Rightarrow \theta_A = +\frac{\omega L^3}{6EI}$ (Ans)

$d_{AB} = \frac{(\omega L^3)}{6EI} \times \left(\frac{3}{4} L \right) = \frac{\omega L^4}{8EI}$ \rightarrow Point A below tangent B?

$N_A = d_{AB} = \frac{\omega L^4}{8EI}$ (downward)

$\int K = \frac{\omega L^4}{8EI}$

Exercise II

$$I = \frac{0.3 \times 0.6^3}{12} = 0.0054 \text{ m}^4$$

Cross-Section: 30x60 cm
Material: $E = 20 \times 10^6 \text{ kPa}$

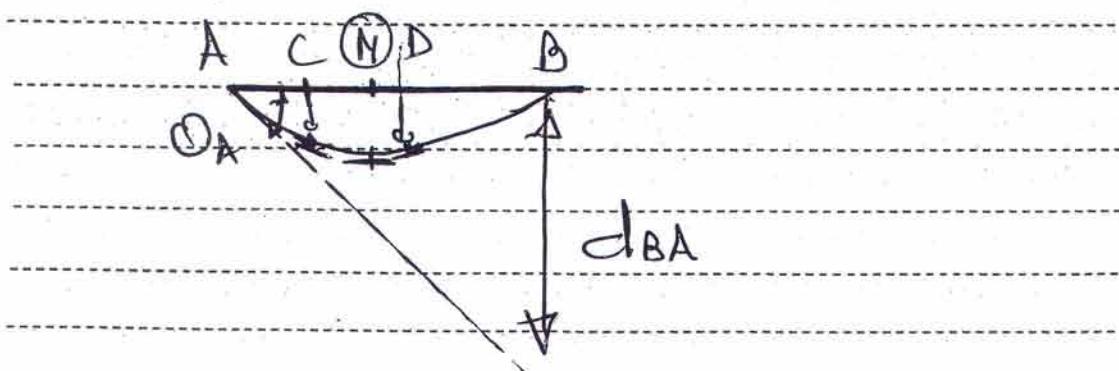
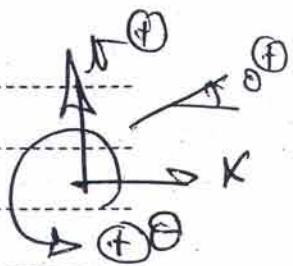
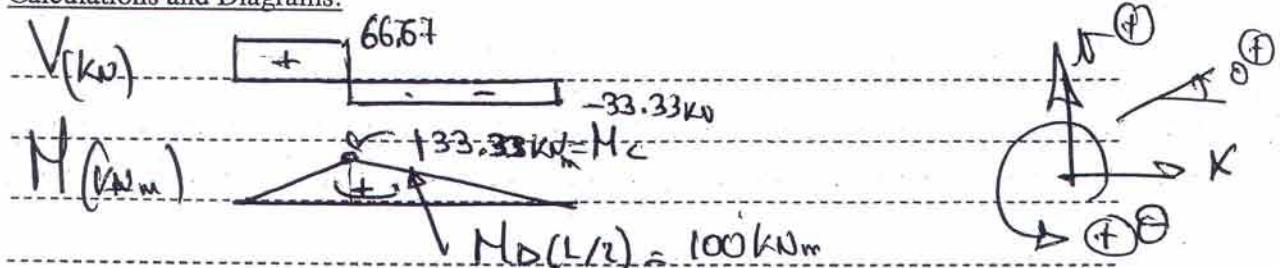
$$EI = 108,000 \text{ kNm}^2$$

Figure II

2. Referring to Figure II, calculate the slopes at A and B, the slope and deflection at C, and the maximum deflection in the beam.

