QUIZ [1]
SPRING 2012
(Tuesday, 27th March 2012)
CIVE410 – STRUCTURES II
CLOSED BOOK, 75 MINUTES

Name: SOLUTION ID#: HB

NOTES

- 3 PROBLEMS – 8 PAGES.
- READ PROBLEM STATEMENTS CAREFULLY.
- ASSUME ANY MISSING DATA YOU MAY REQUIRE. WRITE YOUR COMMENTS BELOW.
- ALL YOUR ANSWERS SHOULD BE PROVIDED ON THE QUESTION SHEETS.
- WELL ORGANIZED AND SYSTEMATIC SOLUTION WILL BE CREDITED. NEATNESS COUNTS.
- *TRY NOT* TO USE THE BACK OF THE SHEETS FOR ANSWERS UNLESS YOU RAN OUT OF SPACE.
- NOTE THE PERCENTAGE OF POINTS ASSIGNED FOR EACH PROBLEM.
- CHECK BOXES ARE FOR YOU TO CONFIRM YOU HAVE SOLVED A QUESTION ☑

YOUR COMMENT(S)

- -

- -

DO NOT WRITE IN THE SPACE BELOW

MY COMMENT(S)

- -

- -

YOUR GRADE

Problem I: ____ /20
Problem II: ____ /35
Problem III: ____ /45
Other: ____

Good Luck & Best Wishes
Dr. Hisham S. Basha

Problem I: *(20 points)*

Check *stability and determinacy* for the structures shown in the accompanying Figures.

Calculations and/or Diagrams:

[A] Two dimensional Frame :-

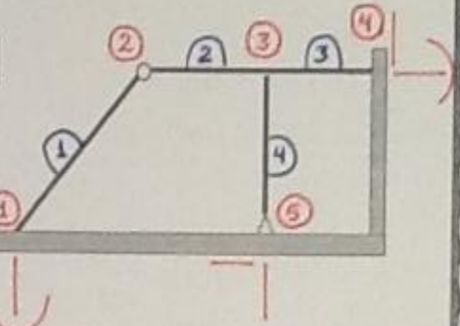
$i = (3m + r) - (3j + h)$.

$m = 4$, $r = 8$, $j = 5$, $h = 1$ $\Rightarrow$ $i = (12 + 8) - (15 + 1) = 4$

Geometrically Stable.

$\Rightarrow$ Stable and indet. of the 4^{th} Degree.

[A]



[B] Two dimensional Truss :-

$i = (m + r) - (2j)$.

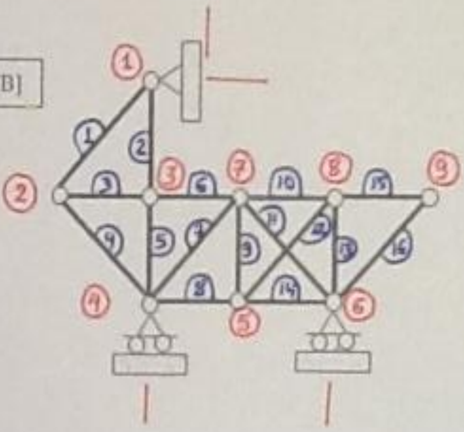
$m = 16$, $r = 4$, $j = 9$ $\Rightarrow$ $i = (16 + 4) - 18 = 2$

Geometrically Stable

$\Rightarrow$ Stable and indet. of the 2^{nd} degree

Once externally & once internally?

[B]



[C] Two dimensional Frame :-

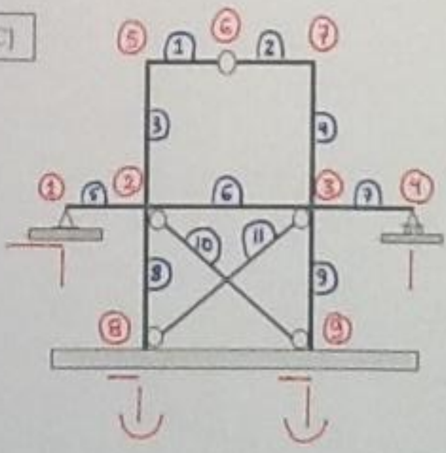
$i = (3m + r) - (3j + h)$.

$m = 11$, $r = 9$, $j = 9$, $h = 5$ $\Rightarrow$ $i = (33 + 9) - (27 + 5) = 42 - 32 = 10$

Geometrically Stable.

