Problems on Viscous Flow

"Advanced Fluid Mechanics Problems" by Shapiro and Sonin
Problems 6.1, 6.3, 6.7, 6.10, 6.16, 6.20, 6.23

Problem 6.1

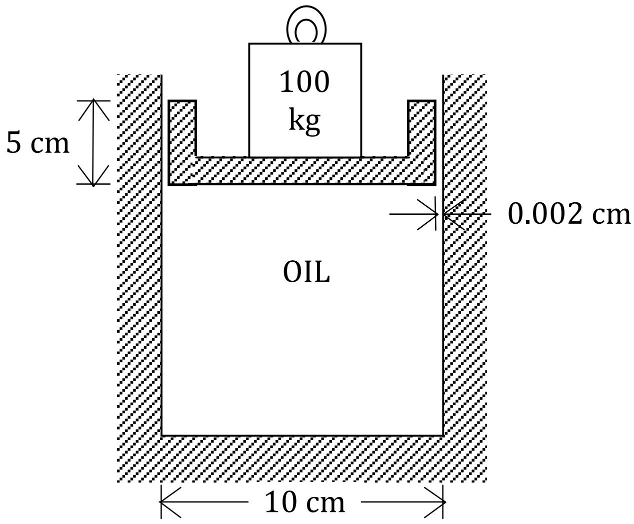

Figure 1: Schematic of Problem 6.1

Oil is confined in a 10 cm diameter cylinder by a piston with a clearance of 0.002 cm. The piston is 5 cm long, and the oil has a viscosity coefficient of 0.05 kg/m s and a density of 920 kg/m^3.

A total mass of 100 kg is applied to the piston. Estimate the leakage rate of oil past the piston, in liters/day. Justify any approximations you use in arriving at your estimate.

Problem 6.3

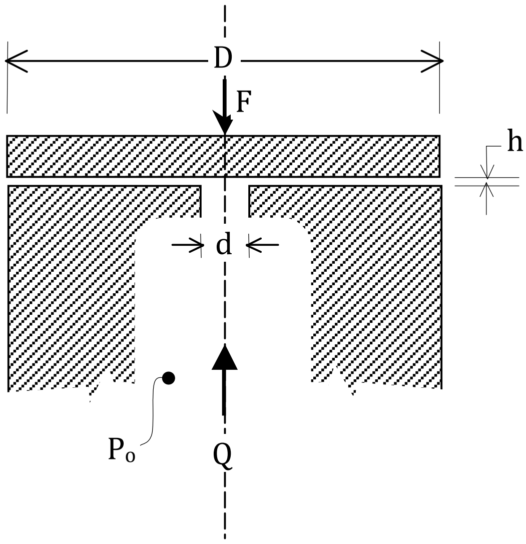

Figure 2: Schematic of Problem 6.3

The sketch shows the type of thrust bearing used in the 200-inch telescope at Palomar. Oil is supplied at high pressure and flows radially outwards between two parallel circular plates. We are interested in obtaining design formulas so that, given the total support load F, the supply gage pressure, p_o, and the desired gap, h, we can determine the necessary disk diameter D, and the required volume flow rate Q, for oil of known viscosity μ and density ρ, the hole diameter being d. In what follows, you make take for granted that $h << d$.

(a) Determine how the ratio of inertial forces to viscous forces acting in the fluid varies with radius r.

(b) Establish an order-of-magnitude criterion which shows in what range the flow may be regarded as inertia-free. Express your criterion in terms of Q and h together with any other necessary quantities.

(c) Assuming that the criterion in (b) for inertia free flow is satisfied, that streamlines are parallel to the plates except very near the injection hole, and that $d/D << 1$, find expressions for F and Q in terms of p_o and the specified quantities.

Problem 6.7

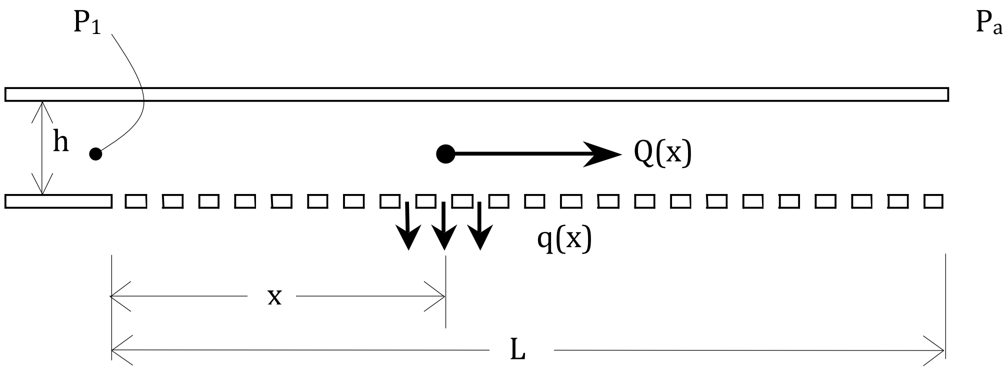

Figure 3: Schematic of Problem 6.7

An incompressible fluid of density ρ and viscosity μ flows through a rectangular channel of length L, width w, and height h (w and $L >> h$). Fluid emerges from the channel outlet to the atmosphere at pressure p_a.

