Problems on Inviscid Flow

"Advanced Fluid Mechanics Problems" by Shapiro and Sonin
Problems 4.4, 4.7, 4.8, 4.9, 4.10, 4.13, 4.18, 4.19, 4.21, 4.23, 4.24, 4.28.

Problem 4.4

A nozzle with exit area A_2 is mounted at the end of a pipe of area A_1, as shown. The nozzle converges gradually and we assume the flow in it is (i) approximately uniform over any particular station x, (ii) incompressible and (iii) inviscid. Gravitational effects are, furthermore, taken as negligible. The volume flow rate in the nozzle is given as Q and the ambient pressure is p_a.
(a) Derive an expression for the gage pressure at a station where the area is $A(x)$.
(b) Show, by integrating the x-component of the pressure force on the nozzle's interior walls, that the net $x-$component of the force on the nozzle due to the flow is independent of the specific nozzle contour and is given by

$$F = \rho Q^2 \frac{(A_1 - A_2)^2}{2A_1A_2^2}$$

(c) The expression in (b) predicts that F is in the positive x direction regardless of whether the nozzle is converging or diverging. Explain.

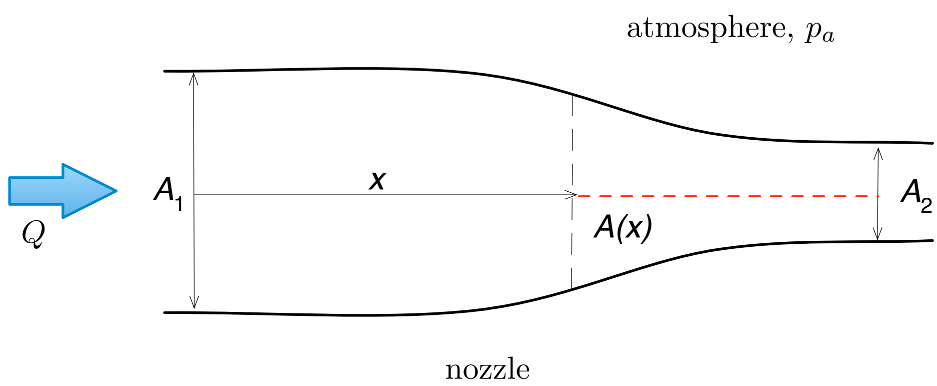

Figure 1: Schematic of Problem 4.4

Problem 4.7

An engine carburetor consists of a duct of diameter D with a small metering jet of diameter d through which fuel enters from the float bowl. The fuel in the bowl is at the same level as the jet. The engine draws air into the duct from the ambient atmosphere, which is at pressure p_a and density ρ_a. The fuel streams breaks into droplets after it exits from the metering jet, and vaporized before it reached the engine.

Neglecting friction and compressibility and assuming no losses in the flow of air from the ambient atmosphere to the metering jet, find d/D for a fuel-air mass flow ratio of α and densities ρ_a and ρ_f.