$\Rightarrow$ Stable and indet. of the 10^{th} degree $\Rightarrow$ (6 ext. & 4 int.).

[C]



Problem [2] (35%)

[A] The Post-and-Bracket ABCD shown in Figure (II-a) is a composite system consisting of Column ABC and a truss CDB as shown. The structure is subjected to the given loads. Neglect axial deformation in column ABC.

- [1] Determine the vertical displacement of joint *D*. *(15 points)* ☐
- [2] Determine the horizontal displacement of joint *C*. *(10 points)* ☐

[B] Remove all loads on the truss. Determine the *vertical* displacement of joint *D* of the steel truss shown in the accompanying figure due to an increase of temperature in member *CD* only of 60°C and a fabrication error in member *CE* being *5 mm* too short. *(10 points)* ☐

Use $\alpha = 1.08(10^{-5})/°C$ and E= 200GPa (1GPa=10^6kN/m^2).

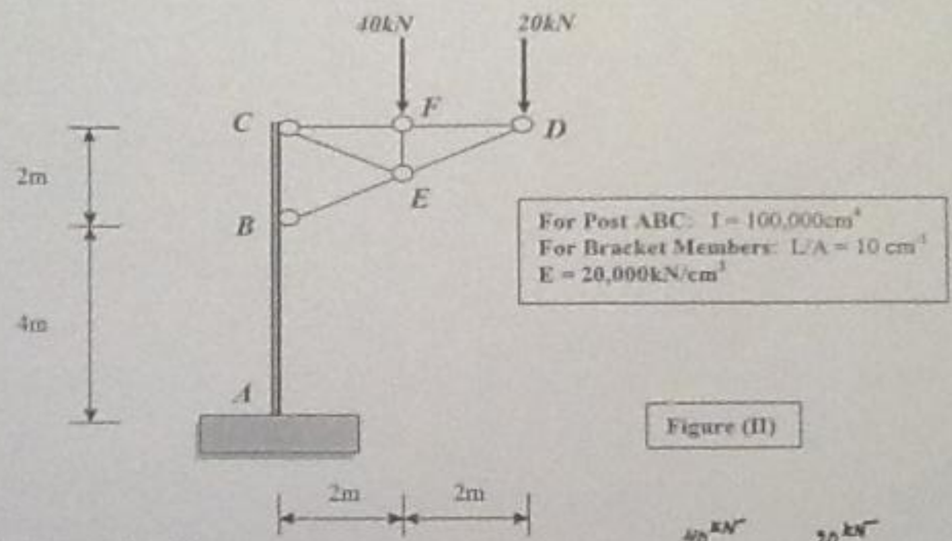
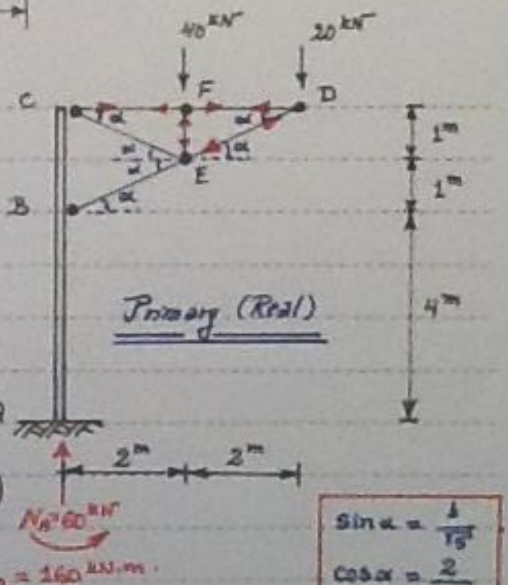


Figure (II)

Calculations and/or Diagrams:

[A] Structure is determinate.



Joint (D) ⇒ $F_{ED} \cdot \sin\alpha = 20$ kN

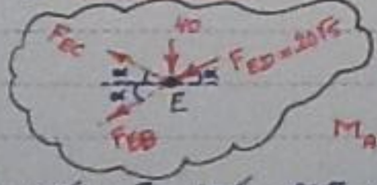
$F_{ED} = 20\sqrt{5}$ kN. (C).

$F_{FD} = F_{ED} \cdot \cos\alpha = 40$ kN (T).

Joint (F) ⇒ $F_{EF} = 40$ kN (C).

