MECH 411 Introduction to Fluid Mechanics
American University of Beirut, Fall 2004-2005

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Handout # PS6

Problem Set 6 Solution

Textbook: 6.15, 6.62, 6.91, 6.103, 6.118

Required Sections: 6.1, 6.2, 6.3, 6.4, 6.6 (Effect of Rough Walls and Moody Chart only), 6.7, 6.8 (no turbulent flow solution), 6.9, 6.10 (series and parallel only.)

Problem 6.15

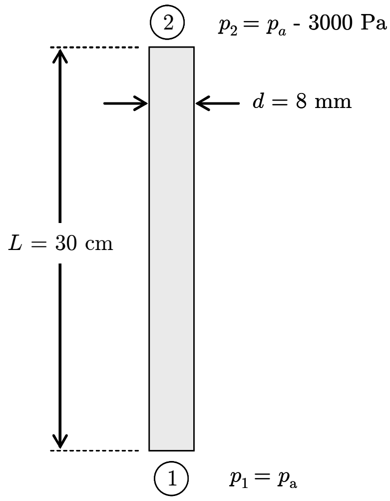

Figure 1: Problem 6.15

(a) $\rho = 1200$ kg/m^3, $\mu = 6$ kg/m.s, $d = 0.008$ m, $L = 0.3$ m.

We assume that the flow is straw is operating between between (1) atmospheric pressure $p_a = 101.4$ kPa and (2) lung pressure $p_l = p_a - 3$ kPa. Conservation of energy between (1) and (2)

$$\frac{\Delta p}{\rho g} + \Delta z = h_f \tag{1}$$

where $\Delta p = p_1 - p_2 = p_a - p_l$, $\Delta z = z_1 - z_2 = -L$ (we assume the student is drinking the straw in a vertical position.), then

$$h_f = \frac{p_a - p_l}{\rho g} - L \tag{2}$$

The head loss is given by

$$h_f = f \frac{L}{d} \frac{V^2}{2g} = f \frac{L}{d} \frac{8Q^2}{\pi^2 d^4 g} \tag{3}$$

where $Q = V \frac{\pi d^2}{4}$ is volume flow rate.

The friction factor, assuming Laminar flow is,

$$f = \frac{64}{\mathrm{Re}} = \frac{64\nu}{Vd} = \frac{16\nu\pi d}{Q} \tag{4}$$

Combining Eqs. (3) and (4), we get

$$Q = \frac{\pi \rho g d^4}{128\mu L} h_f = \frac{\pi \rho g d^4}{128\mu L} \left(\frac{p_a - p_l}{\rho g} - L \right)$$

For $L = 30$ cm, the volume flow rate is $Q = -0.029$ cm^3/sec; the student would be unable to drink the milk shake.

(b) For $L = 15$ cm, the volume flow rate is $Q = 0.138$ cm^3/sec. We can easily verify that Re< 2000; the flow is laminar as we assumed.

Problem 6.62

$L = 2000$ ft, $z2 - z1 = 120$ ft, $Q = 3$ ft^3/s. Pipe: Cast iron $d = 6$ in, pump efficiency 0.75. Power of pump in hp?

From Table 6.1 Page 367, the surface roughness for cast iron is $\epsilon = 0.00085$ ft.

The energy equation between (1) and (2) is

$$\frac{\Delta p}{\rho g} + \Delta z + \frac{\Delta(V^2)}{2g} = h_f + \sum h_m - h_p \tag{5}$$

where $\Delta() = ()_1 - ()_2$, h_f is the friction head loss, and h_p is the head delivered by the pump. The minor losses $\sum h_m$ are neglected in this problem.

Referring to Figure P6.62 in book, we assume (1) and (2) correspond to locations on the surfaces of the two tanks so that $p_1 = p_2 = p_a$. Then

$$h_p = h_f - (z_1 - z_2)$$

With $h_f = f(L/d)\,(V^2/2g)$, we get

$$h_p = f\frac{L}{d}\frac{V^2}{2g} - (z_1 - z_2)$$

The average velocity is $V = 4Q/\pi d^2 = 15.28$ ft/s. For water $\mu = 2.09E - 5$ slug/ft.s and $\rho = 1.94$ slug/ft^3. Then $\mathrm{Re}_d = Vd/\nu = 7.09 \times 10^5$. The flow is turbulent. The friction factorcan be obtained from (a) Moody Chart Fig. 6.13 page 366, (b) Eq. 6.48 page 365, or (c) Eq. 6.49 page 366. We shall use Eq. 6.49 because it is easier to use

$$\frac{1}{f^{1/2}} \simeq -1.8\log\left[\frac{6.9}{\mathrm{Re}_d} + \left(\frac{\epsilon/d}{3.7}\right)^{1.11}\right] \tag{6}$$

leading to $f = 0.02274136153$, $h_f = 330$ ft. Then $h_p = h_f - (z_1 - z_2) = 330 + 120 = 450$ ft.

