QUIZ 2

Spring 2001-2002

(Thursday, May 23, 2002)

CVEV 051 – STRUCTURES I

CLOSED BOOK, 1 ½ HOURS

Name: — — – — — – **ID#:** | | | | | | | | | | | | | | |

NOTES

- 2 PROBLEMS – 3 SHORT QUESTIONS – 11 PAGES.
- ALL YOUR ANSWERS SHOULD BE PROVIDED ON THE QUESTION SHEETS.
- **ONE EXTRA SHEET IS PROVIDED AT THE END.**
- **ASK FOR ADDITIONAL SHEETS IF YOU NEED MORE SPACE.**
- SOME ANSWERS MAY REQUIRE MUCH LESS THAN THE SPACE PROVIDED.
- ***DO NOT*** USE THE BACK OF THE SHEETS FOR ANSWERS.
- DRAFT BOOKLET WILL BE PROVIDED; BUT DO NOT USE FOR ANSWERS.
- BOTH QUESTION SHEETS AND DRAFT BOOKLET SHOULD BE RETURNED.

YOUR COMMENT(S)

- -

- -

DO NOT WRITE IN THE SPACE BELOW

MY COMMENT(S)

- -

- -

YOUR GRADE

Problem I: 60 /60
Problem II: 25 /25
Questions: 15 /15
Other: +2 → Trivia

TOTAL: 100 /100
+2

Problem I: *(60 points)*

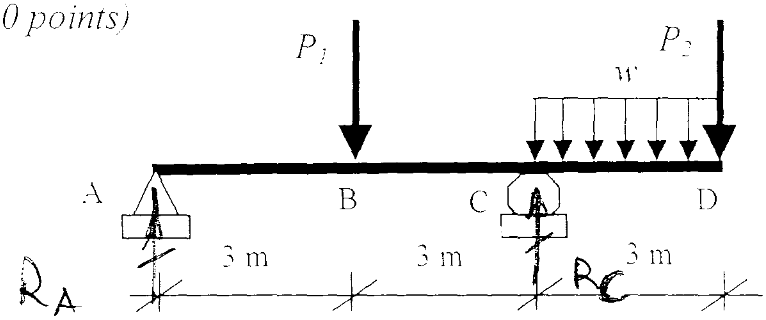


Figure I

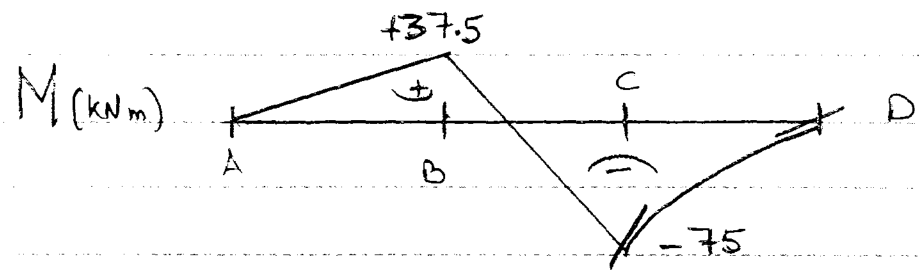
Referring to Figure I, let ***EI*=50,000 kN.m²** throughout the beam. Neglect the own weight of the beam.

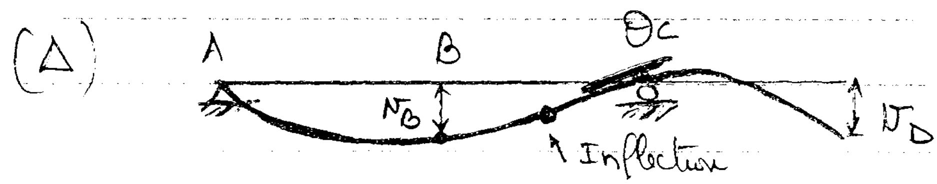
USE THE MOMENT-AREA METHOD THROUGHOUT THIS PROBLEM.