$F_{FC} = F_{FD} = 40$ kN (T).

Joint (E) ⇒



$\sin\alpha = \frac{1}{\sqrt{5}}$

$\cos\alpha = \frac{2}{\sqrt{5}}$

$\Sigma X = 0 \Rightarrow F_{EC} \cdot \cos\alpha + F_{EB} \cdot \cos\alpha + 20\sqrt{5} \cdot \cos\alpha = 0 \Rightarrow F_{EC} + F_{EB} = -20\sqrt{5}$ ...(1)

$\Sigma Y = 0 \Rightarrow F_{EC} \cdot \sin\alpha - 40 - 20\sqrt{5} \cdot \sin\alpha - F_{EB} \cdot \sin\alpha = 0 \Rightarrow (F_{EC} - F_{EB})\sin\alpha = 60$ ...(2)

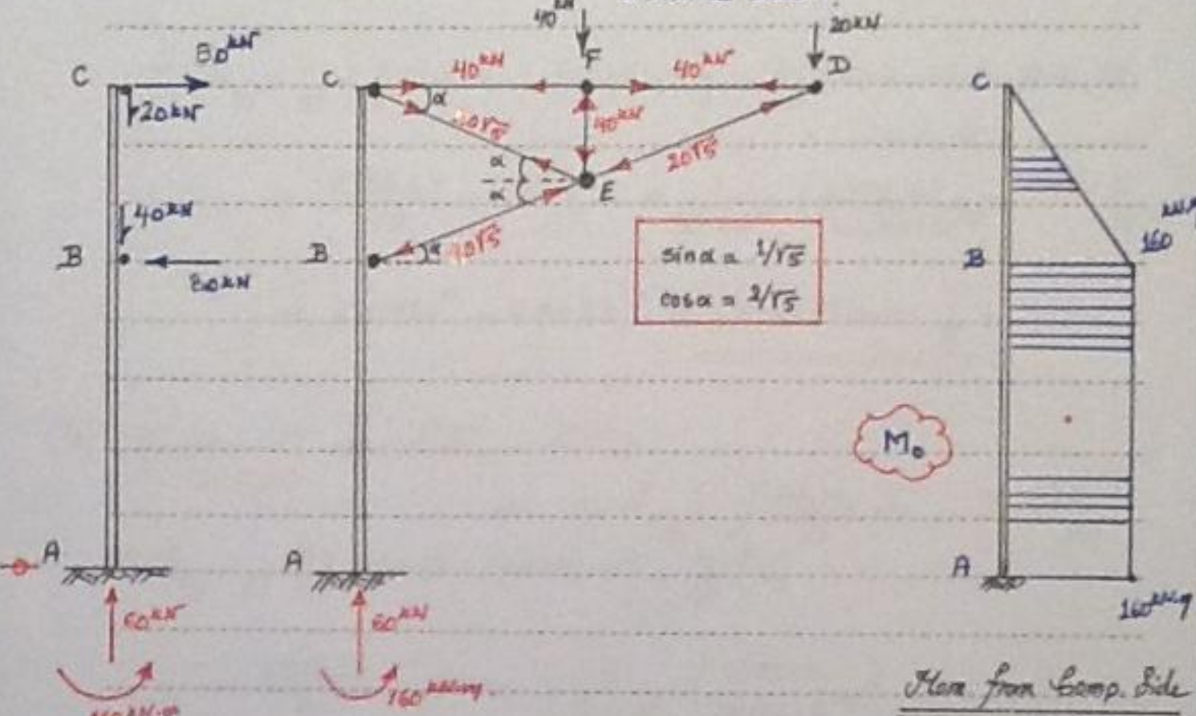
or $F_{EC} - F_{EB} = 60\sqrt{5}$ ...(2)

Calculations and Diagrams (cont'd):

