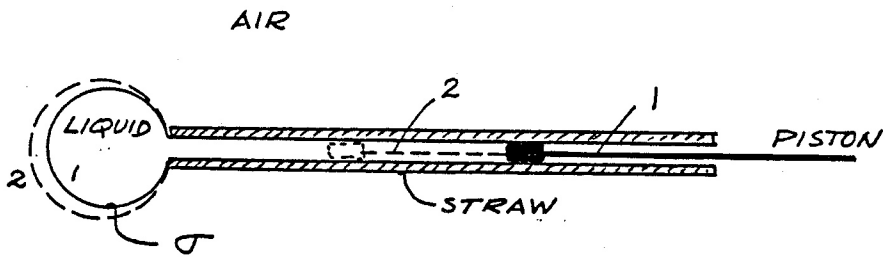
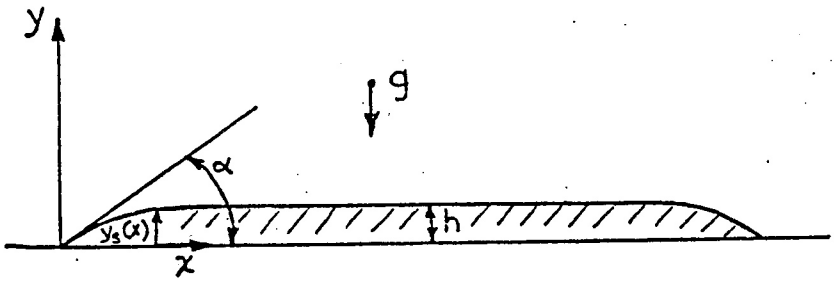
2. SURFACE TENSION

Problem 2.1



A liquid drop is held at the end of a straw, as sketched. its volume controlled by the position of the piston. Gravity is negligible. Show that if the drop's volume is changed by a (slow) displacement of the piston, the net work done on the system comprised of the liquid and its bounding surface is equal to the product of the surface tension coefficient and the incremental change in the system's surface area. Explain why it follows that the surface tension coefficient can be interpreted as an internal energy per unit surface area.

Problem 2.2



A liquid of denisty ρ and surface tension σ has been spilled on a horizontal plate so that it forms a very large puddle whose depth (in the central parts) is h. Consider the region near the edge of the puddle, which can be viewed to a good aproximation as <u>two-dimensional</u>. If the contact angle is α, derive an expression for the shape of the liquid surface $y_s(x)$.

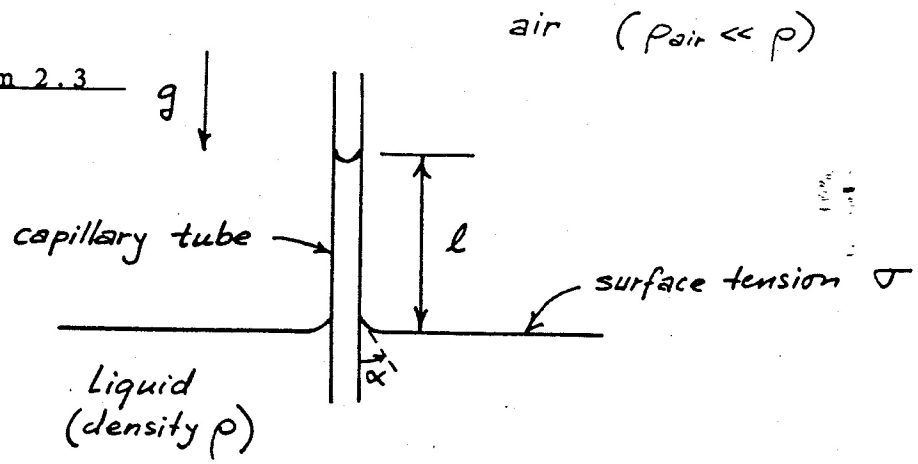
Assume for simplicity that α is small, so that the radius of curvature of the surface is large compared with h and can be approximated by

