DEFLECTION OF BEAMS (2)

CIVE311 – STRUCTURES I

(Wednesday, April 13, 2005)

Exercise I

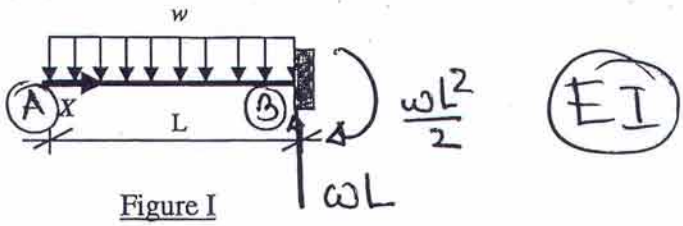
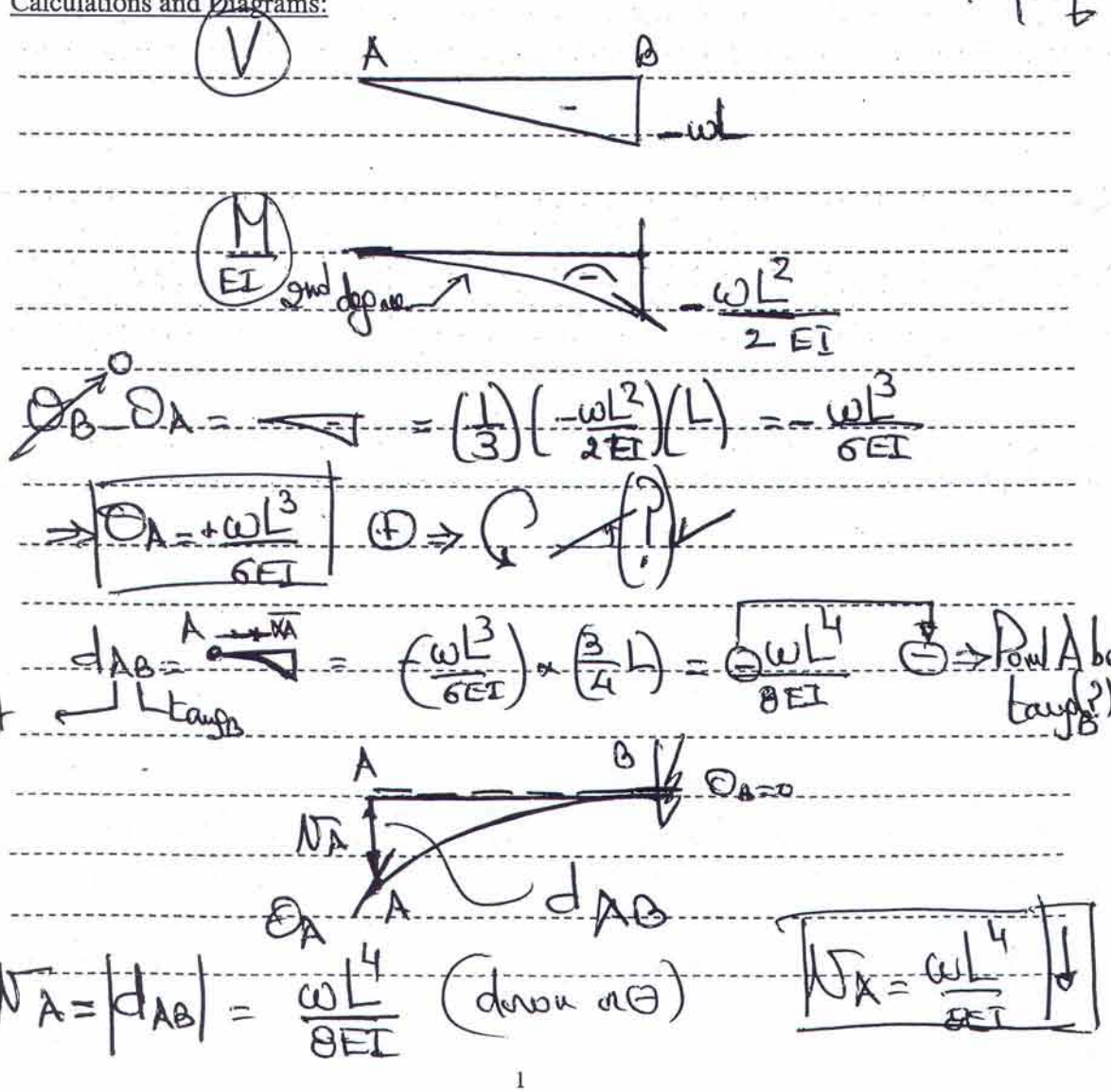