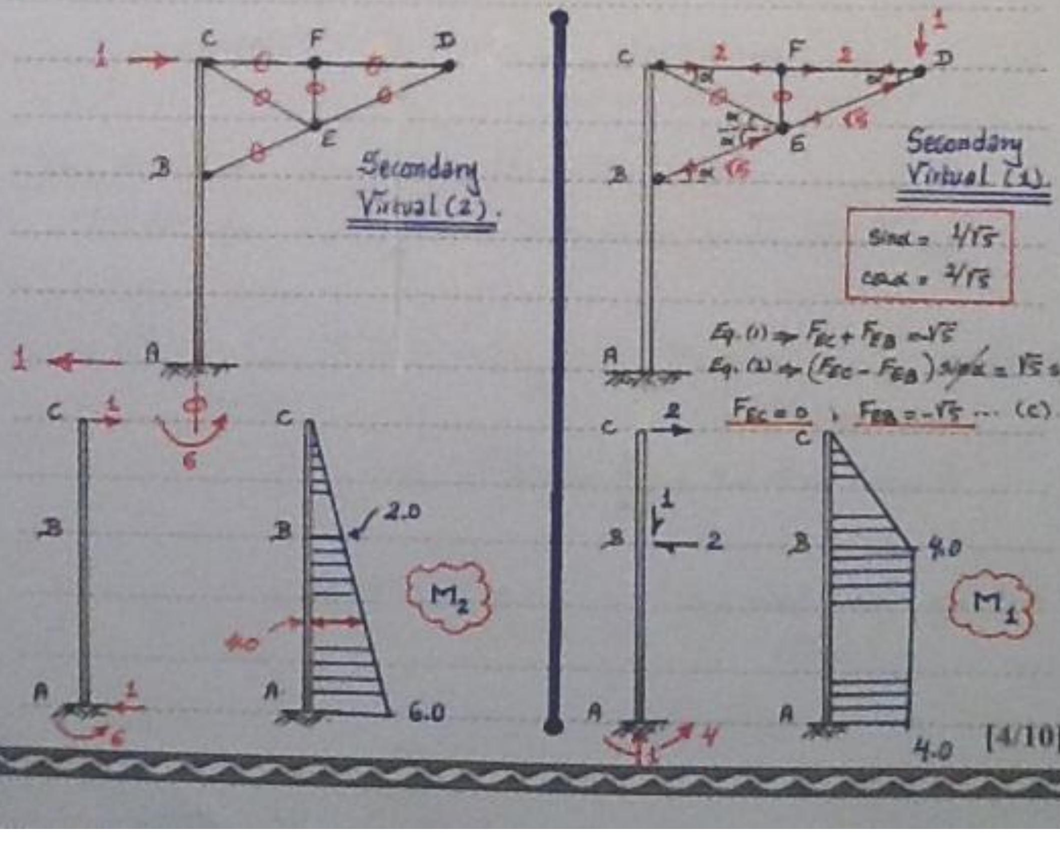
Eqns (1) & (2) $\Rightarrow$ $F_{EC} = 20\sqrt{5}$ .... (T).

substituting $\rightarrow$ $F_{EB} = \ominus 40\sqrt{5}$ $\Rightarrow$ $F_{EB} = 40\sqrt{5}$ .... (C).

indicates opp. to assumed sense!



Mom. from Comp. Side



Calculations and Diagrams (cont'd):

[1] Principle of Virtual Work.

System (1) $\Rightarrow \Sigma W_{ext} = \Sigma W_{int}$

$$\Rightarrow 1 \times \delta_{V_D} = \int \frac{M_0 M_1 \, dx}{EI} + \Sigma \frac{N_0 N_1 L}{EA}$$

$$\therefore \delta_{V_D} = \frac{1}{EI}\left\{\frac{2}{3}(160\times4) + (160\times4)\times4\right\} + \frac{L}{EA}\left(40\times2 + 40\times2 + 20\sqrt{5}\times\sqrt{5} + 40\sqrt{5}\times\sqrt{5}\right)$$

$$= \frac{2986.67}{EI} + \frac{460\times L}{EA} = \frac{1}{200\times10^6}\left(\frac{2986.67}{100{,}000\times10^{-8}} + 460\times\frac{10}{10^{-2}}\right)$$

$$= 0.01493^m + 0.0023^m = +0.01723\text{ m} \downarrow$$

[2] System (2) $\Rightarrow \Sigma W_{ext} = \Sigma W_{int}$

$$\Rightarrow 1 \times \delta_{H_D} = \int \frac{M_0 M_2}{EI} dx + \cancel{\Sigma \frac{N_0 N_2 L}{EA}}^{\,0}$$

$$\therefore \delta_{H_D} = \frac{1}{EI}\left\{\frac{2}{3}(160\times2) + (160\times4)\times4\right\} + \frac{L}{EA}(0)$$

$$= \frac{2773.33}{EI} + \text{Zero} = \frac{1}{200\times10^6}\left(\frac{2773.33}{100{,}000\times10^{-8}}\right)$$

$$= 0.01387^m = +0.01387\text{m} \rightarrow$$

[B] Due to Temp. Change & a Fabrication error.

