MECH 314 Introduction to Fluid Mechanics February 16, 2009
American University of Beirut, Spring 2009 Handout # 1

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**Continuum Viewpoint and the Defining Aspect of a Fluid, and other stuff**

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1. The concept of a fluid: *matter that undergoes continuous deformation under the action of tangential stress.*

2. The continuum hypothesis and the underlying assumptions: check the handout on the continuum hypothesis.

3. Flow description: Lagrangian vs. Eulerian. Streamlines, streaklines, and pathlines. *Lagrangian description* of a fluid flow involves determination or measurement of properties of fluid particles as they move in time. Since fluid particles in general move relative to each other, Lagrangian flow description involves dividing the body of fluid into a large number of particles. Each particles $i$ is then characterized by $\mathbf{x}_i(t), \mathbf{u}_i(t), \rho_i(t), T_i(t)$, etc. For most applications, Lagrangian description imposes great difficulties on mathematical analysis. It is however very useful to employ the Lagrangian description in the physical understanding of fluid mechanics by asking questions such as: what are the forces acting on a small fluid particles as it moves? For certain flow conditions, what are the properties that are conserved as the particle moves?. Lagrangian description is also employed in various computational fluid dynamics methods such as vortex methods. The trajectory traversed by a fluid element is a *pathline*.

   Alternatively one could describe fluid flow by monitoring the properties at fixed locations. In *Eulerian description*, all properties such as velocity, temperature, pressure etc. are function of time and space, $\Pi = \Pi(\mathbf{x}, t)$, where $\mathbf{x}$ is the position vector in the system of coordinates. Both $t$ and $\mathbf{x}$ are independent variables in this case. This description enables use of the powerful tools of vector calculus for predicting flow behavior. At a certain time instant, a line whose tangent is everywhere parallel to the velocity is a *streamline*. Streamlines are solutions of the equation:

$$\frac{dx}{u(\mathbf{x}, t)} = \frac{dy}{v(\mathbf{x}, t)} = \frac{dz}{w(\mathbf{x}, t)}$$

   Now since Lagrangian description is close to physical intuition, it is useful to convert results obtain from Lagrangian description to Eulerian description. Here are two examples. In the first example, we know that the time rate of change of a property $\Pi$ associated with a small fluid element as it moves over time of $dt$ is $d\Pi/dt$. Now how can we represent the time rate of change of $\Pi$ associated with a moving particle in Eulerian description? In Eulerian description $\Pi = \Pi(x, y, z, t)$ so that $d\Pi =$

$\frac{\partial \Pi}{\partial t}dt + \frac{\partial \Pi}{\partial x}dx + \frac{\partial \Pi}{\partial y}dy + \frac{\partial \Pi}{\partial z}dz$, where $dx, dy, dz$ are respectively the $x, y, z$ components of the distance traveled by the particle over time $dt$. Dividing by $dt$, we get $\frac{d\Pi}{dt} = \frac{\partial \Pi}{\partial t} + \frac{\partial \Pi}{\partial x}\frac{dx}{dt} + \frac{\partial \Pi}{\partial y}\frac{dy}{dt} + \frac{\partial \Pi}{\partial z}\frac{dz}{dt}$. Therefore

$$\frac{d\Pi}{dt} \equiv \frac{D\Pi}{dt} = \frac{\partial \Pi}{\partial t} + (\mathbf{u} \cdot \nabla)\Pi$$

where the differential operator $D$ is used to remind us that it denotes changes in the particle property as the particle moves. $D/Dt$ is called substantial derivative, or material derivative, or total derivative.

In the second example, we know that in Lagrangian description the mass of a small fluid element is conserved as it moves in time, i.e. $\frac{D(\delta m)}{Dt} = 0$ or $\frac{D(\rho\,\delta V)}{Dt} = 0$. Luckily the chain rule applies for $D$ so that $\delta V \frac{D\rho}{Dt} + \rho \frac{D(\delta V)}{Dt} = 0$ or $\frac{D\rho}{Dt} + \rho \frac{1}{\delta V}\frac{D(\delta V)}{Dt} = 0$. It may be shown that $\lim_{\delta V \to 0} \frac{1}{\delta V}\frac{D(\delta V)}{Dt} = \nabla \cdot \mathbf{u}$, so that the continuity equation (differential form of the conservation of mass) is obtained

$$\begin{aligned} &\frac{D\rho}{Dt} + \rho\,\nabla \cdot \mathbf{u} = 0 \\ \text{or} \quad &\frac{\partial \rho}{\partial t} + (\mathbf{u} \cdot \nabla)\rho + \rho\,\nabla \cdot \mathbf{u} = 0 \end{aligned}$$

A *streak line* consists of all the fluid elements that at some earlier instant passed through a certain point in space; thus when a dye is discharged slowly at some fixed point in a moving fluid, the visible line produced in the fluid is a streak line.

4. Thermodynamic properties of a fluid. Transport properties of a fluid: viscosity and thermal conductivity. Newtonian vs. Nonnewtonian fluids.

5. Forces acting on a continuum: body forces, surface forces, and surface tension.

**Surface Tension**
Surface tension gives rise to various familiar phenomena in liquid behavior such as the formation and rise of soap bubbles, liquid level in a capillary tube rising above that of the pool in which the tube is placed, the breakup of a jet of water into droplets. Surface tension physics is currently being employed in some very exciting technologies such as ink jet printing and drug delivery. What is gives rise to surface tension?

Whenever there is an interface between a liquid and a gas or a liquid and a solid. The liquid atoms close the interface experience uneven attractive forces. For example, in a liquid-gas interface, the liquid molecules at the interface experiences stronger attractive forces from the liquid molecules neighboring them that from neighboring gas molecules. As a result the liquid molecules at the interface are attracted inward and normal to the interface. This uneven force distribution causes the interface to experience tension and the surface of the liquid behaves as if it were in tension like a stretched membrane. From energy consideration, the liquid molecules near the surface have higher energy than those interior since work must be done to bring a molecule from the interior to the surface. Since the free energy of a system always tends to a minimum, the interface must tend to a minimum, i.e. contract.

Surface tension, having the units of force per unit length (or energy per unit area) is given by

$$\sigma = \frac{\partial H}{\partial A}$$

for constant temperature and volume, where $H$ is the Helmholtz free energy and $A$ is the surface area. Note that a decrease in both $H$ (minimizing the surface energy) and $A$ (surface contraction) yield a positive surface tension. The surface tension for water and mercury at 20 C is 0.073 and 0.4865 N/m respectively.

Now if we approach the problem of a liquid-vapor interface experiencing surface tension from a force balance point of view, we reach the conclusion that balancing force pointing outwards (from the inside of the concave surface to its outside) must exist. Indeed there will be a pressure difference across the interface with higher pressure on the inside of the curved surface. The relationship between surface tension and this pressure difference (Young-Laplace equation) is

$$\Delta p = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right)$$

where $R_1$ and $R_2$ are the principal radii of curvature of the surface (radii of curvature along any two orthogonal tangents.) This equation is applicable to arbitrarily shaped surfaces where the radii of curvature may change spatially. In the special case of a liquid (spherical) droplet of radius $R$, the pressure on the inside of the droplet is higher than that on the outside by $\Delta p = 2\sigma/R$.

6. Conservation laws: mass, momentum, energy. Relation to Newton's second law and laws of thermodynamics.

Read: White Chapter 1