American University of Beirut
Department of Mechanical Engineering
MECH 314 – Fluid Mechanics
Quiz 1
24 February 2016

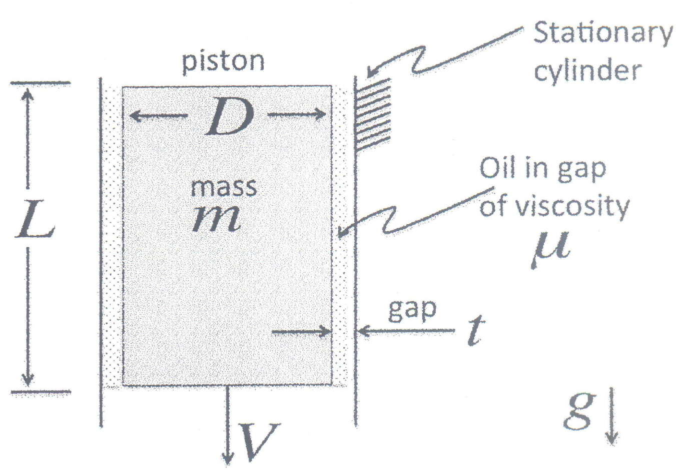
1) 35 %

Consider the piston-cylinder arrangement shown below. The piston's height $L = 13cm$; its <u>diameter</u> $D = 20cm$; and its mass $m = 0.53\ kg$. The gap clearance between the piston and cylinder is uniform, $t = 0.24mm$, and it is filled with oil of viscosity $\mu = 0.61\ pa.s$. The piston slides steadily downward under the effect of its own weigh at a constant velocity V. It is possible to assume that the velocity distribution within the oil film is linear.

(a) Derive a *formula* (symbols only) for the velocity (V) in terms of the other problem parameters. Do not substitute with numbers for this part. (20%)

(b) Substitute with numbers to find the piston's sliding velocity in mm/sec. (5%)

$y \downarrow$



Steady Speed

$\sum F_y = 0$

$mg = \tau A$

$mg = \mu \frac{du}{dy}(\pi D L) = \mu \frac{V}{t}(\pi D L)$

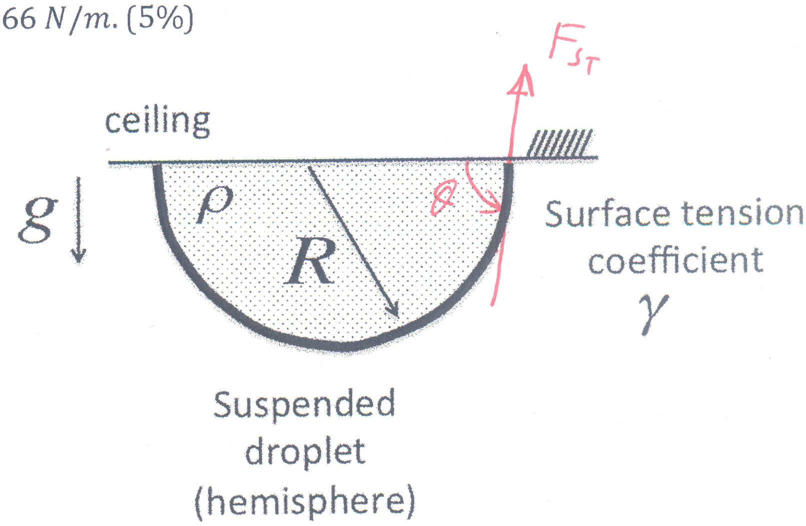
$\Rightarrow V = \frac{mgt}{\pi \mu D L} = \frac{0.53 \times 9.8 \times 0.24 \times 10^{-3}}{\pi (0.61)(0.2)(0.13)} = 25\ \frac{mm}{sec}$

2) 25%

A symmetric hemispherical water droplet of radius R, and density ρ, is hanging upside down on a ceiling as shown in the figure. The surface tension coefficient is γ. If the droplet exceeds a maximum size it can fall or break up.