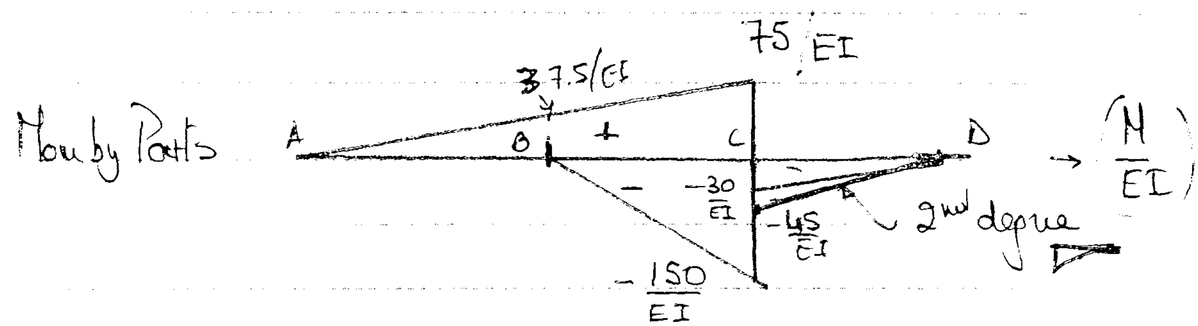
1. Let ***w*=10 kN/m**, ***P*₁=50 kN**, and ***P*₂=10 kN**
   Compute the slope at C (ϑ_C), the deflections at B and D (v_B and v_D), and the maximum downward deflection between A and C (v_{max}). *(45 points)*
   NOTE: You can calculate slopes and deflection in whichever order you find suitable.

   Calculations and Diagrams: $R_A = 12.5$ kN (↑) $R_C = 77.5$ kN (↑)

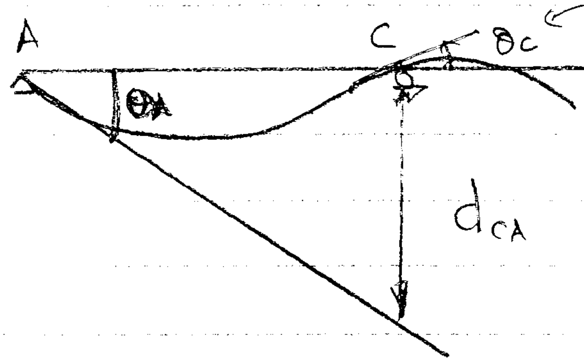
   Moment Diagram: Same as Quiz #1 (S 2001-2002)







Calculations and/or Diagrams (cont'd):



$\bigcirc$ Correct below

$\rightarrow$ C.~~X~~.W

C.W

$$EI\, d_{CA} = \left(\frac{1}{2}\right)(75\times 6)\times\left(\frac{1}{3}6\right) + \left(\frac{1}{2}\right)(-150\times 3)\left(\frac{1}{3}\times 3\right) = \overset{\oplus}{225}$$

Ok point above

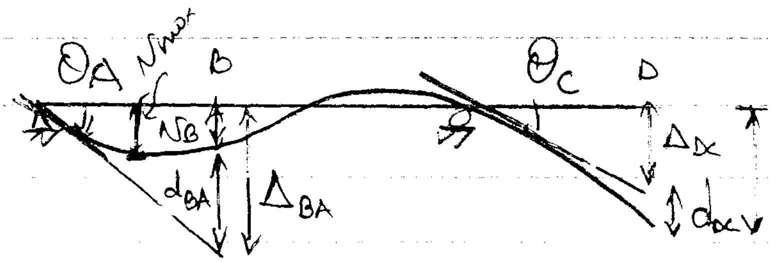
$d_{CA} = 4.5\times 10^{-3}$ m

$|\theta_A| = \frac{|d_{CA}|}{6} = 0.75\times 10^{-3}$ rad $\qquad \theta_A = \underset{\text{clockwise}}{(-)}\ 0.75\times 10^{-3}$ rad

$$EI(\theta_C - \theta_A) = \underbrace{\left(\frac{1}{2}\right)(75\times 6) + \left(\frac{1}{2}\right)(-150\times 3)}_{\phi} \Rightarrow \boxed{\theta_C = \theta_A = -0.75\times 10^{-3}\text{ rad}}$$

