Fall 2017 Dr. Oweis

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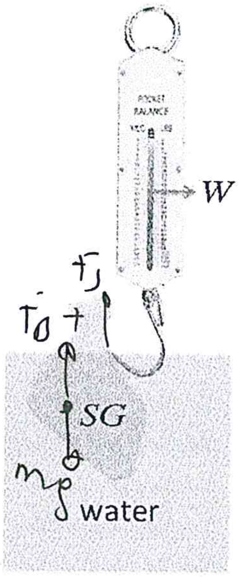
Department of Mechanical Engineering
MECH 314 – Fluid Mechanics
Quiz 1
4 October 2017

1) 5% (Final answer) $\rho_w = 1000$

You use a spring–hook weighing scale as shown and measure your own weight when you are fully submerged in a freshwater pool to be 3 kg. Estimating your body volume to be roughly 0.1 m³, what is your average bodily density?



$F_s + F_B = W$

$3\,kg \times \cancel{9.8} + 0.1 \times 1000 \times \cancel{9.8} = 0.1 \times \rho_B \times \cancel{9.8}$

$\rho_B = \frac{103}{0.1} = 1030$

ρ_{body} = 1030 kg/m³

CIRCLE ANY PART ON THE FORMULA SHEET YOU USE IN YOUR SOLUTION

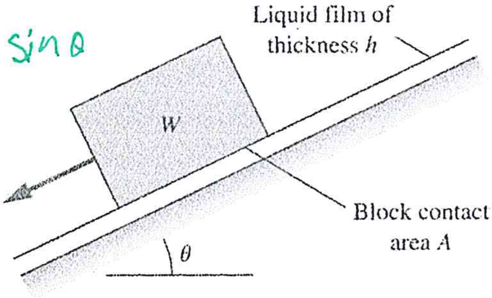
2) 5% (Final Answer & units)

The block of weight **W** is a cube of side length 15 cm and slides down an inclined surface ($\theta = 30°$) at constant speed. There is a liquid film of thickness (**h** = 0.05 mm) and dynamic viscosity (μ = 0.25 Pa.sec) between the surface and the sliding block. The shear stress within the liquid film is constant everywhere and equal to $\tau = \mu \frac{V}{h}$. What is the sliding velocity (**V**) in mm/sec if the weight (**W**=7 N).

$$\tau A = \frac{0.25 \times V}{0.05 \times 10^{-3}} \times (0.15)^2 = mg \sin\theta$$

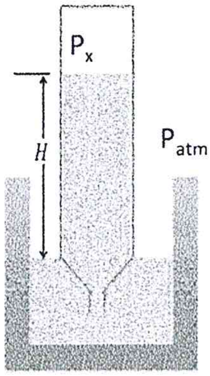
$$= 7 \sin 30$$

$V =$



V = 31.1 (mm/sec)

3) 10% (Final answer & units)
In our class demo on atmospheric pressure we dipped an inverted a water bottle into a cup as shown in the figure. If the water height H = 28 cm, its density is 1000 kg/m³, then what is the pressure P_x at the top of the bottle?



Fill in the blanks below for the pressure P_x as absolute, gage, and vacuum.

In your answers below make sure to write the **correct units**. Do **not** substitute a numerical value for P_{atm}, and just leave it as P_{atm}.

P_x = $P_{atm} - \rho g H$ (absolute) $P_{atm} - 2744$ Pa

P_x = -2744 Pa (gage)

P_x = $+2744$ Pa (vacuum)

Nor vertical rule

4) ~~5)~~ 50 ~~40%~~

Gate **AB** is cylindrical in shape of diameter ($D = 3.8\ m$) and weighs 200 kg. It is used to hold the water in the reservoir to the left. It is pinned at point (**A**) and is held in place by the external horizontal force (**F**) as shown. The gate is free to move at point (**B**) with no friction. If the height of the water in the reservoir ($H = 10.4\ m$), what is the magnitude of the force (**F**) required to keep the gate in place. It is useful to know that the width of the gate into the page ($W = 1.75\ m$). Neglect the effect of the atmospheric pressure in your computation because it acts on both side of the gate. You are free to solve this problem using the method of your choice.

$\Sigma M_A = 0$ Static Equilib.

