Final Exam

- This exam is an open book exam.
- All problems have equal weight.
- You have 3 hours.

Problem 1

Air is blown downwards at a volume flow rate Q through the pipe of diameter d as shown in Fig 1. After leaving the pipe, air flows radially outwards (in an axisymmetric fashion) between two parallel disks of diameter D and separation h. Given: $D = 10$ cm, $h = 5$ mm, $d = 1$ cm, $Q = 0.05$ m^3/s, $\rho = 1.225$ kg/m^3, $\mu = 1.789 \times 10^{-5}$ Pa.s.
(a) Find the average velocity $V(r)$ as a function of radial position r. What is Reynolds number (based on channel height h) as a function of radial position r? For the given parameters, is the flow laminar or turbulent? Would the flow be modeled as a viscous or inviscid flow ?
(b) Find the pressure distribution $p(r)$ as a function of radial position r.
(c) What is the force (due to pressure) acting on the lower disk ? [Hint: account only for pressure variation from $r = d/2$ to $r = D/2$.]

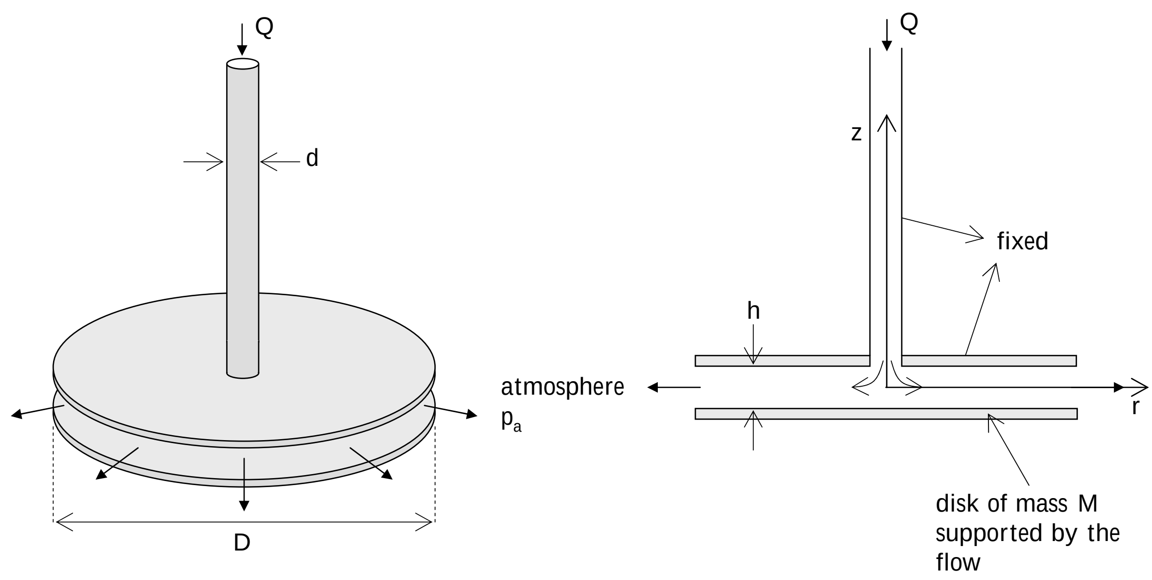

Figure 1: Schematic for Problem 1.

Problem 2

A piston in the cylinder of an internal combustion engine slides back and forth on a thin film of oil that fills that gap between the piston surface and the cylinder wall, as depicted in Fig. 2. The gap thickness h is uniform along the length L and circumference of the piston and is much smaller than both of these dimensions. We wish to determine the velocity of the oil in the gap. Assume that the inertia of the oil is negligible compared to the pressure and viscous forces, and choose a reference system fixed in the moving piston as shown in the figure. In this coordinate system, the cylinder wall moves with the speed u_c:

$$u_c = u_m \sin \omega t$$

where u_m is the maximum piston speed and ω is the angular frequency of the crankshaft. The pressure below the piston is atmosphere pressure p_a while that above the piston in the combustion chamber, p_c, is

$$p_c = p_a + \frac{p_m}{2}(1 - \cos \omega t)$$

where p_m is the maximum pressure increase in the cylinder, which occurs at $\omega t = \pi$. Assuming that the oil flow in the gap is a combination of plane Couette and Poiseuille flows,

(a) derive an expression for the velocity distribution $u(y, t)$ in terms of the parameters p_m, u_m, ω, h, L and the viscosity μ.