+ Mud point

Calculations and Diagrams:



$$EI d_{BA} = \frac{1}{2} \left(\frac{1}{2} \right) \left(133.33 \right) \left(2 \right) \times \left(4.66 \right) + \left(\frac{1}{2} \right) \left(133.33 \right) \left(1 \right) \times \left(\frac{2}{3} \times 4 \right)$$

$$= 1333.26$$

$d_{BA} = 0.01235 \text{ rad} \Rightarrow \text{Point B above Axis A?} \checkmark$

$$|\theta_A| = 0.01235 / 6 = 0.00206 \text{ rad} \Rightarrow \boxed{\theta_A = -0.00206}$$

$$\theta_B - \theta_A = \frac{1}{6} \rightarrow \theta_B - (-0.00206) = \frac{1}{6} (133.33) (6)$$

$$\boxed{\theta_B = (+0.00165) \text{ rad}} \quad \text{g(?) } \checkmark$$

Calculations and Diagrams (cont'd):

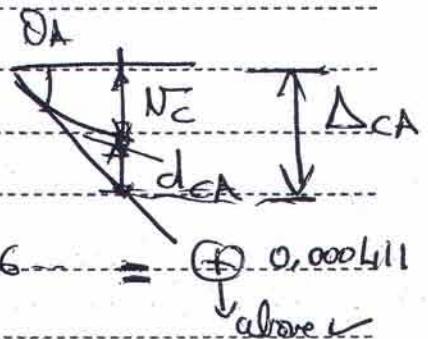
• $\theta_c - \theta_A = \frac{1}{2} \rightarrow \theta_c = \left(\frac{1}{2}\right) \left(\frac{133.33}{EI}\right) \times 2 = 0.00206$
 $\boxed{\theta_c = +0.000893} \quad \boxed{?} \quad \checkmark$

• Next point θ_D :

$$\theta_B - \theta_D = \frac{100}{3} \rightarrow \theta_D = 0.00165 - \left(\frac{1}{2}\right) \left(\frac{100}{EI}\right) (3)$$

$$\boxed{\theta_D = -0.00026} \quad \boxed{?} \quad \checkmark$$

• $|N_C| = |\Delta_{CA}| - |d_{CA}|$



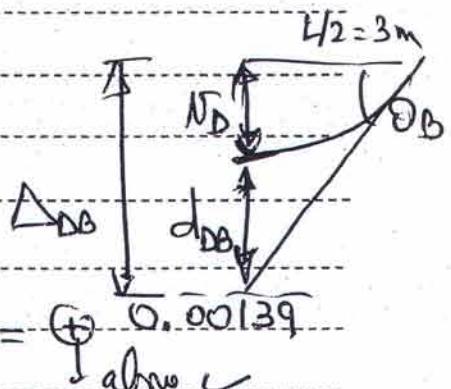
$$|\Delta_{CA}| = \theta_A \times 2 = 0.00412$$

$$|d_{CA}| = \frac{1}{2} = \left(\frac{1}{2}\right) \left(\frac{133.33}{EI}\right) (2) = 0.666 = +0.000411$$

above ✓

$$\boxed{|N_C| = 0.0033 \quad (\downarrow)}$$

• $|N_D| = |\Delta_{DB}| - |d_{DB}|$



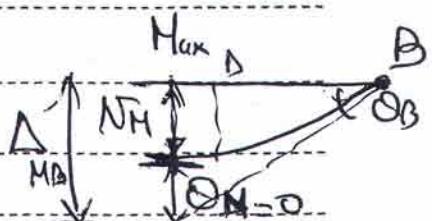
$$|\Delta_{DB}| = \theta_B \times 3 = 0.00495$$

$$|d_{DB}| = \frac{100}{3} = \left(\frac{1}{2}\right) \left(\frac{100}{EI}\right) (3) \left(\frac{1}{3} \times 3\right) = +0.00139$$

above ✓

$$\boxed{|N_D| = 0.00356 \quad (\downarrow)}$$

• ~~$N_{H,N}$~~ $\Rightarrow \theta_N = 0$



$$+0.00165$$

$$\theta_B - \theta_N = \left(\frac{1}{2}\right) \left(\frac{33.33 X_H}{EI}\right) (X_H)$$

$$\Rightarrow X_H = 3.27 \text{ m} \quad (\approx 3.27 \text{ m})$$

$$H = \frac{133.33}{4} X_H \quad X_H = ?$$

$$= 33.33 X_H$$

$$= 109 \text{ kNm}$$

$$|N_N| = |\Delta_{MB}| - |d_{MB}| = 0.00375 \text{ m}$$

$$\downarrow$$

$$0.00165 \times 3.27 = 0.005395$$

$$\rightarrow \left(\frac{1}{2}\right) \left(\frac{109}{EI}\right) (3.27) \times \left(\frac{1}{3} \times 3.27\right) = 0.001798$$

$$\boxed{|N|_{max} = 0.00375 \text{ m}}$$