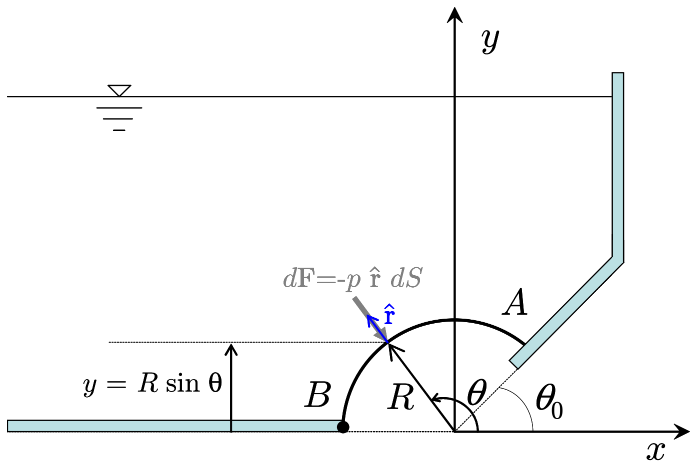
Selected Problems from Problem Set 2

Problem 2.97



$\hat{\mathbf{r}} = \cos\theta\,\hat{\mathbf{x}} + \sin\theta\,\hat{\mathbf{y}}$

$dS = R\,d\theta$

The gauge pressure at y is

$$p_g(y) = p(y) - p_a = \rho\,g\,(H - y) = \rho\,g\,(H - R\,\sin\theta) \tag{1}$$

The unit outward normal vector is

$$\hat{\mathbf{n}} = \hat{\mathbf{r}} = \cos\theta\,\hat{\mathbf{x}} + \sin\theta\,\hat{\mathbf{y}} \tag{2}$$

The force due to pressure distribution is

$$\begin{aligned}
\mathbf{F} &= \int d\mathbf{F} = \int_{\mathcal{S}} -p_g(y)\,\hat{\mathbf{n}}\,d\mathcal{S} \\
&= \int d\mathbf{F} = -WR\int_{\pi/4}^{\pi} \rho\,g\,(H - R\,\sin\theta)\,(\cos\theta\,\hat{\mathbf{x}} + \sin\theta\,\hat{\mathbf{y}})\,d\theta \\
&= -\rho g W R\left(\frac{R}{4} - \frac{H}{\sqrt{2}}\right)\hat{\mathbf{x}} + \rho g W R\left(R\left(\frac{3\pi}{8} + \frac{1}{4}\right) - H\left(1 + \frac{1}{\sqrt{2}}\right)\right)\hat{\mathbf{y}} \qquad (3)
\end{aligned}$$

Noting that the position vector on the gate is $\mathbf{r} = x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} = R(\cos\theta\,\hat{\mathbf{x}} + \sin\theta\,\hat{\mathbf{y}})$, the moment about the center of coordinates is

$$\mathbf{M}_O = \int \mathbf{r} \times d\mathbf{F} = \mathbf{R}^* \times \mathbf{F}$$

where $\mathbf{R}^* = x^*\,\hat{\mathbf{x}} + y^*\,\hat{\mathbf{y}}$ is the center of pressure. Then satisfying the x and y components of the above equation yields

$$\begin{aligned}
\int -x\,dF_y &= -x^* F_y \\
\int y\,dF_x &= y^* F_x
\end{aligned}$$

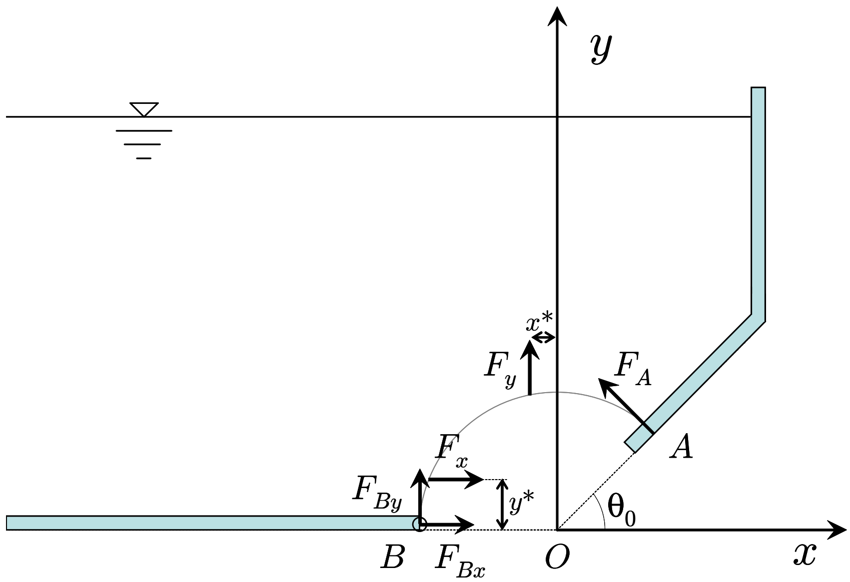
Notice that since $d\mathbf{F}$ is always along the radial coordinate, the moment about O is zero, i.e. $x^* F_y = y^*\, dF_x$. The center of pressure $\mathbf{R}^* = x^*\,\hat{\mathbf{x}} + y^*\,\hat{\mathbf{y}}$ is determined as follows