$$R = \frac{1}{\left| \frac{d^2 y_s}{dx^2} \right|} \quad .$$

ans: $$y_s = h \left[1 - \exp(-\sqrt{\rho g/\sigma}\, x) \right]$$

$$h = \tan \alpha \sqrt{\sigma/\rho g}$$

Problem 2.3



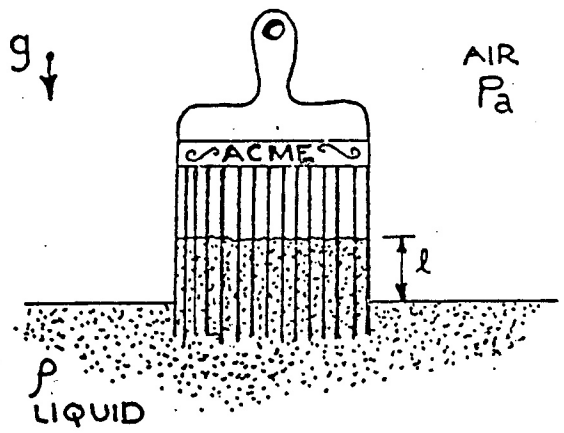
A capillary tube with a small radius a is held vertically in air with its bottom immersed in a large body of liquid. The surface tension of the air-liquid combination is σ, and the contact angle of the air-liquid-tube wall combination is α.

(a) Show that if $l >> a$, the capillary rise l is given by

$$l = 2\sigma \cos \alpha / \rho g a$$

(b) How would this expression change if the system were comprised of two plates seperated by 2a, instead of being a tube with radius a?

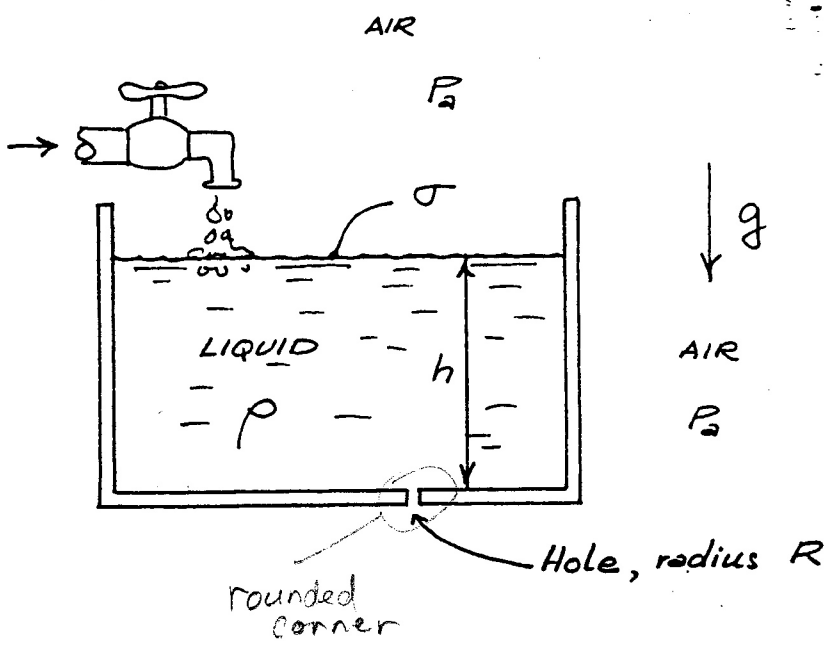
Problem 2.4



A brush is dipped into paint thinner of density ρ. If the brush hairs have a radius **a**, the porosity or ratio of void volume to total volume of the brush is ϵ, the surface tension coefficient of the liquid is σ, and the contact angle between the brush hairs and the liquid is α, derive an expression for the rise **l** of the liquid into the brush.

Assume that l is large compared with the distance between the hairs.