Figure I

1. Referring to Figure I, calculate the slope and deflection at the tip, using the Moment-Area theorems.

Calculations and Diagrams:



Exercise II

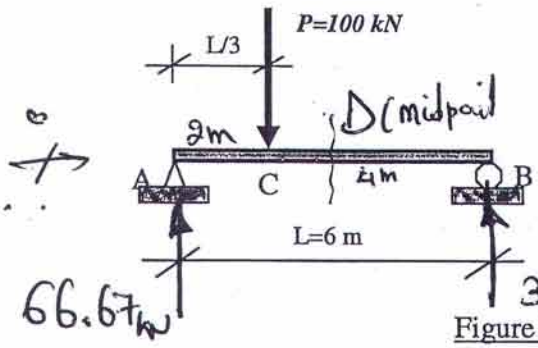


Figure II

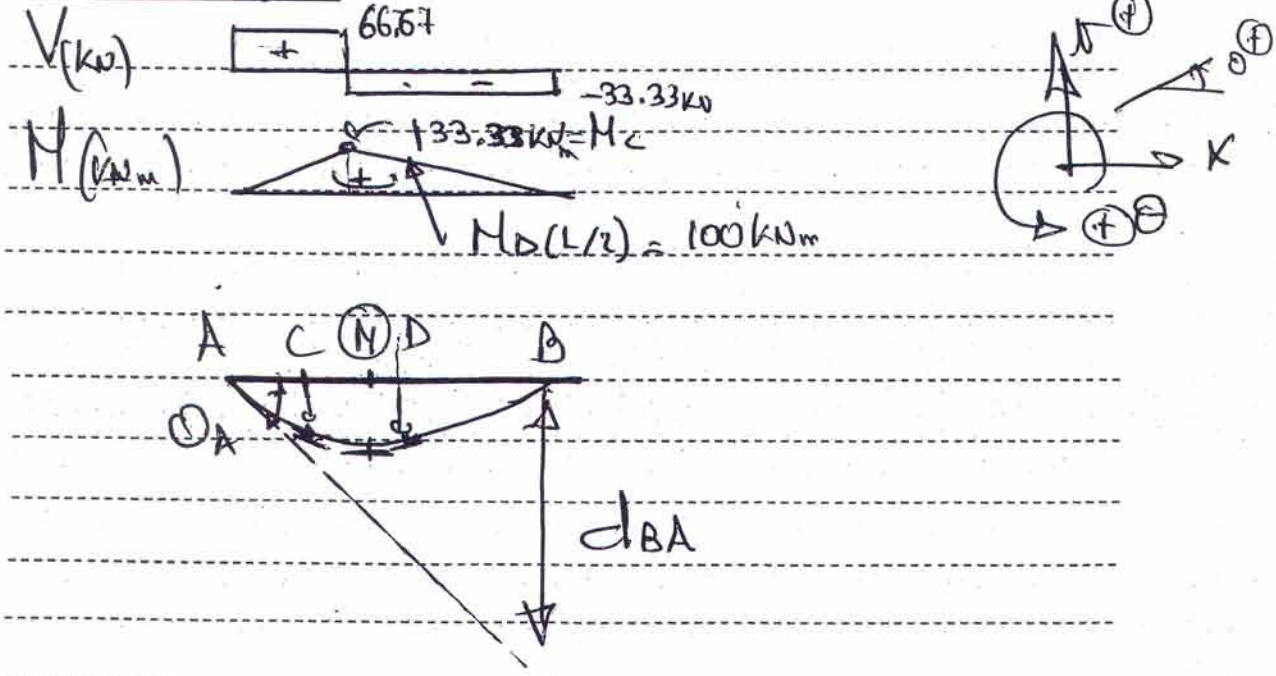
$I = \frac{0.3 \times 0.6^3}{12} = 0.0054\ m^4$

Cross-Section: 30x60 cm
Material: E= 20×10^6 kPa

$EI = 108{,}000\ kNm^2$

2. Referring to Figure II, calculate the slopes at A and B, the slope and deflection at C, and the maximum deflection in the beam. + Mid point

Calculations and Diagrams:



$EI\ d_{BA} = \text{(area of M/EI diagram between A and B)} = \left(\frac{1}{2}\right)\left(\frac{133.33}{2}\right)(2)\times\left(4.66\right) + \left(\frac{1}{2}\right)\left(133.33\right)(4)\times\left(\frac{2}{3}\times 4\right)$

$= 1333.26$

$d_{BA} = 0.01235$ (+) ⇒ Point B above tang A (?)