(a) Derive a formula for the maximum possible droplet radius R_{max} that can remain suspended on the ceiling. (20%)

(b) Compute the numerical value in mm of R_{max} given that the surface tension coefficient $\gamma = 0.066\ N/m$. (5%)



Static droplet (hanging on ceiling)

Weight = surface tension

$mg = \gamma L \Rightarrow \rho \forall g = \gamma(\pi D)\sin\theta$

$\rho \times \left(\frac{1}{\cancel{2}} \times \frac{\cancel{4}^{\,2}}{3}\cancel{\pi} R^{\cancel{3}\,2}\right) g = \gamma(\cancel{2}\cancel{\pi}\cancel{R})\sin\theta$

Maximum R when $\sin\theta = 1$

$\Rightarrow R_{max} = \left(\frac{3\gamma}{\rho g}\right)^{\frac{1}{2}} = \sqrt{\frac{3\times0.066}{1000\times9.8}} = 4.5\ mm$

$\rightarrow M = \frac{\rho g W h^3}{6} + \rho g W\left(\frac{hL^2}{2} - 0.9L^4\right) = 18{,}947$ kN.m

3) 40% (+10% extra)

Dam **AB** is used to hold water in a lake of depth $h = 22.5\ m$, as shown in the figure. Consider the dam to have a uniform width into the page $W = 1\ m$. Notice the horizontal length from **A** to **B** is $L = 2.4\ m$. The dam has a parabolic shape of the form $y = 3.6\ x^2$, where x, y are both measured in meters. Neglecting the effect of the atmospheric pressure, you are asked to compute the water pressure force on the dam using the <u>*mathematically beautiful way*</u> that we discussed in class where you start by evaluating the water pressure force on the elemental area dA shown in the figure. (Do NOT use the quick formulas, as you won't gain credit.)

(a) Derive a <u>formula</u> for the horizontal component of the water pressure force F_x on dam **AB** (. (20%)

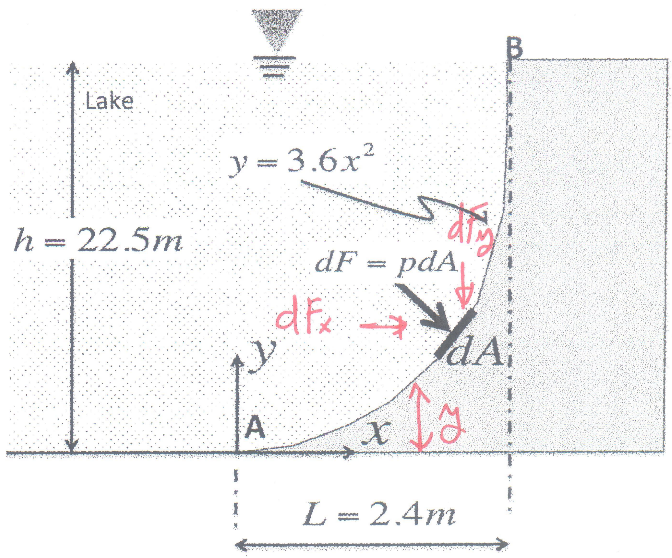
(b) Derive a <u>formula</u> for the vertical component of the water pressure force F_y on dam **AB** (. (15%)

(c) Substitute for the variables in the formulas in (a & b) above to find the magnitude of F_x, F_y in kN. (5%)

(d) You are asked to evaluate the moment experienced by the dam about point **A** due to the distributed water pressure force. Derive the formula for the moment in terms of the problem parameters. Then evaluate the moment using the provided numerical parameters, in kN.m. (extra 10%; granted only if answer is right)

$F_x = \int_A dF_x = \int_{y_1}^{y_2} p\, w\, dy$

$F_y = \int_A dF_y = \int_{x_1}^{x_2} p\, W\, dx$



$P = \rho g (h - y)$

$\Rightarrow F_x = \int_{y=0}^{y=h} \rho g (h-y)\, w\, dy$

$= \rho g W\left[hy - \frac{y^2}{2}\right]_0^h = \rho g W \frac{h^2}{2} = \frac{1000 \times 9.8 \times 1 \times 22.5}{2} = 2480.6$ kN

$F_y = \int_{x=0}^{x=L} \rho g (h-y) W\, dx = \int_{x=0}^{x=L} \rho g (h - 3.6x^2) W\, dx = \rho g W\left[hx - \frac{3.6x^3}{3}\right]_0^L$

$F_y = \rho g W[hL - 1.2L^3] = 366.6$ kN

$dM = y\, dF_x + x\, dF_y = \rho g W(hy - y^2)dy + \rho g W(hx - 3.6x^3)dx$

$M = \int dM = \rho g W\left[\frac{hy^2}{2} - \frac{y^3}{3}\right]_0^h + \rho g W\left[\frac{hx^2}{2} - \frac{3.6x^4}{4}\right]_0^L$ (cont'd) Top of Page