Problem 2.5

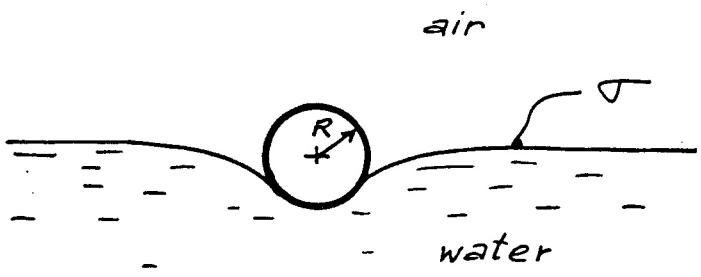


A container is being filled with liquid of density ρ. A small, sharp-edged hole of radius R penetrates the container's bottom. The surface tension between the liquid and the ambient air is σ, and the contact angle for the air/liquid/container combination is α (measured from the wall through the liquid to the interface).

(a) Find the critical liquid depth h_c at which liquid first begins to flow through the hole in the bottom. Assume that $R << h$. (Hint: Is the expression different depending on whether α is greater or smaller than $\pi/2$?)

(b) Evaluate h_c for the case when the liquid is water at $20^{\circ}C$, $R = 0.1mm$, $\sigma = 0.07$ N/m, and $\alpha = 120^{\circ}$.

Problem 2.6



The water strider, or pond skater, is a slender insect, about 1/2 cm long, that runs or "skates" over the surfaces of ponds and streams. It stays easily on the surface because its feet (tarsi) are equipped with numerous fine, nonwetting hairs.

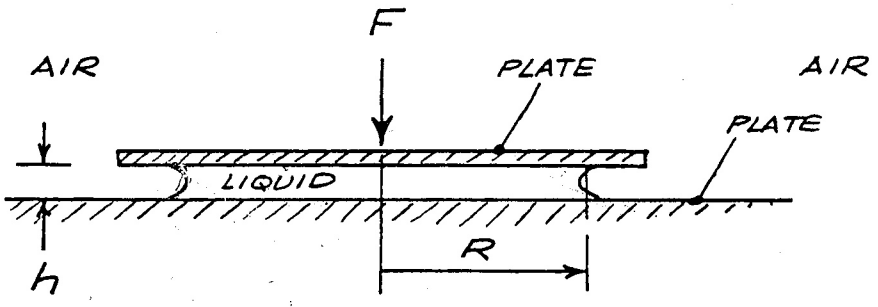
Suppose we model one of these hairs as a long cylinder of radius R made of completely nonwetting material (contact angle 180 degrees), and assume that it is set down on the water with its axis parallel to the surface, as sketched. The surface tension is 0.07 N/m.

(a) Show that as the cylinder, or hair, is brought into contact with the water and then depressed into it, the lift force exerted on it by surface tension first increases, then reaches a maximum at a certain depression, and finally decreases as the cylinder is depressed further. What is the maximum value of the surface-tension-induced lift force per unit cylinder length?

(b) What is the criterion for the gravitational effects to have a negligible effect on the (maximum) total lift force? Is it likely that this criterion is satisfied for the pond skater's tarsi?

(c) If a pond skater weighs 0.05 gram (note that this is only a guess, not a figure based on observation of the real insect), what minimum total length of hair must it have on its feet to keep on top of the water?

Problem 2.7



A drop of liquid of volume V is squeezed between two parallel smooth plates until the liquid thickness h is very small compared with the liquid's radial extent R. The liquid/plate/air contact angle is α, and the liquid/air surface tension is σ. Gravitational effects are negligible.

(a) Derive an expression for the downward force F required to hold the plates in position. Express F in terms of V, α, σ and R.

(b) If $\alpha = \pi$ radians (a perfectly nonwetting situation) and T = 0.07 N/m, say (representing a clean air-water interface), what downward force is required to press a 3 mm^3 drop of liquid into a thin disc of radius R = 2 cm?

$$-F = \frac{2\sigma(\pi R^2)^2}{V}\cos\alpha\left(1 + \frac{V}{\pi R^3}\tan\alpha\right)$$

$$V = \pi R^2 h$$

$$F = 73.7\,N$$