$\Rightarrow$ clockwise

Correct for θ_C



- $|\Delta_{BA}| = |\theta_A| \times 3 = 2.25\times 10^{-3}$ m

$EI\, d_{BA} = \left(\frac{1}{2}\right)(37.5\times 3)\left(\frac{1}{3}\times 3\right) = 56.25 \Rightarrow d_{BA} \overset{\oplus}{=} 1.125\times 10^{-3}$ m Ok point above

$|v_B| = |\Delta_{BA}| - |d_{BA}| = 1.125\times 10^{-3}$ m $\boxed{(\downarrow) = 1.125 \text{ mm} = v_B}$

- $|\Delta_{DC}| = |\theta_C| \times 3 = 2.25\times 10^{-3}$ m

$EI\, d_{DC} = \left(\frac{1}{2}\right)(-30\times 3)\left(\frac{2}{3}\times 3\right) + \left(\frac{1}{3}\right)(-45\times 3)\left(\frac{3}{4}\times 3\right) = -191.25 \Rightarrow d_{DC} = 3.825\text{ m}\times 10^{-3}$ $\ominus$ below

$|v_D| = |\Delta_{DC}| + |d_{DC}| = 6.075\times 10^{-3}$ m $\boxed{\downarrow = 6.075 \text{ mm} = v_D}$

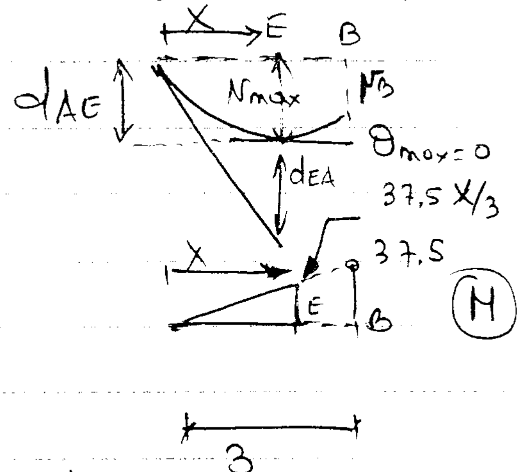
Calculations and/or Diagrams (cont'd):

$v_{max}(A \to C)$ Assume between $A \to B$ @ E

$\theta_E = 0$

$\cancel{\theta_E}^{0} - \theta_A = \left(\frac{1}{2}\right)\left(\frac{37.5}{EI}\frac{X}{3} \times X\right)$

$\Rightarrow X = 2.45\ m$ $(< 3\ m\ ✓)$



Can do $|v_{max_E}| = |\Delta_{EA}| - |d_{EA}|$

OR $|d_{AE}| = |v_{max_E}|$

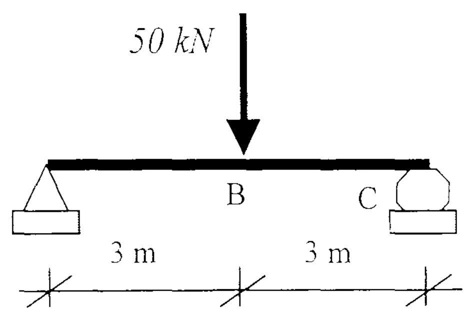
$EI\, d_{AE} = \frac{1}{2}\left(\frac{37.5}{3} \times 2.45 \times 2.45\right)\left(\times \frac{2}{3} \times 2.45\right) \overset{+}{=} 61.28$ above

$\Rightarrow d_{AE} = 0.001226\,m \Rightarrow$ $\boxed{\Downarrow v_{max} = v_E = 1.23\ mm}$

2. Compare v_B and v_{max} from Part 1 and briefly comment. (*5 points*)
Compare v_B and v_{max} from Part 1 to the mid-span deflection (v_B) of the beam (same ***EI*=50,000 kN.m^2**) shown below and briefly comment. (*10 points*)

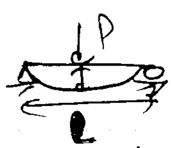
NOTES:

- In answering the questions above, you should use the values calculated as well as your engineering judgement; i.e. if it happens that the values obtained are not logical or do not make sense when compared with each other, this may be a hint that you may have done something wrong somewhere (?). If you do not have time to review and correct, make the proper judgement, answer accordingly, and note this in your comment.
- In calculating v_B below, you should take advantage of symmetry and of the fact that the slope is zero at B. This will simplify your calculation of v_B to a single line only (or, if you know the formula, you may use is directly).