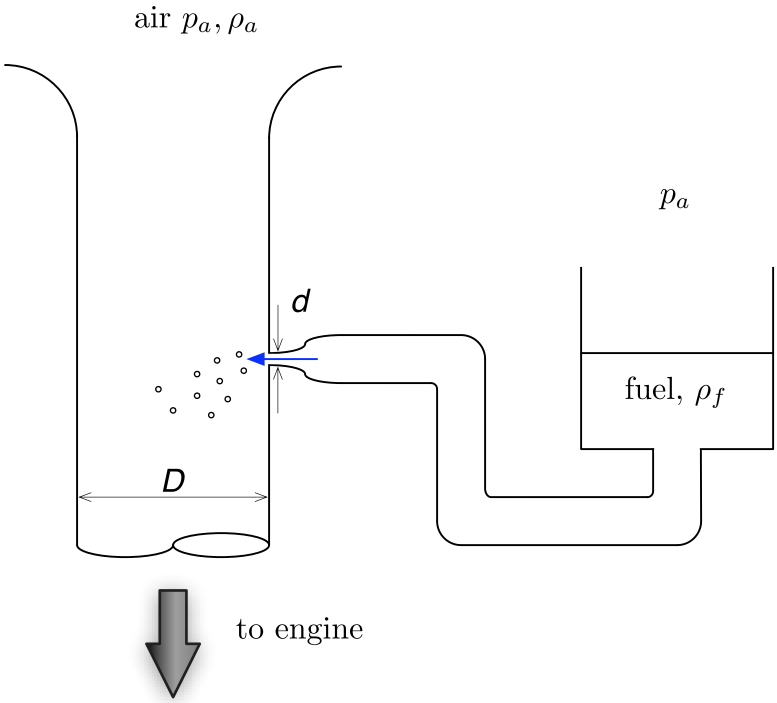

Figure 2: Schematic of Problem 4.7

Problem 4.8

A constant force F is applied to a simple cylindrical bellows of diameters D_1. The air flows out of the bellows via a nozzle of diameter D_2, to the ambient atmosphere.
(a) If the air flow is incompressible (density ρ) and inviscid, derive an expression for the time it takes to exhaust a volume V of air from the bellows.
(b) Compute this time for STP air if $V = 1$ liter, $D_1 = 10$ cm, $D_2 = 1$ cm and $F = 2$ kgf.

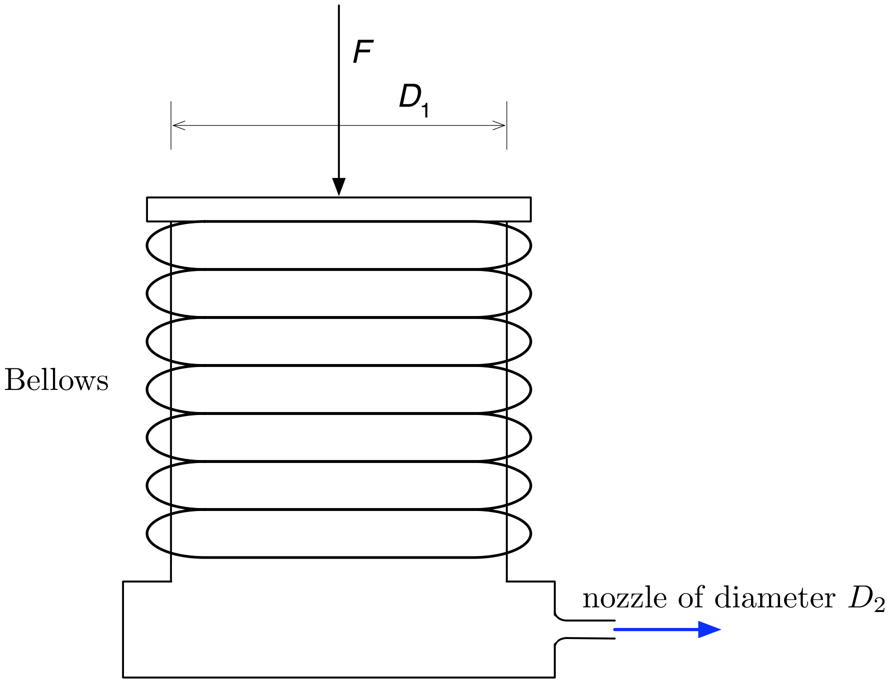

Figure 3: Schematic of Problem 4.8

Problem 4.9

Consider a furnace of height H with a tall cylindrical smoke stack of diameter d ($d << H$) and height h ($h >> H$). Air, an ideal gas ($p = \rho RT$), enters the furnace at atmospheric density and temperature and at local atmospheric pressure. Between stations 1 and 2, heat is added at constant pressure and the air temperature is raised by an amount Δt. Thereafter, heat addition is negligible and the air rises through the stack at a sensibly constant density.

(a) On the assumption that viscous effects are negligible, derive an expression for the steady mass flow rate of air drawn by a stack of given height, h, in terms of the temperature rise in the furnace.