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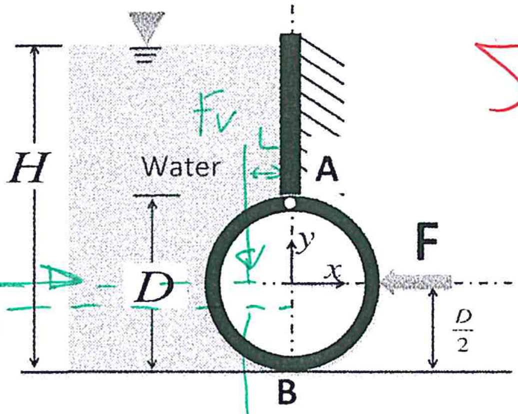


$F_H\left(\frac{D}{2} + |y_{cp}|\right) + F_v L$

$= F \times \frac{D}{2}$

$L = \frac{4R}{3\pi} = \frac{2D}{3\pi} = \frac{2\times 3.8}{3\times\pi} = 0.806\ m$

$\frac{4R}{3\pi} = L$

$F_H = \rho g h_G A_{proj} = 9.8\times 10^3 \times (10.4 - \frac{3.8}{2})(3.8\times 1.75)$

$= 553.945\ kN$

$y_{cp} = -\frac{\rho g \sin\theta\ I_{xx}}{F}$; $I_{xx} = \frac{BL^3}{12} = \frac{1.75\times 3.8^3}{12}$

$= 8\ m^4$

$|y_{cp}| = \frac{9.8\times 10^3 \times 1 \times 8}{553.945\times 10^3} = 0.142\ m$

$F_v = \rho_w g V = 9.8\times 10^3 \times \left(\frac{\pi D^2}{4} \times W\right)\times \frac{1}{2} = 9.8\times 10^3 \left(\frac{\pi}{4}(3.8)^2 \times 1.\frac{}{2}\right)$

$= 97.25\ kN$ ← points upwards.

Sub: $553.95 \times (1.9 + 0.142) \cancel{+} - 97.25 \times 0.806) = F \times 1.9$

$F = \cancel{636.6\ kN}$ 554 kN

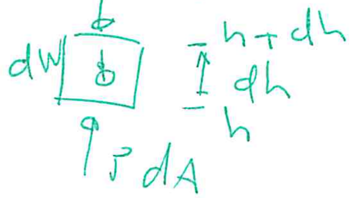
5) ~~6)~~ 30 ~~40%~~

The stratosphere extends from the surface of the earth ($h = 0\ km$) to roughly an altitude of ($h = 14\ km$), where the atmospheric pressure is known to be ~~10~~ 15 kPa, approximately. Let the temperature distribution in degrees Celsius in the stratosphere vary linearly as ($T = 15 - 5h$) with (h) measured in km. Starting with an elemental control volume of atmospheric air, derive a formula for the pressure distribution in the stratosphere region, then compute the pressure at the surface of the earth.

Hint: you may assume that the atmospheric air is an ideal gas with universal gas constant ($R = 287\ J/kg.K$). Also, T(Kelvin) = T(Celsius) + 273.

$T(h) = 15 - 5h \Rightarrow T(h) = 273 + 15 - 5h = \underbrace{288 - 5h}_{\text{Kelvin}}$

CV:



$P \uparrow$; dA : ~~width into page~~ action area

~~P+dP~~

$P\,dA = dW + (P+dP)\,dA$

$P\,\cancel{dA} = \rho g\,\cancel{dA}\,dh + (P+dP)\,\cancel{dA} \Rightarrow \boxed{dP = -\rho g\,dh}$

But $\frac{P}{\rho} = RT \Rightarrow \rho = \frac{P}{RT} \Rightarrow dP = -\frac{P}{RT}\,g\,dh$

but $T = 288 - 5h \Rightarrow dP = -P\,\frac{g}{R} \cdot \frac{dh}{(288-5h)}$

$\Rightarrow \frac{dP}{P} = -\frac{g}{R}\,\frac{dh}{(288-5h)}$ $\left\{ \begin{array}{l} \text{let } k = 288 - 5h \\ dk = -5\,dh \end{array} \right\}$

$\Rightarrow \frac{dP}{P} = +\frac{g}{5R}\,\frac{dk}{k} \overset{\int}{\Rightarrow} \ln P = \frac{g}{5R} \ln k + C$

get (C) : $\left. \begin{array}{l} P(h = 14 km) = 15 kPa \\ k = 288 - 5 \times 14 = 218 \end{array} \right\} \Rightarrow \ln 15 = \frac{9.8}{5 \times 0.287} \ln 218 + C$

$\Rightarrow C = -34.07$

$\Rightarrow \ln P = \frac{g}{5R} \ln(288 - 5h) - 34.07$

$P(h=0) = 99.9\ kPa$