(b) Derive an expression for the time-averaged volumetric flow rate $\bar{Q}$ of oil past the piston.

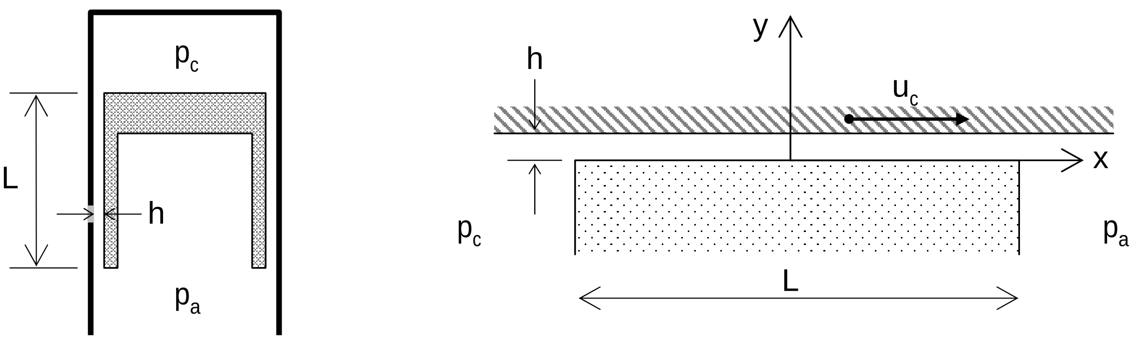

Figure 2: Schematic for Problem 2.

Problem 3

The drag (F) on a submarine moving well below the free surface is to be determined by a test on a model scaled to 1/20 th of the prototype. The test is to be carried out in a water tunnel. Determine the ratio of model to prototype drag (F_m/F_p) needed for finding the prototype drag when the speed of the prototype is 2.57 m/s. The kinematic viscosity of sea water is 1.3×10^{-6} m^2/s, and the density is 1010 kg/m^3 at the depth of the prototype. The water in the tunnel has a temperature of 50 °C.
If the model submarine were to be tested in a wind tunnel where, at free stream, $p = 2000$ kPa, and $T = 50$°C, what will the ratio F_m/F_p have to be ?

Note: Only the Reynolds number and the Euler number ($\Delta p/\rho V^2 \equiv (F/L^2)/\rho V^2$) are important for dynamic similarity, where L =submarine length. Explain briefly why the Mach number and the Froude number may be neglected.

Problem 4

Water at 10°C is to flow from reservoir A to reservoir B through a cast-iron pipe of length 20 m at a rate of $Q = 0.0020$ m^3/s as shown in Fig. 3. The system contains a sharp-edged entrance and six regular threaded 90 ° elbows. Determine the pipe diameter needed, taking $K_{elbow} = 1.5$ [Hint: the friction factor lies between 0.030 and 0.035.]

