MECH314 QUIZ 2
CLOSED BOOK – NO FORMULAE SHEET ALLOWED

If needed, consider the atmospheric pressure P_{atm} equivalent to 100 kPa, $g = 9.8\ m/s^2$, and the water density as $1000\ kg/m^3$.

Problem 1 [40 pts]

The water syringe shown in the figure is similar to those used in the medical field for intravenous injections. For the current position shown with H = 5 cm, what should the applied plunger force **F** be to achieve a volumetric flow rate of 15 ml/sec. (1 ml) is equivalent to ($1\times10^{-6}\,m^3$). You may neglect friction in the system. The cylinder diameter is D_{cyl} = 20 mm and the jet dia. D = 1 mm.

Problem 2 [40 pts]

A number of FEA students have suggested that AUB should install a water fountain next to the Bechtel Building to beautify Lower Campus. It is expected that the fountain will be continuously operated only during school days (approx. 150 days per year). In one suggested design, the fountain will have 20 identical, vertical jets similar to the one shown in the figure – with each jet driven by its own pump. The electrically powered pump transfers water from a constant level reservoir to the fountain as shown. The jet leaves the pipe end with a diameter D = 12 mm, and velocity V = 20 m/s. Take H_1 to be 185 cm, H_2 100 cm, and H_3 75 cm. The friction head loss in the whole system is approximately estimated to be 45 cm.

A. Estimate the power consumption for one pump.
B. Compute the electric power consumption per pump if the pump efficiency is 80%.
C. What are the electricity costs in USD for running this 20-jet fountain over one year. It costs AUB approximately 3 cents per kilo Watt-Hour (kW-h). One USD is equivalent to 100 cents.

Problem 3 [20 pts]

a) One can use the Navier Stokes equation to study the flow details inside a pump or a fan like the one shown in the figure. Write the appropriate boundary conditions at points A and B. Point A is on the tip of the rotating blade and point B lies on the stationary casing.
b) Illustrate the difference between *local* and *convective* accelerations in a fluid flow.

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USEFUL FORMULAS

Bernoulli: $p_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2$

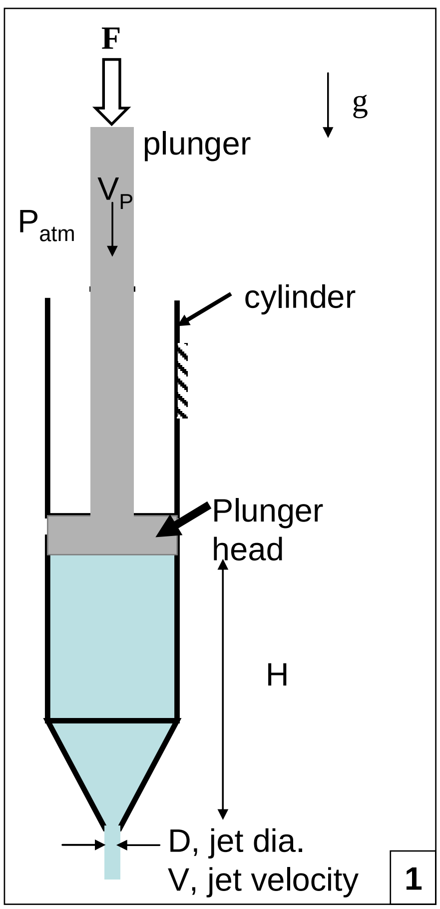
Energy Equation: $p_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = p_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2 + \rho g h_f + \rho g h_t - \rho g h_p$

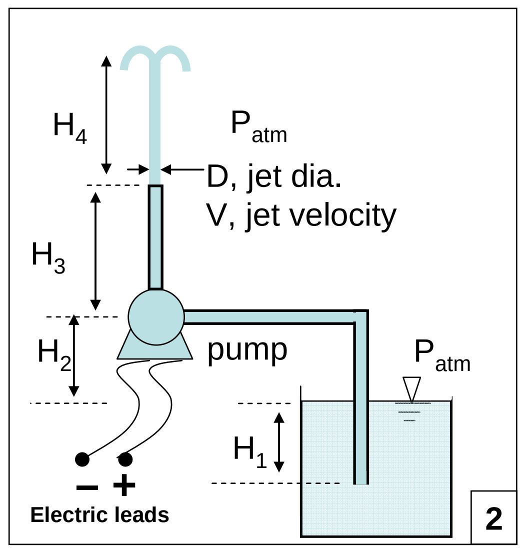
Efficiency: $\eta_{pump/turbine} = \dfrac{useful\ power\ output}{paid\ power\ input}$