The bottom plate is *slightly porous*, and some fluid leaks out through it. The local leakage volume flow rate, $q(x)$, per unit area of plate, depends on the local pressure difference across the plate according to

$$q(x) = k\,[p(a) - p_a]$$

where k is the plate "permeability" and $p(x)$ is the local pressure inside the channel (k is constant). In all that follows, it is assumed that the flow is steady and inertia-free and that the leakage rate is "small" in the sense that the leakage velocity is small compared to the local mean horizontal velocity.

(a) Obtain a differential equation that relates the gradient in the horizontal volume flow rate dQ/dx at any station x to the local pressure.

(b) Obtain a differential equation that predicts the pressure distribution inside the channel as a function of position x, the channel geometry, fluid properties, and atmospheric pressure p_a. State the boundary conditions necessary to solve this differential equation.

(c) Consider the limiting case of small total fluid leakage, where the horizontal fluid volume flow rate at the channel end differs only slightly from that at the inlet. Estimate the point $x^\star$ in the channel where the gage pressure is approximately $(p_1 - p_a)/2$.

(d) Now consider the opposite limiting case where the leakage rate is sufficiently large, or the channel length so large, the the horizontal volume flow rate at the channel end is very small compared with that at the inlet. Use an order of magnitude analysis of your differential equation in part (b) to estimate the point $x^\star$ in the channel where the gage pressure is approximately $(p_1 - p_a)/2$.

(e) Solve the differential equation in (b) and show that your solution reduces to your answers in (c) and (d) in the appropriate limits.

Problem 6.10

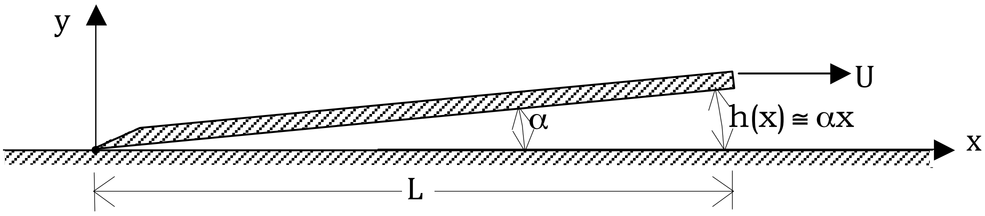

Figure 4: Schematic of Problem 6.10

The sketch shows a "doctor blade" which, in various industrial processes, is used for scraping a viscous fluid off a roller. When the angle α is small, the roller surface may be regarded as plane in the region near the contact point.

(a) From order-of-magnitude considerations, establish the criteria for treating the flow as locally-Couette, that is, an inertia-free.

(b) Assuming the flow to be locally-Couette, derive a formula for $(p(x) - p_L)$, where $p(x)$ is the pressure at the location x and p_L is the pressure at $x = L$.

Problem 6.16

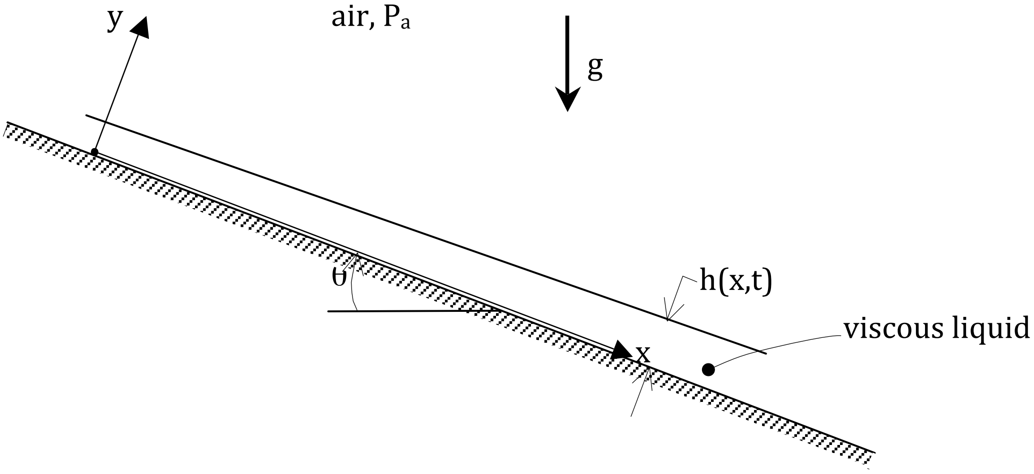

Figure 5: Schematic of Problem 6.16

A rigid plane surface is inclined at an angle θ relative to the horizontal and wetted by a thin layer of highly viscous liquid which begins to flow down the incline.

