MECH 314, SPRING 2010, Drop Quiz 1 - Section 4

Problem 4.18 (White)

Conservation of mass in differential form is

$\frac{D\rho}{Dt} + \rho\nabla\cdot\mathbf{u} = 0$

or

$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\mathbf{u}) = 0$

Noting that $\rho = \rho(t)$ and $\mathbf{u} = u(x,t)\hat{\mathbf{x}}$ where $u(x,t) = V(1 - x/L(t))$, then $\frac{d\rho}{dt} + \rho\frac{\partial u(x,t)}{\partial x} = 0 \Rightarrow \frac{d\rho}{\rho} = \frac{V}{L(t)}dt$

Note, however, that $\rho_0 L_0 = \rho L$, then $\frac{d\rho}{\rho^2} = \frac{V}{\rho_0 L_0}dt$

Integrating from $t = 0$, at which $\rho = \rho_0$ to t, we get $\frac{1}{\rho_0} - \frac{1}{\rho} = \frac{Vt}{\rho_0 L_0}$ so that

$\rho = \frac{\rho_0}{1-Vt/L_0}$.

Note that at $t = 0$, $\rho = \rho_0$ and when $t = L_0/V$ we get $\rho = \infty$, why?