$$\text{P.V.W} \Rightarrow 1 \times \delta_{V_D} = \Sigma \frac{N_1 \times \alpha . \Delta T . L}{EA} + \Sigma N_1 \times \delta L_{Fab_{(CE)}}$$

$$\Rightarrow \left(\delta_{V_D}\right)_{Temp\&Fab.} = 2\times2\times\alpha\times\Delta T\times\frac{L}{EA} + \underbrace{\text{Zero}}_{(N_1)_{CE} = \text{zero}} \qquad \Delta T = \oplus 60^\circ C$$

$$= \oplus 4\times1.08\times10^{-5}\times\oplus60\times L = 4\times1.08\times10^{-5}\times60\times2$$

$$= +5.184\times10^{-3}\text{ m} \downarrow$$

(very small, may neglect its effect).

Problem III: *(45 points)*

[a] For the steel beam shown in the accompanying figure (Figure III-a) use the Flexibility Approach (Virtual Work Method) for analysis of indeterminate structures and solve the following; knowing that $E = 20{,}000\ kN/cm^2$, and $I_{Beam} = 40{,}000\ cm^4$. Neglect axial deformation in Beam ABCD.

[1] Determine the support reactions at *B* and *C*.
Use the reactions at B and C as your redundant in your solution. *(25 points)* ☐

[2] Draw the Shearing Force Diagram (SFD) and the Bending Moment Diagram (BMD) for the beam *ABCD*. *(10 points)* ☐

[b] If the roller support at *B* is removed and replaced by a steel cable *BE* at *B*, as shown in figure (Figure III-b), determine the force in the cable *BE* and the reaction at *C*; knowing that $E = 20{,}000\ kN/cm^2$, Cross-sectional $Area_{(BE)} = 5\ cm^2$ and $I_{Beam} = 40{,}000\ cm^4$.
Neglect axial deformation in Beam ABCD only. *(10 points)* ☐

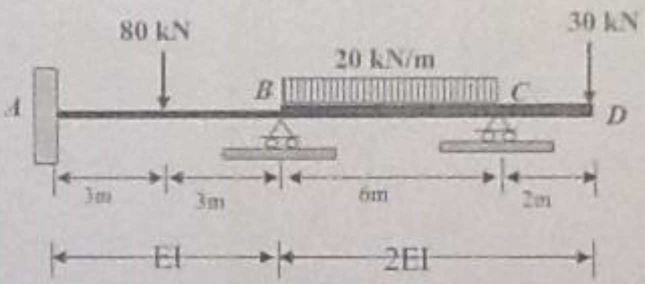


Figure (III-a)

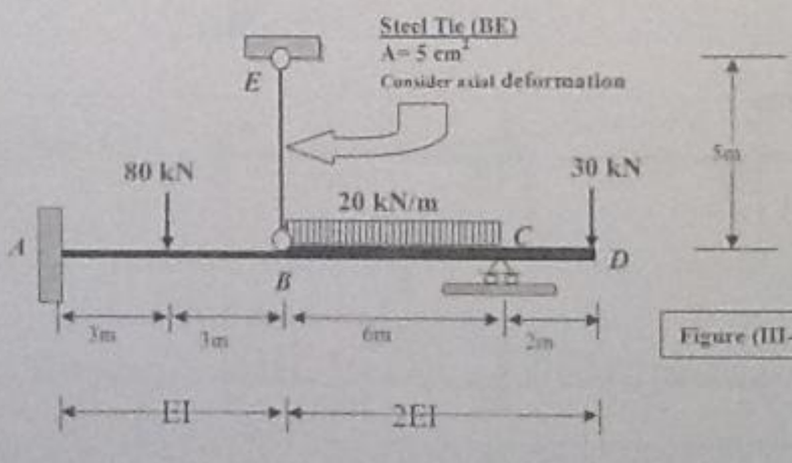


Figure (III-b)

Calculations and Diagrams:

[1] Release the supports at B and C.

