MECH 314, SPRING 2010, Drop Quiz 1 - Section 3

Problem 4.17 (White)

(a) The x-component of the velocity inside a boundary layer may be approximated as

$$u = U\left(\frac{2y}{\delta} - \frac{y^2}{\delta^2}\right)$$

where U is the free stream velocity and δ is the boundary layer thickness, $\delta = C\sqrt{x}$ where $C = 5\sqrt{\nu/U}$, where $\nu = \mu/\rho$, μ being the viscosity. Noting that Reynolds number is $\mathtt{Re}_x = Ux/\nu$, we get $\delta/x = 5/\sqrt{\mathtt{Re}_x}$.

For constant density, the conservation of mass, in differential form, reduces to $\nabla \cdot \mathbf{u} = 0$, which in 2D is expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Then

$\int_0^y \frac{\partial v}{\partial y}\, dy = -\int_0^y \frac{\partial u}{\partial x}\, dy$ leading to

$v = U\left(\frac{y^2}{2Cx^{3/2}} - \frac{y^3}{3C^2x^2}\right) = U\frac{5}{\sqrt{\mathtt{Re}_x}}\frac{y^2}{\delta^2}\left(\frac{1}{2} - \frac{y}{3\delta}\right)$ for $y < \delta$ and $v = 0$ outside the boundary layer.

(b) Note that the vertical component of the velocity reaches a maximum at $y = \delta$. Note also that $v = 0$ for $y > \delta$, which implies that v is discontinuous at $y = \delta$. The u and v velocity profiles are plotted in Figure 1. For more accurate solution of the boundary layer problem, refer to Sclichting, Boundary Layer Theory.

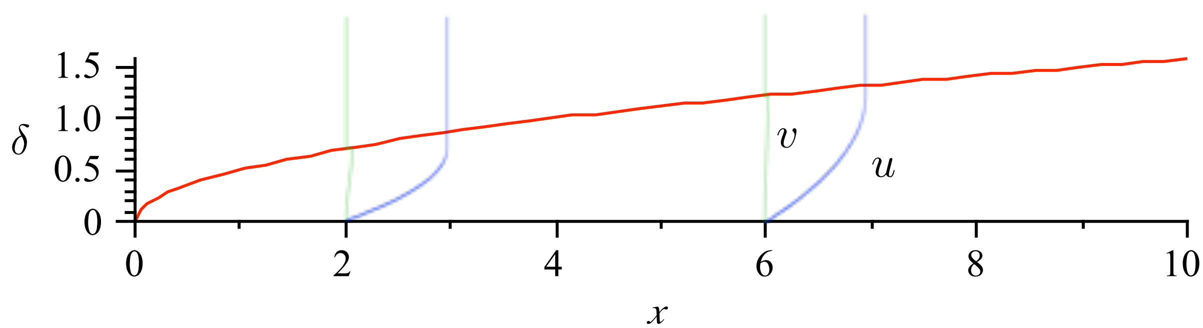

Figure 1. Growth of boundary layer thickness with $x, (U = 1, \nu = 0.01)$

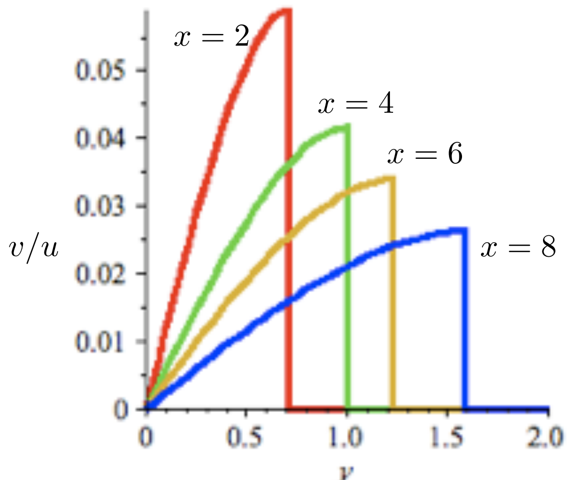

Figure 2. v/u at different downstream locations. Notice how small v is compared to u, especially at large x. Notice also the discontinuity in the profile due to the approximate u profile given to you.

(c) The streamlines, shown in Figure 3, are funny looking. They intersect! This is again because of the approximate velocity profile given to us.

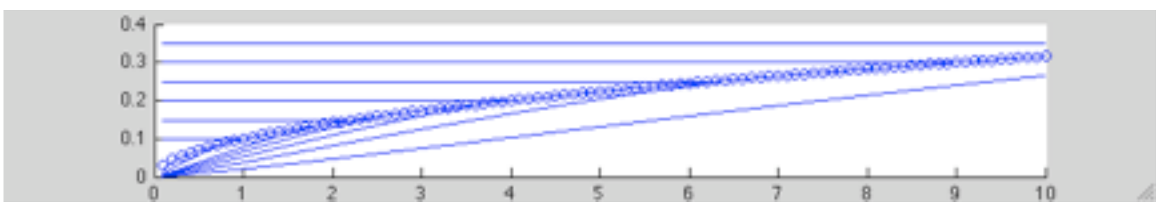

Figure 3. Streamlines (note the boundary layer edge).