$$x^* = \frac{\int_{\pi/4}^{\pi} x\, dF_y}{F_y} = \frac{2}{3}\,\frac{R\left(R\sqrt{2} - 3\,H\right)}{4\sqrt{2}H - 3\,R\pi - 2\,R + 8\,H} \tag{4}$$

$$y^* = \frac{\int_{\pi/4}^{\pi} y\, dF_x}{F_x} = -\frac{1}{3}\,\frac{R\left(R\sqrt{2} - 3\,H\right)}{2\sqrt{2}H - R} \tag{5}$$

With $R = 2$ m, $W = 4$ m, $H = 4$ m, $\rho g = 10050$ N/m^3, we get

$$F_x = 140.4 \text{ kN} \tag{6}$$
$$F_y = -239.5 \text{ kN} \tag{7}$$
$$x^* = -0.385 \text{ m} \tag{8}$$
$$y^* = 0.6565 \text{ m} \tag{9}$$



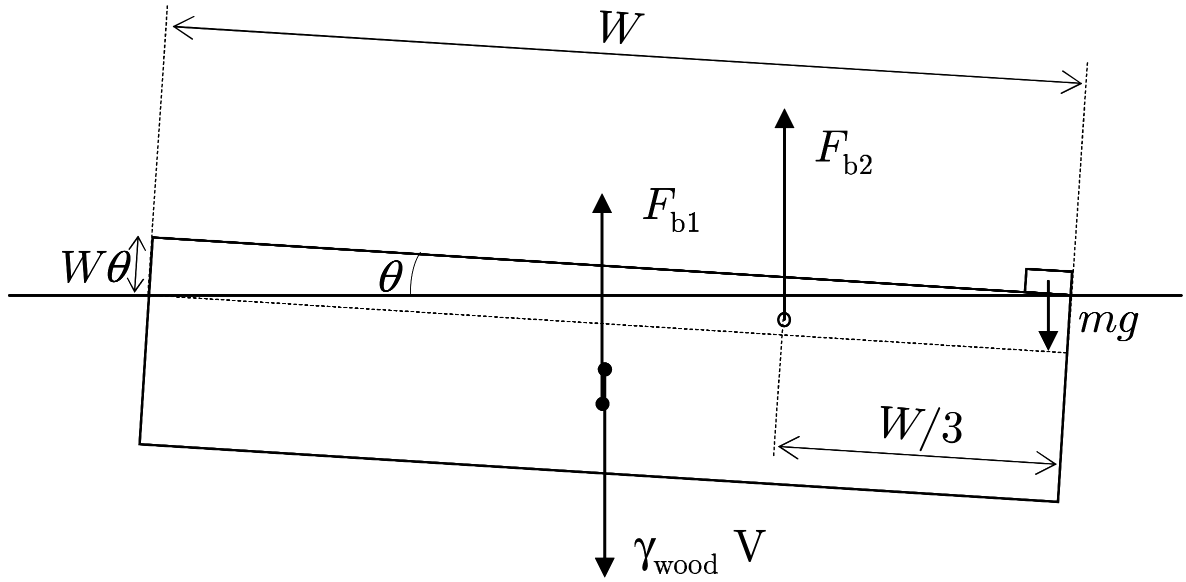
To get the reaction forces at A and B, we employ $\sum \mathbf{F} = \mathbf{0}$ and $\sum \mathbf{M} = 0$ about any chosen point. Then

$$\begin{aligned}
& F_{Ax} + F_{Bx} = -F_x \Rightarrow F_{Bx} = -F_x + F_A \cos\theta_0 \\
& F_{Ay} + F_{By} = -F_y \Rightarrow F_{By} = -F_y - F_A \sin\theta_0 \\
\text{moments about } B \quad & F_y(R + x^*) - F_x y^* + F_A R(1 + 2\cos\theta_0)\sin\theta_0 = 0
\end{aligned}$$

Solving for F_{Ax}, F_{Ay}, F_{Bx} and F_{By}, we get

$$\begin{aligned}
F_A &= 140.3 \text{ kN} \\
F_{Ax} &= -99.2 \text{ kN} \\
F_{Ay} &= 99.2 \text{ kN} \\
F_{Bx} &= -41 \text{ kN} \\
F_{By} &= 140 \text{ kN}
\end{aligned}$$

