Conservation of Momentum for a Linearly Accelerating Control Volume

In White's book, the conservation of moment in a reference frame undergoing linear acceleration **a** with respect to an inertial reference frame, which may be conveniently chosen to be the ground[1], is given by equation (3.49):

$$\sum \mathbf{F} - \int_{CV} \mathbf{a}\, dm = \frac{\partial}{\partial t}\int_{CV} \rho \mathbf{u}_{/\text{NIF}}\, d\mathcal{V} + \int_{CS} \rho \mathbf{u}_{/\text{NIF}} (\mathbf{u}_r \cdot \hat{\mathbf{n}})\, dS$$

where
$\sum \mathbf{F}$ is the sum of forces acting on the control volume,
$\mathbf{a}$ is the acceleration of the non-inertial reference frame with respect to the inertial reference frame,
$\mathbf{u}_{/\text{NIF}}$ is the flow velocity relative to the accelerating reference frame, and
$\mathbf{u}_r$ is the flow velocity relative to the control surface.

An alternative derivation is obtained as follows. The flow velocity $\mathbf{u}$ in the inertial reference frame may be expressed as $\mathbf{u} = \mathbf{u}_{\text{NIF}} + \mathbf{u}_{/\text{NIF}}$, where $\mathbf{u}_{\text{NIF}}$ is the velocity of the non-inertial frame with respect to the inertial frame and $\mathbf{u}_{/\text{NIF}}$ is the flow velocity relative to the non-inertial frame of reference. The conservation of momentum for a control volume fixed in the non-inertial frame is

$$\sum \mathbf{F} = \frac{\partial}{\partial t}\int_{CV} \rho \mathbf{u}\, d\mathcal{V} + \int_{CS} \rho \mathbf{u} (\mathbf{u}_r \cdot \hat{\mathbf{n}})\, dS$$

substituting $\mathbf{u} = \mathbf{u}_{\text{NIF}} + \mathbf{u}_{/\text{NIF}}$, we get

$$\begin{aligned}
\sum \mathbf{F} &= \frac{\partial}{\partial t}\int_{CV} \rho(\mathbf{u}_{\text{NIF}} + \mathbf{u}_{/\text{NIF}})\, d\mathcal{V} + \int_{CS} \rho(\mathbf{u}_{\text{NIF}} + \mathbf{u}_{/\text{NIF}})(\mathbf{u}_r \cdot \hat{\mathbf{n}})\, dS \\
&= \frac{\partial}{\partial t}\int_{CV} \rho \mathbf{u}_{\text{NIF}}\, d\mathcal{V} + \int_{CS} \rho \mathbf{u}_{\text{NIF}} (\mathbf{u}_r \cdot \hat{\mathbf{n}})\, dS + \frac{\partial}{\partial t}\int_{CV} \rho \mathbf{u}_{/\text{NIF}}\, d\mathcal{V} + \int_{CS} \rho \mathbf{u}_{/\text{NIF}} (\mathbf{u}_r \cdot \hat{\mathbf{n}})\, dS \\
&= \int_{CV} \frac{d\mathbf{u}_{\text{NIF}}}{dt} \rho\, d\mathcal{V} + \mathbf{u}_{\text{NIF}} \left(\frac{\partial}{\partial t}\int_{CV} \rho\, d\mathcal{V} + \int_{CS} \rho (\mathbf{u}_r \cdot \hat{\mathbf{n}})\, dS \right) \\
&\quad + \frac{\partial}{\partial t}\int_{CV} \rho \mathbf{u}_{/\text{NIF}}\, d\mathcal{V} + \int_{CS} \rho \mathbf{u}_{/\text{NIF}} (\mathbf{u}_r \cdot \hat{\mathbf{n}})\, dS
\end{aligned}$$

Noting that the term between brackets is equal to zero by conservation of mass, we get

$$\sum \mathbf{F} - \int_{CV} \mathbf{a}\, dm = \frac{\partial}{\partial t}\int_{CV} \rho \mathbf{u}_{/\text{NIF}}\, d\mathcal{V} + \int_{CS} \rho \mathbf{u}_{/\text{NIF}} (\mathbf{u}_r \cdot \hat{\mathbf{n}})\, dS$$

[1] For large scale problems such as in oceanography and meteorology, earth is actually a non-inrtial reference frame, but for small scale problems, it is ok to assume earth as an inertial reference frame.