(b) If the chimney were capped off at the top, what would be the pressure differential across the cap, assuming that ΔT would not be altered by the flow stoppage ?

Note: The height h of the stack is small compared with the length RT_a/g over which the atmosphere density falls by 1/e. Hence gravitational density changes are neglected in this problem.

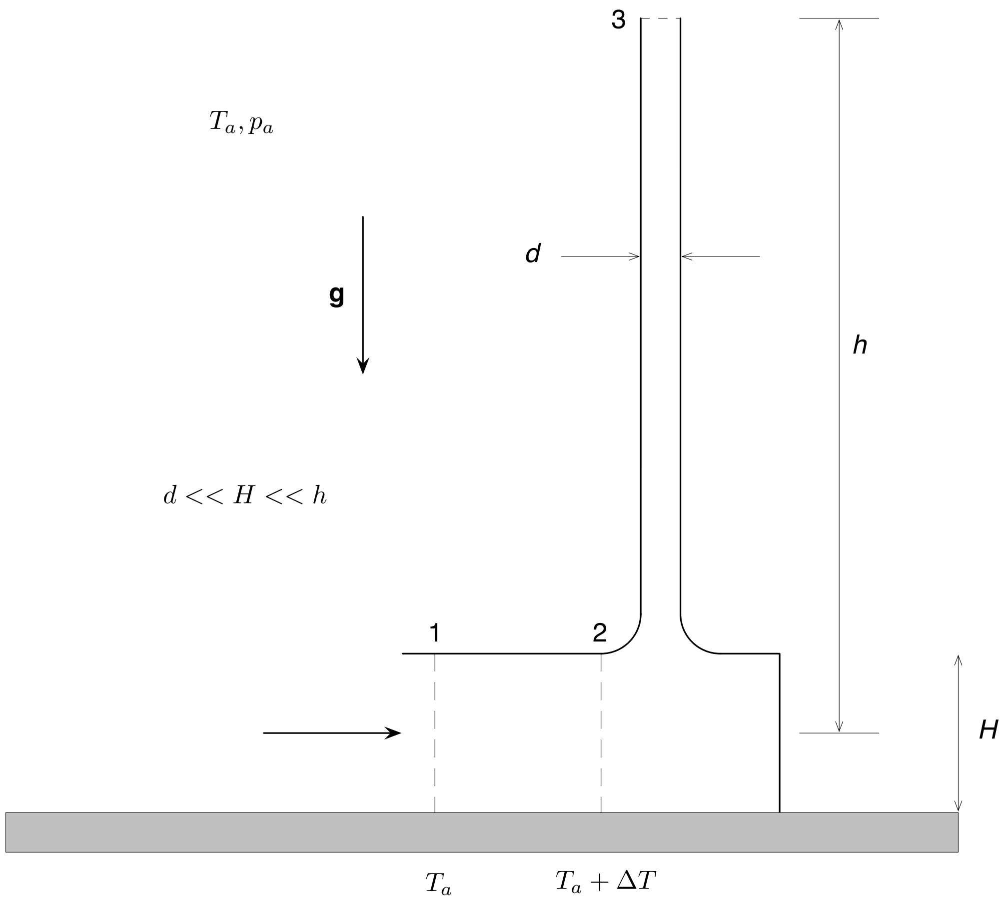

Figure 4: Schematic of Problem 4.9

Problem 4.10
A vessel of volume V originally contains a perfect gas at pressure $p_i > p_a$ and temperature $T = T_a$ where p_a and T_a are pressure and temperature of the ambient air outside the vessel. At $t = 0$, a small, streamlined nozzle with exit area A is opened, and the gas slowly escapes to the ambient atmosphere. The gas obeys the law $p = \rho RT$, where p is the absolute pressure and T the absolute temperature, and the flow is quasi-steady , i.e. the flow out through the nozzle changes so slowly that the unsteady term in Euler's equation is negligible compared to other terms.