Problem 2.119



We assume that the angle θ is small so that the floating volume is

$$\mathcal{V}_{float} = \frac{(W\theta)(W)}{2}(D)$$

where D is the depth. The weight of the piece of wood must balance the buoyant force

$$\begin{aligned} & \gamma_{wood}\mathcal{V}_{wood} + mg = \gamma_{water}\mathcal{V}_{submerged} \\ \Rightarrow\quad & \gamma_{wood}HWD + mg = \gamma_{water}\left(HWD - \frac{W^2 D\theta}{2}\right) \end{aligned} \tag{10}$$

where mg is the weight placed at the end of wooden beam.

For equilibrium the sum of moments about the center of gravity of the wooden beam must be zero. We split the buoyant force into two components. Component F_{b1} due to a rectangular volume of fluid and component F_{b2} due to triangular volume of fluid. The component F_{b1} due to the rectangular volume of water passes throught the center of gravity of the wooden beam. This is because the angle θ is small. Then the moment induced by the weight placed at the edge of the wooden beam must be balanced by F_{b2} as shown in the Figure. Then

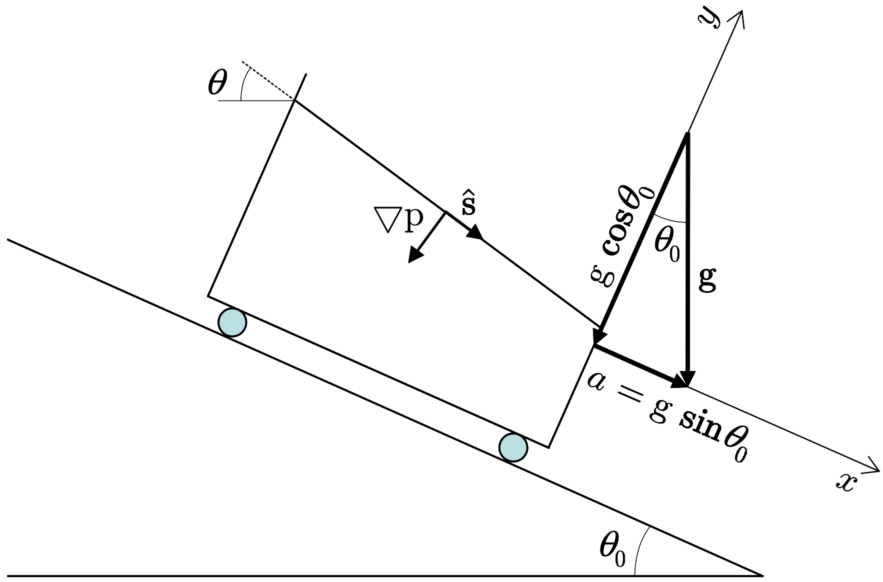
$$\begin{aligned} mg\frac{W}{2} &= F_{b2}\left(\frac{W}{2} - \frac{W}{3}\right) \\ &= \gamma_{water}(D)\frac{(W)(W\theta)}{2}\left(\frac{W}{6}\right) \\ \Rightarrow \theta &\simeq \frac{6mg}{\gamma_{water}DW^2} \end{aligned}$$

Setting $mg = 5$ lbf, $W = 9$ ft, $D = 4/12$ ft, $\gamma_{water} = 62.4$ lbf/ft^3, we get $\theta = 0.0178$ rad or 1.02^o.

Substituting in equation (10), we get $\gamma_{wood} = 42.4$ lbf/ft^3. The specific gravity of wood is then $SG \simeq \gamma_{wood}/\gamma_{water} = 0.68$.

Problem 2.147

We start by applying Newton's second law per unit volume



$$\sum \mathbf{F} = \rho\, \mathbf{a}$$
$$\Rightarrow \quad -\nabla p + \rho \mathbf{g} = \rho\, \mathbf{a}$$

The free surface is characterized by uniform pressure, p_a. Then the pressure gradient is normal to the free surface. If $\hat{\mathbf{s}}$ is the unit vector along the free surface, then $\nabla p \cdot \hat{\mathbf{s}} = 0$, then

$$\mathbf{g} \cdot \hat{\mathbf{s}} = \mathbf{a} \cdot \hat{\mathbf{s}}$$

Note that

$$\mathbf{g} \cdot \hat{\mathbf{x}} = \mathbf{a} \cdot \hat{\mathbf{x}} = a = g \sin\theta_0$$

Then we must have

$$\hat{\mathbf{s}} = \hat{\mathbf{x}}$$

i.e. the water surface is aligned with the x coordinate.