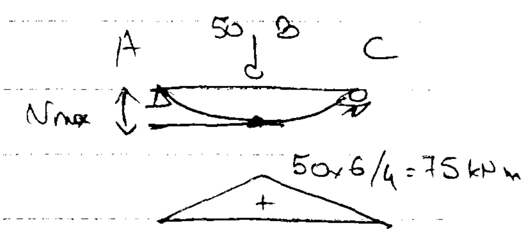
Calculations and Diagrams:

- (↓) v_B = 1.125 mm
  (↓) v_{max} = 1.226 mm } v_{max} slightly higher than v_B

- 

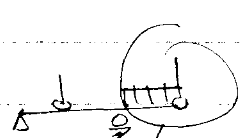
$v_{max} = \frac{PL^3}{48EI} = 4.5\times10^{-3}$ m

or $|t_{A/B}| = |v_{max}|_B = \left(\frac{1}{2}\right)(75\times3)\left(\frac{2}{3}\times3\right)/EI$



$v_B = 4.5\times10^{-3}$ m = 4.5 mm

v_B > (v_{max} / v_B)



expected since right part reduces deflection @ B

Problem II: *(25 points)*

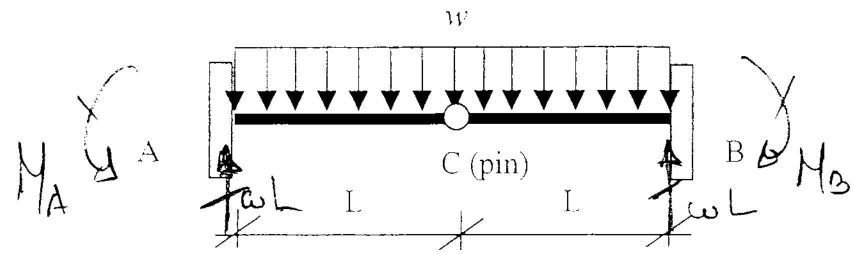


Figure II

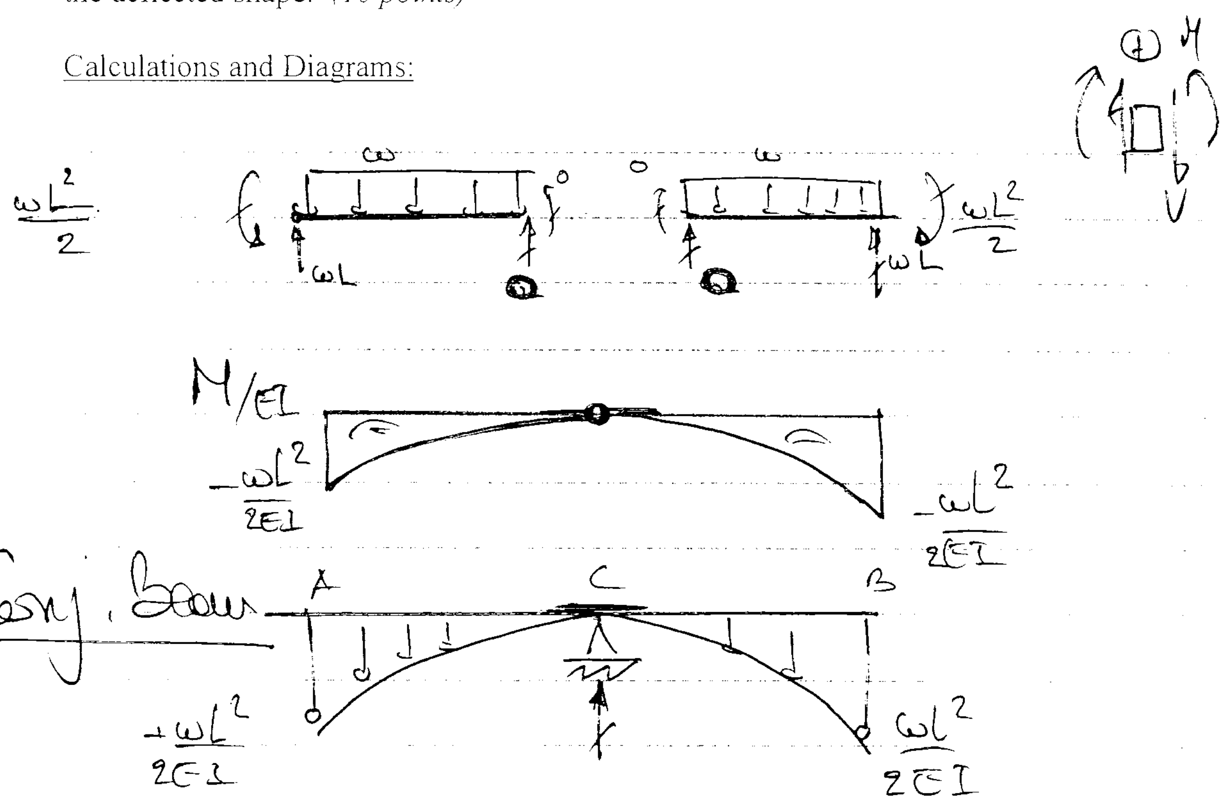
NOTE: This should be a simple/quick problem (15 mn. maximum)