In what follows, you may assume that the gage pressure $p' = p - p_a$ is always small in the sense that $p'/p_a << 1$ and use approximations accordingly. Gravitational effects are to be neglected.

Obtain a solution for the gage pressure $p'(t)$ in the vessel as a function of time for the following two limiting cases:
(a) The flow rate out is so small, and the heat transfer to the gas inside so effective, that the gas inside essentially maintains itself at the ambient temperature T_a. The flow out the nozzle is, however, inviscid.
(b) The depressurization is rapid, and the vessel thermally insulated, and as a result the gas inside the vessel expands isentropically, i.e. the absolute pressure and density in the vessel at any instant are related by $p/p_i = (\rho/\rho_i)^\gamma$ where γ is the ratio of the gas's specific heats. The flow out the nozzle is also isentropic (inviscid).

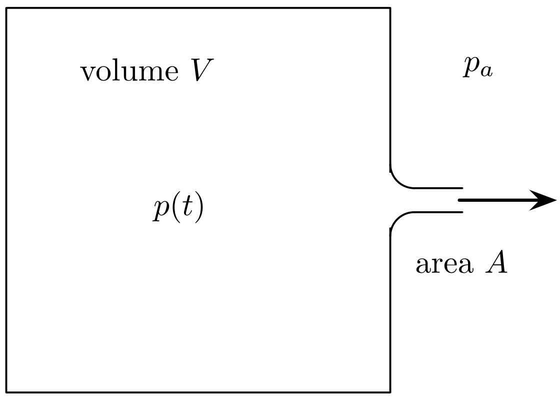

Figure 5: Schematic of Problem 4.10

Problem 4.13

The sketch shows a steady, free-surface, non-viscous, two-dimensional flow of water over the bed of a channel. The free surface is at uniform atmospheric pressure, p_a. At any section the channel makes the local angle α with the horizontal, the local water velocity is V, and the local depth of water above the bed is h.

The angle α is everywhere small, and rates of change of α are also small. This permits the assumption that $\cos\alpha \simeq 1$, (ii) the vertical components of fluid acceleration are negligible, and (iii) the velocity V is uniform over each cross section.

(a) Demonstrate that the slop of the free surface is given by

$$\frac{dz}{dx} \simeq \frac{V^2/gh}{V^2/gh - 1} \tan\alpha$$

(b) discuss the difference between "'subcritical' flow ($V^2 < gh$) and "supercritical" flow ($V^2 > gh$).

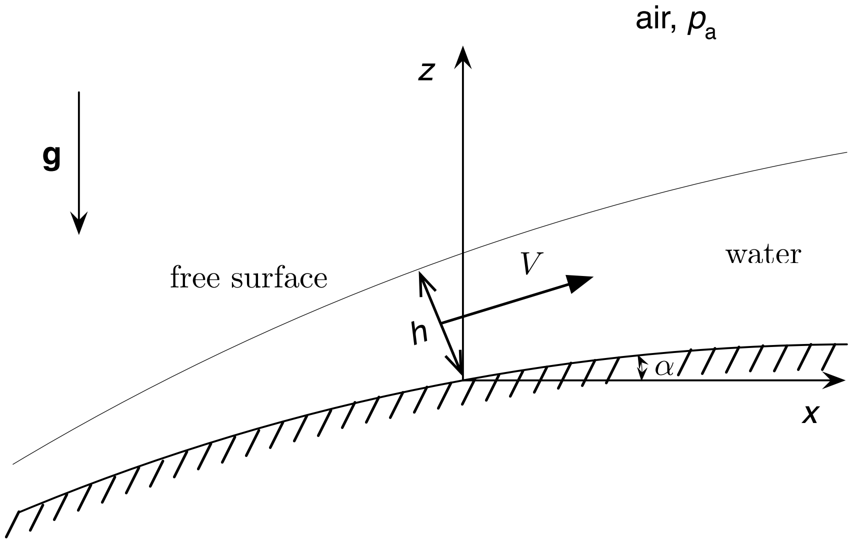

Figure 6: Schematic of Problem 4.13