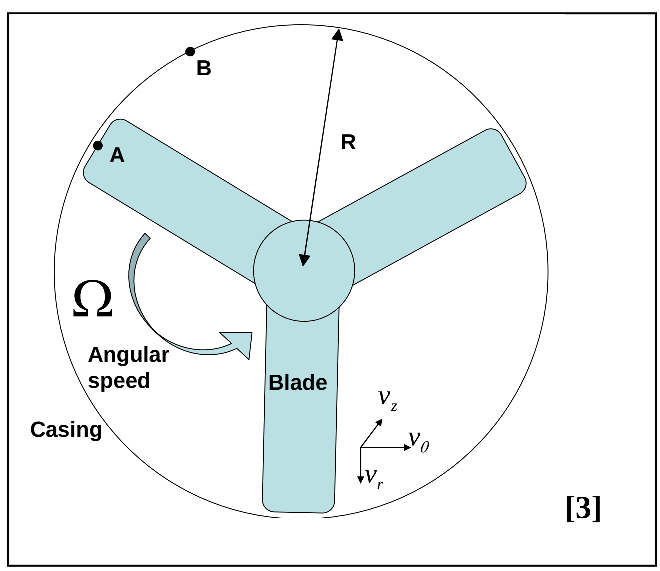
Mass conservation: $\dfrac{\partial (m_{cv})}{\partial t} = \sum_{inlet} \dot{m} - \sum_{outlet} \dot{m}$

Steady momentum conservation: $\sum \vec{F} = \sum_{outlet} \dot{m}\vec{V} - \sum_{inlet} \dot{m}\vec{V}$

Power = torque x angular speed = force x velocity = Q x Δp; where Q: volumetric flow rate







$\underline{P3}$

(a) @ A no slip:

$$v_{r_A} = v_{A_{\text{Blade tip}}}$$

$$v_{\theta_A} = v_{\text{Blade tip}} = R\Omega$$

$$v_z = v_{\text{Blade tip}}$$

B $\quad v_{r_B} = v_{\theta_B} = v_{z_B} = 0$

(b) Convective acceleration: change in velocity of fluid ~~due~~ with spatial distance

Local: change in velocity with time.

(P2) Energy Conservation for CV

(a) Inlet: free surface of reservoir $V_{in}=0$, $P_{in}=P_{atm}$, $z_{in}=0$

Outlet: jet exit; $V_{out}=20\frac{m}{s}$, $P_{out}=P_{atm}$, $Z_{out}=H_2+H_3$

$$\cancel{P_{in}+\frac{1}{2}\rho V_{in}^2+\rho g z_{in}} = \cancel{P_{out}}+\frac{1}{2}\rho V_0^2+\rho g z_{out}+\rho g h_f-\rho g h_p$$

$$0=\frac{1}{2}\times 1000\times 20^2+1000\times 9.8\times(1.75+0.45)-1000\times 9.8\times h_p$$

$h_p = 22.6$ m pump head

$$\Delta P_{pump} = \rho g h_p = 1000\times 9.8\times 22.6 = 221.56\ kPa$$

$$Power = Q\cdot\Delta P = (VA)\,\Delta P_{pump}$$

$$= 20\times\frac{\pi}{4}\times(12\times10^{-3})^2\times 221.56\times10^3$$

$= 501.2$ Watts per pump

(b) ~~Ppump~~ $P_{electric} = \frac{P_{ump}}{\eta} = \frac{501.2}{0.8} = 626.4\ W$

(c) $Cost = kW\text{-}h \times \frac{\$}{kWh} = time\times Power\times\frac{\$}{kW\text{-}h}$

$$cost = (626.4\times10^{-3})\,kW\times(150\times24\ h)(20\ jets)\times 0.03\ \frac{\$}{kWh}$$

$= 1353.1$ \$ per year

$D_1 = 20\ mm\ ,\ D_2 = 1\ mm$

P1 $\quad V_1 D_1^2 = V_2 D_2^2 \Rightarrow V_2 = V_1 \left(\frac{D_1}{D_2}\right)^2$

$\frac{\dot{m}}{\rho} = 15 \times 10^{-6}\ m^3 = V_1\ \frac{\pi}{4} D_1^2 = V_1\ \frac{\pi}{4} (20 \times 10^{-3})^2$

$V_1 = 0.0477\ \frac{m}{s}\ ;\ V_2 = 400 V_1 = 19.1\ \frac{m}{s}$

$F = P_1 A_1 - P_{atm} A_1$

Streamline 1-2 Bernoulli

$P_1 + \frac{1}{2}\rho V_1^2 + \rho g z_1 = P_2 + \frac{1}{2}\rho V_2^2 + \rho g z_2$

$P_1 + \frac{1}{2} \times 1000 \times (4.77 \times 10^{-2})^2 + 1000 \times 9.8 \times 0.05$

$= P_{atm} + \frac{1}{2} \times 1000 \times (19.1)^2 + 0$

$P_1 - P_{atm} = 181.9 \times 10^3\ Pa$

$F = 181.9 \times 10^3 \times \frac{\pi}{4}(0.02)^2 = 57.1\ N$

$\equiv 5.8$ kg force