<u>Math 201 - Final (Spring 15)</u>

T. Tlas

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- Write the answers to questions 2, 3 and 6 on their sheets. The other three questions have extra sheets for you to write your answers on them. Any part of your answers written on the wrong sheet will not be graded. Note that a sheet of paper has two sides, you can write on both of them.
- There are 6 problems in total. Some questions have several parts to them. Make sure that you attempt them all.
- This is a closed book exam and no calculators are allowed.

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Name :

ID # :

Section :

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$Q1$	
$Q2$	
$Q3$	
$Q4$	
$Q5$	
$Q6$	
$TOTAL$	

Problem 1

(5 points each) Which of the following series converge and which diverge? Those which converge, do they converge absolutely or conditionally? When possible find the sum of the series.

i-

$$\sum_{n=0}^{\infty} \frac{(-2)^n}{3^n n!}$$

ii-

$$\sum_{n=0}^{\infty} \frac{(-1)^{n+1}}{4^n}$$

iii-

$$\sum_{n=0}^{\infty} (-1)^n \frac{(n!)^2}{(2n)!}$$

iv-

$$\sum_{n=0}^{\infty} \frac{n^2+2}{n+3}$$

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ADDITIONAL SHEET FOR PROBLEM 1 ANSWER

Problem 2

(10 points) Evaluate the following integral

$$\int_0^1 \sin(x^2)dx$$

with an error less than 0.01. Is your answer an over- or an under-estimate?

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Problem 3

(16 points) Find the minimum value of the function $f(x, y) = x^2 + 4y^2$ subject to the constaint $y^2 - x^2 = 1$.

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Problem 4

(12 points each) Let $f(x,y) = x^2y$, set up the integral of this function in the orders $dxdy$, $dydx$ as well as in polar coordinates over the following two regions:

i- R_1 : The region is in the first quadrant and is below the line $y = 2$, to the left of the line $x = 3$, above the line $y = 0$ and outsite the circle $x^2 + y^2 = 1$.

ii- R_2 : The region is inside the circle $x^2 + y^2 = 4$ but to the left of the line $x = 1$.

Note that in this problem you are *not* required to calculate the integrals, only to set them up (i.e. indicate the limits of integration). Also, note that for each region you must set up the integral in three ways.

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ADDITIONAL SHEET FOR PROBLEM 4 ANSWER

Problem 5

(10 points each) Integrate the function $f(x, y, z) = \frac{1}{\sqrt{x^2+y^2}}$ over the following two regions:

i- R_1 is the region bounded from above by $z = 2$, from below by $z = 0$ and on the sides by $x^2 + y^2 = 1$.

ii- R_2 is the region in the first octant inside the sphere $x^2 + y^2 + z^2 = 1$ but above the cone $x^2 + y^2 - z^2 = 0$.

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ADDITIONAL SHEET FOR PROBLEM 5 ANSWER

Problem 6

(10 points) Suppose you have a semi-infinite rod along the positive x-axis whose mass density is given by $\rho(x)$. Is it possible to choose ρ in such a way that the center of mass of the part of the rod from a onwards is located at $a + L$, for any a? In other words, is it possible to choose ρ such that, if you cut the rod somewhere, then, no matter where the cut was made, the infinite piece has its center of mass a distance L from the cut?

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