The power of the pump is therefore

$$\mathcal{P} = \frac{\rho Q g h_p}{\xi} = 204 \text{ hp}$$

where $\xi = 0.75$ is the pump efficiency.

Problem 6.91

$L = 0.6$ m, $V = 2$ m/s. Channel: isoceles cross section: $a = 2$ cm, $\beta = 80^o$. SAE Oil at 20 C: $\rho = 870$ kg/m^3, $\mu = 0.104$ kg/m.s from From Table A.3 page 811.

The Perimeter of the cross section is $\mathcal{P} = 2a(1 + \sin(\beta/2)) = 0.06571$ m.
The cross section area is $A = a^2 \sin(\beta/2)\, \cos(\beta/2) = 0.000197$ m^2.
The hydraulic diameter is $d_h = 4A/\mathcal{P} = 0.012$ m.

For Reynolds number is Re$= V d_h/\nu = 200.58$: Laminar flow.

For Laminar flow in triangular duct with $\theta = \beta/2 = 40^o$, From Table 6.4 page 383, the friction factor $f = 52.9/\mathrm{Re} = 0.2637$.

The friction head loss is $h_f = f(L/d_h)\,(V^2/2g) = 2.6933$ m.

The pressure drop is obtained from the energy equation

$$\frac{\Delta p}{\rho g} + \Delta z = h_f$$

With $\Delta z = 0$, then the pressure drop is $\Delta p = \rho g h_f = 22.96$ kPa.

Problem 6.103

From Table 6.1 Page 367, the surface roughness for cast iron is $\epsilon = 0.00085$ ft.

Energy equation from (1) at surface to (2) at surface according to Figure P6.103 page 436

$$\frac{\Delta p}{\rho g} + \Delta z + \frac{\Delta(V^2)}{2g} = h_f + \sum h_m - h_p + h_t \tag{7}$$

For $\Delta z = z_1 - z_2 = 45$ ft, pump head $h_p = 0$, turbine head $h_t = 0$, and $p_1 = p_2 = p_a$. Also $V_1 \simeq V_2 = 0$ (recall that (1) and (2) are at surfaces of tanks and not correspond to flow inside pipe) then

$$\begin{aligned} \Delta z &= h_f + \sum h_m \\ &= \frac{V_a^2}{2g}\frac{f_a L_a}{d_a} + \frac{V_b^2}{2g}\frac{f_b L_b}{d_b} + \frac{V_a^2 K_{ent}}{2g} + \frac{V_a^2 K_{exp}}{2g} + \frac{V_b^2 K_{exit}}{2g} \\ &= \frac{V_a^2}{2g}\left(\frac{f_a L_a}{d_a} + \frac{d_a^4}{d_b^4}\frac{f_b L_b}{d_b} + K_{ent} + K_{exp} + \frac{d_a^4}{d_b^4} K_{exit}\right) \end{aligned} \tag{8}$$

The subscripts a and b refer to the two sections of the pipe. The minor losses are sharp entrance, sudden expansion and sharp exit:

Figure 6.22 page 390 for sharp entrance (sudden contraction with $D = \infty$): $d/D = 0$, we get $K_{ent} = 0.4$.
Figure 6.22 page 390 for sudden expansion: $d/D = 1/2$, we get $K_{exp} = 0.5$.
Figure 6.22 page 390 for sharp exit (sudden expansion with $D = \infty$) : $d/D = 0$, we get $K_{exp} = 1$.

Then

$$\frac{V_a^2}{2g}(240 f_a + 7.5 f_b + 0.9625) = 45 \tag{9}$$

We assume trubulent flow. (We will validate later).

Initial guess: fully rough flow, Moody Chart: so for $\epsilon/d_a = 0.0102$, $f_a = 0.038$ and $\epsilon/d_b = 0.00051$, $f_b = 0.0165$.

Eq. (9): $V_a = 16.85$ ft/s, then $\mathrm{Re}_a = V_a d_a/\nu = 1.303E5$, $\mathrm{Re}_b = V_b d_b/\nu = 65171$.

Update from Eq. (6) $f_a = f_a(\mathrm{Re}_a, \epsilon/d_a) = 0.03866$, $f_b = 0.03188$.

Eq. (9): $V_a = 16.629$ ft/s, then $\mathrm{Re}_a = 1.286E5$, $\mathrm{Re}_b = 64314$.

Update from Eq. (6) $f_a = 0.03866$, $f_b = 0.03189$.

The converged solution is $V_a = 16.628$ ft/s, $Q = 0.09$ ft^3/s.