The beam shown in Figure II is SYMMETRICAL and is indeterminate to the first degree (assuming no axial forces and reactions exist in the beam). Let ***EI*** be the same throughout the beam and neglect its own weight. The vertical reaction at A is obviously **wL** (upward).

1. Draw the conjugate beam with the corresponding M/EI (curvature) loading. Check if this conjugate beam is stable and in equilibrium and briefly comment. *(15 points)*
2. Using the conjugate beam method, compute the deflection and slope at the pin and sketch the deflected shape. *(10 points)*

Calculations and Diagrams:

1. Conj. Beam

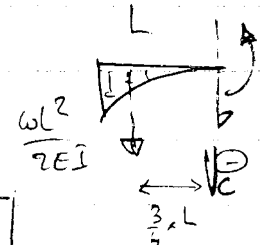


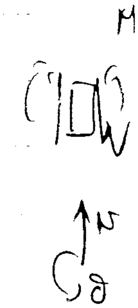
Conditionally stable under current loading
($\Sigma F_y = 0$ ✓ $\Rightarrow R_C =$ ✓ $\Sigma M_c = 0$ ✓)

Calculations and Diagrams (cont'd):

2. v_c + θ_c < $\theta_c^{(-)}$, $\theta_c^{(+)}$ Note $|\theta_c^{(-)}| = |\theta_c^{(+)}|$

Conj: $V_c^{(-)} = (-)\left(\frac{1}{3}\right)\times\left(+\frac{\omega L^2}{2EI}\times L\right) = -\frac{\omega L^3}{6EI}$



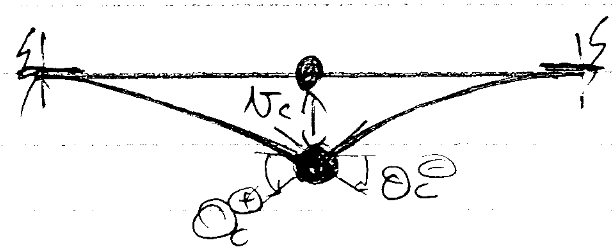


Real: $\boxed{\theta_c^{(-)} = (+)\frac{\omega L^3}{6EI} \Rightarrow \text{c.c.w}}$

$\Rightarrow \boxed{\theta_c^{(+)} = (-)\frac{\omega L^3}{6EI} \Rightarrow \text{c.w.}}$