$|\theta_A| = 0.01235/6 = 0.00206$ rad ⇒ (−) $\theta_A = -0.00206$

$\theta_B - \theta_A = \text{(area of M/EI diagram between A and B)} \Rightarrow \theta_B - (-0.00206) = \left(\frac{1}{2}\right)\left(\frac{133.33}{EI}\right)(6)$

$\theta_B = (+)0.00165$ ↻

↳ ∠ ?(?)

Calculations and Diagrams (cont'd):

- $\theta_C - \theta_A = \triangle \Rightarrow \theta_C = \left(\frac{1}{2}\right)\left(\frac{133.33}{EI}\right)\times 2 - 0.00206$

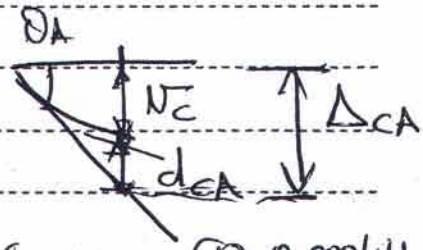
  $\theta_C = +0.000823$ ✓ (?) ✓

- Next find θ_D:

  $\theta_B - \theta_D = {}^{100}\triangle \Rightarrow \theta_D = 0.00165 - \left(\frac{1}{2}\right)\left(\frac{100}{EI}\right)\times(3)$

  $\theta_D = -0.00026$ ✓ (?) ✓

- $|N_C| = |\Delta_{CA}| - |d_{CA}|$

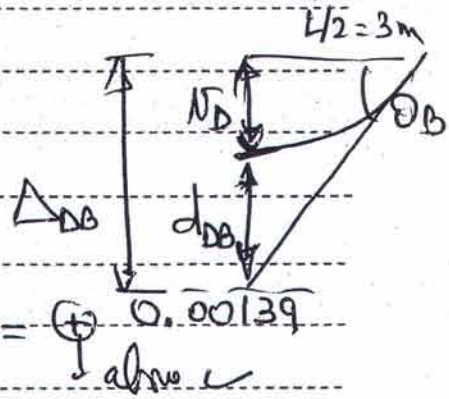


  $|\Delta_{CA}| = \theta_A \times 2 = 0.00412$

  $|d_{CA}| = {}^{C}\triangle = \left(\frac{1}{2}\right)\left(\frac{133.33}{EI}\right)(2)(0.666) = +0.000411$ above ✓

  $\Rightarrow |N_C| = 0.0033\ m\ (\downarrow)$

- $|N_D| = |\Delta_{DB}| - |d_{DB}|$

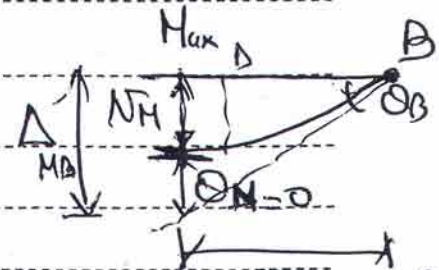


  $|\Delta_{DB}| = \theta_B \times 3 = 0.00495$

  $|d_{DB}| = {}^{100}\triangle_{3} = \left(\frac{1}{2}\right)\left(\frac{100}{EI}\right)(3)\left(\frac{1}{3}\times 3\right) = +0.00139$ above ✓

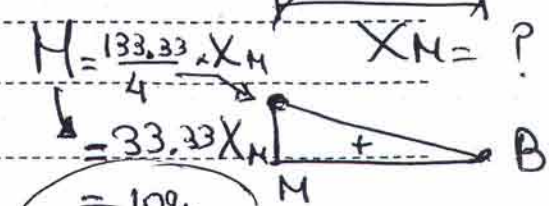
  $|N_D| = 0.00356\ m\ (\downarrow)$

- $M_{@N} \Rightarrow \theta_M = 0$



  $+0.00165 = \theta_B - \theta_M = \left(\frac{1}{2}\right)\left(\frac{33.33 X_M}{EI}\right)(X_M)$

  $\Rightarrow X_M = 3.27\ m$ (>3 ✓, <4 ✓)



  $|N_M| = |\Delta_{MB}| - |d_{MB}|$

  $|\Delta_{MB}| = 0.00165 \times 3.27 = 0.005395$

  $|d_{MB}| = \left(\frac{1}{2}\right)\left(\frac{109}{EI}\right)\left(3.27\right)\left(\frac{1}{3}\times 3.27\right) = 0.001798$

  $= 0.00375\ m$

  $X_M = 3.27\ m$

  $N_{max} = 0.00375\ m$