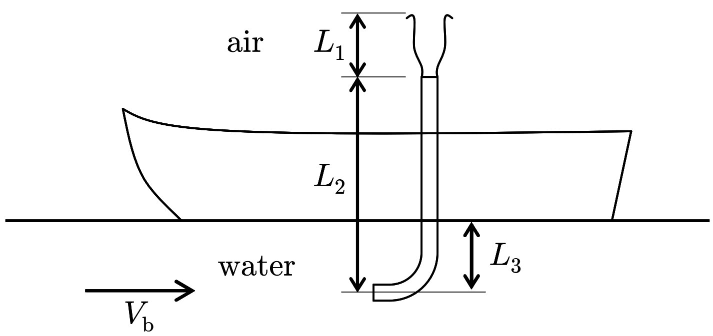
Quiz 2

- This exam is an open book exam.
- You have to solve three out of the four problems included in this exam.
- Please write down the name of your instructor.
- You have 90 minutes.

Problem 1 (33%)

An inventor has suggested an inexpensive, simple method for determining the speed at which a small motor boat is traveling. His invention is nothing more than a pipe with a 90^o elbow at the lower end. When placed in the water, as shown in Figure, a fountain of water flows from the upper end of the pipe. To calculate the boat's speed, the maximum height L_1 that the water stream reaches above the end of the pipe is measured to be 1 m. In addition, the length of the pipe is $L_2 = 1.5$ m and the depth of the opening below the surface $L_3 = 0.5$ m are known. Considering the flow with respect to the boat to be steady, and neglecting the losses in the fountain, calculate the value of the boat speed V_b.



Problem 2 (33%)

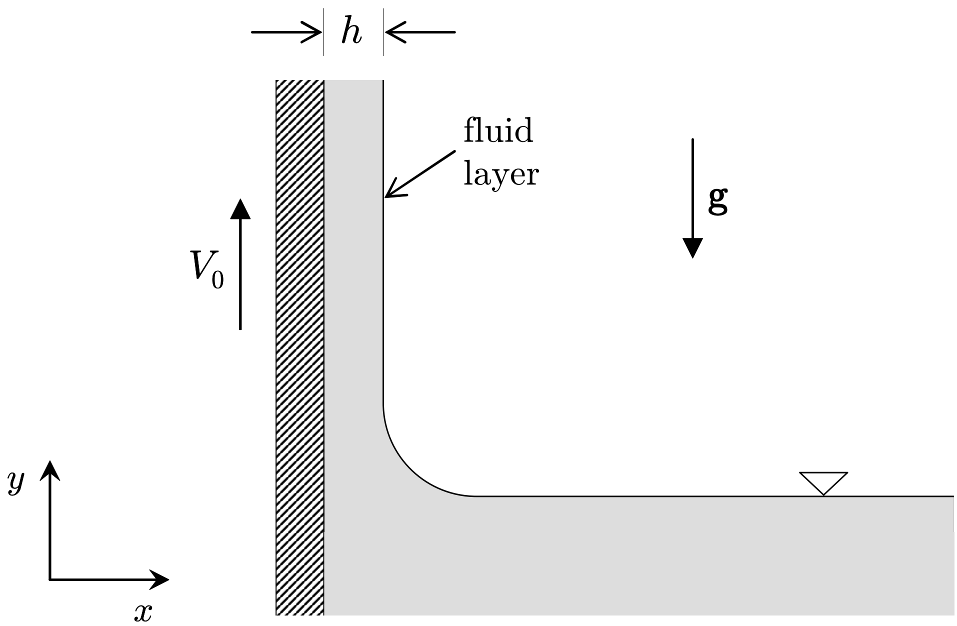
(a) The u and v velocity components of a three-dimensional, incompressible, steady-flow field are given by $u = x^3 + 2z^2$ and $v = y^2 - 3x^2y$. What is the general form of the w velocity component to satisfy continuity.

(b) A two-dimensional incompressible flow field is defined by the stream function

$$\psi = ax^2 - ay^2$$

where $a = 3\ \mathrm{s}^{-1}$. (i) Show that the flow is irrotational. (ii) Determine the velocity potential for this flow.

Problem 3 (33%)



A wide moving belt passes through a container of a viscous liquid. The belt moves vertically upward with constant velocity V_0 as shown. Because of viscous forces, the belt picks up a film of fluid of thickness h. Gravity tends to make the fluid drain down the belt. Assuming that the flow is steady and uniform (with v as the only velocity component in the y direction),

(a) use the Navier-Stokes equations to determine an expression for the average velocity (V) of the fluid film (flow rate per unit width $q = V\,h$) as it is dragged up the belt.

(b) What is the minimum value of V_0 if $\gamma = 8825$ N/m^3, $h = 0.01$ m, and $\mu = 0.85$ kg/m.s.

Problem 4 (33%)

A humming bird is a 1/50 linear scale model of an albatross. Both birds fly in a gravitational field g in air of density ρ_a, have the same average density ρ_b and store the same energy per unit mass of bird, ϵ. The side of each bird can be characterized by its wing span d.

(a) It has been suggested that the frequency f at which the bird flaps its wings should depend at most upon its mass M, ϵ, ρ_b, d, g and ρ_a. Using dimensional analysis, express a dimensionless flapping frequency of a bird in terms of a set of other dimensionless variables.

(b) By varying the gravity g and keeping all other independent variables fixed, it was determined that the flapping frequency is proportional to $\sqrt{g}$. Modify the expression you got in (a) to reflect this experimental fact. How does the flapping frequency depend upon the energy stored per unit mass ϵ ?

(c) At what frequency does a humming bird flap if an albatross flaps at 1 beat per second.