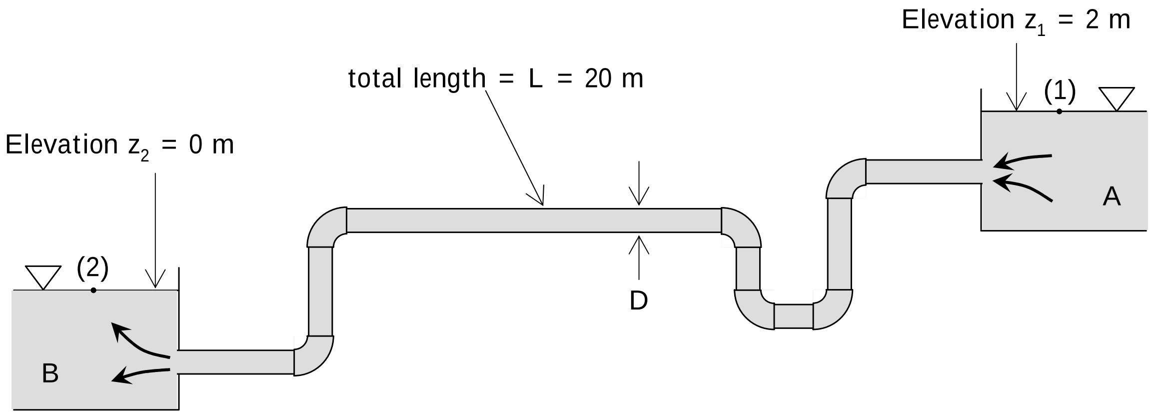

Figure 3: Schematic for Problem 4.

Problem 1 Solution

(a) Taking control volume as shown, conservation of mass

$$Q = 2\pi r h V(r) \Rightarrow V(r) = \frac{Q}{2\pi r h}$$

Reynolds number is then given by

$$Re_h(r) = \frac{\rho V(r) h}{\mu} = \frac{\rho Q}{2\pi \mu r}$$

At $r = R$,

$$Re_h(r = R) = \frac{\rho Q}{2\pi \mu R} = 10898$$

also $Re_h(r = d/2) = 108980$.

(b) Bernoulli along a streamline from point (1) at radius r to point (2) at radius $D/2$

$$p(r) + \frac{1}{2}\rho V(r)^2 = p_a + \frac{1}{2}\rho V_2^2$$

From part (a), $V_2 = V(r = D/2) = \frac{Q}{\pi D h}$, then

$$p(r) - p_a = \frac{\rho Q^2}{2\pi^2 h^2}\left(\frac{1}{D^2} - \frac{1}{4r^2}\right)$$

(c) The upward force on the disk due to pressure is

$$\begin{aligned} F &= -\int_{r_0}^{D/2} (p(r) - p_a) 2\pi r\, dr = -\int_{d/2}^{D/2} \frac{\rho Q^2}{2\pi^2 h^2}\left(\frac{1}{D^2} - \frac{1}{4r^2}\right) 2\pi r\, dr \\ &= -\frac{\rho Q^2}{4\pi h^2}\left[\frac{1}{2}\left(1 - \frac{d^2}{D^2}\right) + \ln\frac{d}{D}\right] \\ &\simeq \frac{\rho Q^2}{4\pi h^2}\left[\ln\frac{D}{d} - \frac{1}{2}\right] \end{aligned}$$

leading to $F = 17.57$ N.

Problem 2 Solution

The flow is fully developed with $v = 0$, so that $p = p(x)$. Momentum in $x-$direction, after neglecting the interia term, and $\partial^2 u/\partial x^2$:

$$\frac{dp}{dx} = \mu \frac{\partial^2 u}{\partial y^2}$$
$$\Rightarrow \quad u = \frac{1}{2\mu}\frac{dp}{dx}y^2 + C_1 y + C_2$$

The boundary conditions are: $u = 0$ at $y = 0$ and $u = u_c$ at $y = h$, we

$$C_2 = 0$$
$$u_c = \frac{1}{2\mu}\frac{dp}{dx}h^2 + C_1 h \Rightarrow C_1 = -\frac{1}{2\mu}\frac{dp}{dx}h + \frac{u_c}{h}$$

The velocity field is then given by

$$u(y,t) = \frac{1}{2\mu}\frac{dp}{dx}(y^2 - hy) + u_c\frac{y}{h}$$

The volume flow rate is

$$Q(t) = \pi D_p \int_0^h u(y,t)\,dy = \pi D_p\left(-\frac{h^3}{12\mu}\frac{dp}{dx} + u_c\frac{h}{2}\right)$$

with $-dp/dx = \frac{p_m}{2L}(1 - \cos\omega t)$ and $u_c = u_m \sin\omega t$,

$$Q(t) = \pi D_p\left(\frac{h^3}{12\mu}\frac{p_m}{2L}(1 - \cos\omega t) + \frac{u_m h}{2}\sin\omega t\right)$$

The time averaged volume flow rate is obtained by averaging over one period

$$\bar{Q} = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} Q(t)\,dt = \pi D_p \frac{h^3}{12\mu}\frac{p_m}{2L}$$