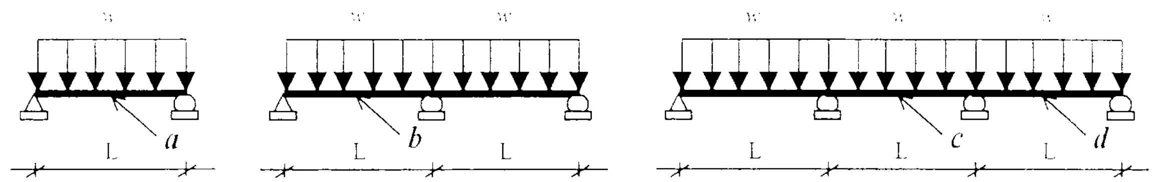
Conj: $M_c = -\frac{\omega L^3}{6EI}\times\frac{3}{4}L = (-)\frac{\omega L^4}{8EI}$ (down ↓)

Real: $\boxed{v_c = -\frac{\omega L^4}{8EI}(\downarrow)}$



NOTE: In the short questions below, *EI* is the same throughout, except when otherwise noted.

Question 1: *(5 points)*

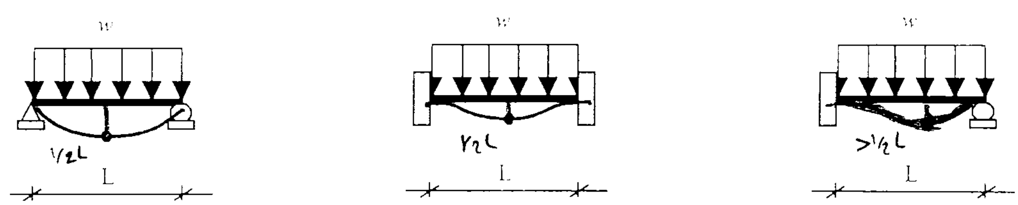


Rank the downward deflections at midspans *a*, *b*, *c*, and *d*, from smallest to largest:

$\delta_c < \delta_b < \delta_d < \delta_a$

Other Version (Largest to Smallest)
a > b > d > c

Question 2: *(5 points)*



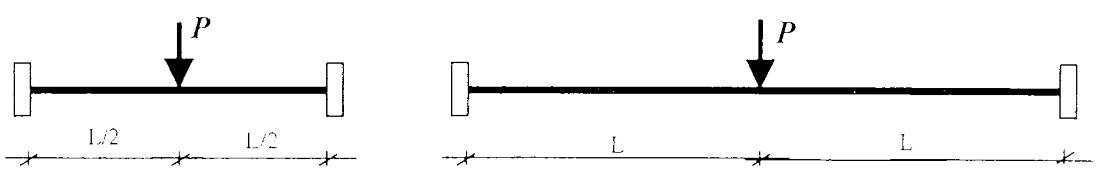
Beam a ***Beam b*** ***Beam c***

Relate the maximum deflections listed below (in random order) to Beams *a*, *b*, and *c*.
Sketch the deflection shape on the beams above, showing exactly/approximately the location of the maximum deflections.

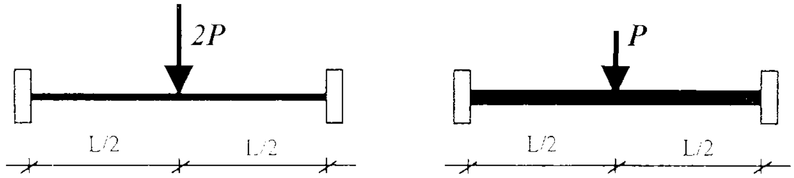
- $(1/384)wL^4/EI$ Beam: b
- $(5/384)wL^4/EI$ Beam: a
- $(1/185)wL^4/EI$ Beam: c

Other Version
c 1/185
a 5/384
b 1/384

Question 3: *(5 points)*



Beam a ***Beam b***



Beam c ***Beam d (2EI)***

Complete/deduce the maximum deflections under the loads for Beams *b*, *c*, and *d*.

- Beam *a*: Deflection = $PL^3/192EI$
- Beam *b*: Deflection = $PL^3/24\,EI$
- Beam *c*: Deflection = $PL^3/96EI$
- Beam *d*: Deflection = $PL^3/384\,EI$