(a) Show that if the flow is two-dimensional and in the inertia-free limit, and if the angle of the inclination is not too small, the local thickness $h(x,t)$ of the liquid layer obeys the equation

$$\frac{\partial h}{\partial t} + c\frac{\partial h}{\partial x} = 0$$

where

$$c = \frac{\rho g h^2}{\mu} \sin\theta$$

(b) Demonstrate that the result of (a) implies that in the region where h decreases in the flow direction, the angle of the free surface relative to the inclined plane will steepen as the fluid flows down the incline, while in a region where h increases in the flow direction, the reverse is true. Does this explain something about what happens to slow-drying paint when it is applied to an inclined surface?

(c) Considering the result of (b) above, do you think that the steady-state solutions of this problem would ever apply in practice? Discuss.

Problem 6.20

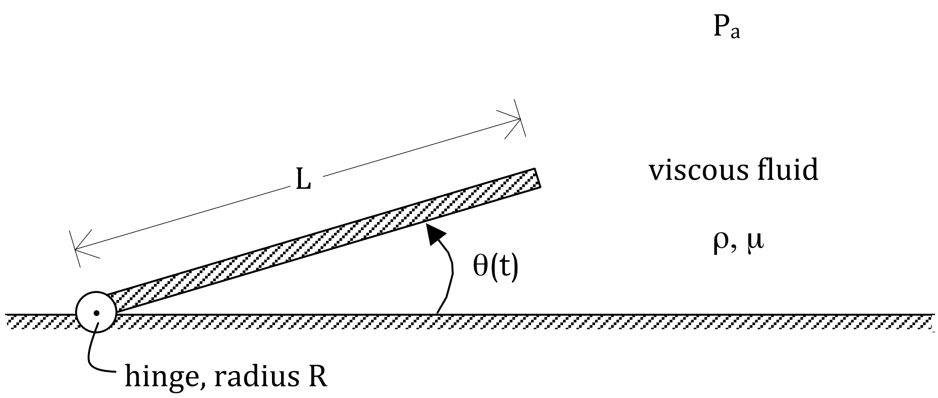

Figure 6: Schematic of Problem 6.20

A flat plate of breadth L and length much greater than its breadth is attached to a plane floor by a hinge. The hinge has a radius R as shown. The plate is initially at a small angle θ_o relative to the floor, and the region between it and the floor is filled with a viscous fluid. Starting at $t = 0$, the plate is forced toward the floor at a constant angular rate $-d\theta/dt = \omega$.

(a) Obtain an expression for the pressure distribution $p(x,t)$ under the plate in the limit of highly viscous (inertia-free) flow. The given quantities are $L, R, \theta_o, \omega, \rho, \mu$ and the atmospheric pressure p_a outside the plate.

(b) Derive an expression for the vertically downward force $F(t)$ which must be applied to the right-hand tip of the plate to make it close down at the specified constant angular rate.

(c) Write down the criteria which must be satisfied for your solutions to apply.

Problem 6.23

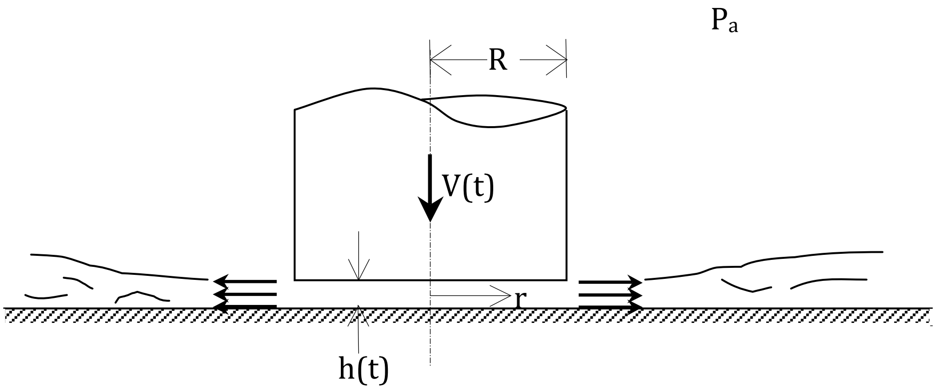

Figure 7: Schematic of Problem 6.23

A circular disc of radius R, its axis vertical, is pressed down with a speed V against a flat, horizontal plate, displacing a liquid of viscosity μ and density ρ which fills the narrow gap between them.

Assuming that the liquid flow is in the highly viscous (inertia-free) limit,

(a) derive an expression for the pressure distribution under the disk, and

(b) derive an expression for the total upward force (lift) exerted by the liquid on the disc. Neglect gravitational (buoyancy) effects.

Express your results in terms of the given quantities and the instantaneous gap height $h(t)$ between the plate and the disc.