⇒ Reactions at A;

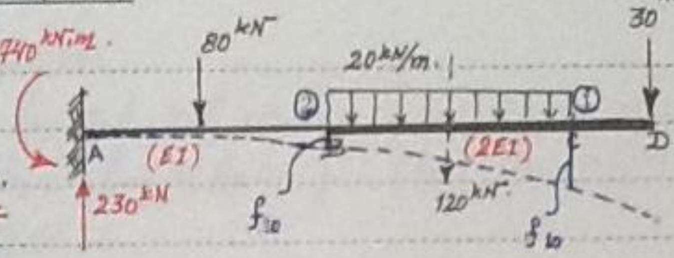
1) $\Sigma Y = 0 \Rightarrow Y_A = 230\ kN \uparrow$.

2) $\Sigma X = 0 \Rightarrow X_A = zero$.

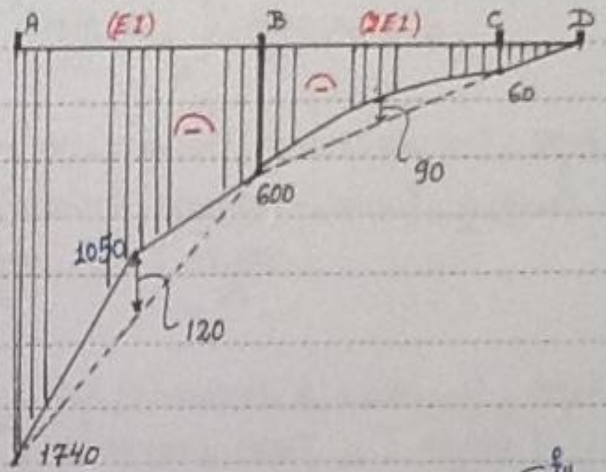
3) $\Sigma M_A = 0 \Rightarrow M_A = -1{,}740\ kN.m$.

Calculations and Diagrams (cont'd):

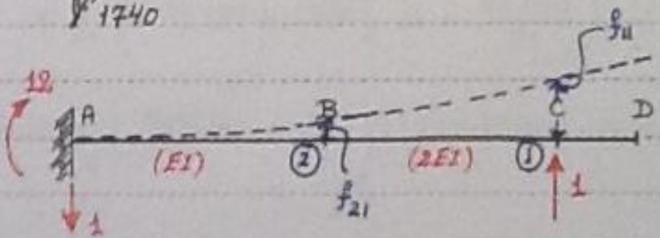
Released Main System.



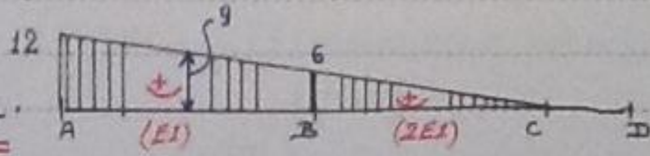
M_0 - Diagram.



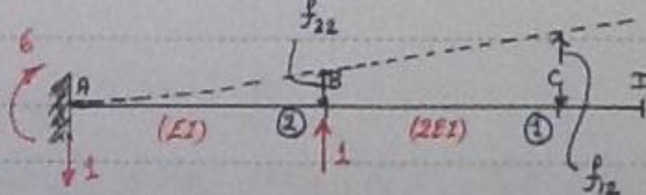
Virtual System [1]



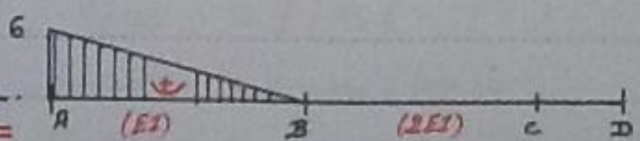
M_1 - Diagram.



Virtual System [2]



M_2 - Diagram.



Calculations and Diagrams (cont'd):

Compatibility Equations :-

$$\left.\begin{array}{l} f_{10}+x_1 f_{11}+x_2 f_{12}=0 \\ f_{20}+x_1 f_{21}+x_2 f_{22}=0 \end{array}\right] \Rightarrow \begin{Bmatrix} f_{10} \\ f_{20} \end{Bmatrix}+\begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}\begin{Bmatrix} x_1 \\ x_1 \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

where $f_{ij}=\int \frac{M_i M_j dx}{EI}$

$$\Rightarrow f_{10}=\int \frac{M_1 M_0\, dx}{EI}={}_A\int^B \frac{M_1 M_0\, dx}{EI}+{}_B\int^C \frac{M_1 M_0\, dx}{2EI}+0$$

$$=\frac{1}{EI}\left\{\frac{-\cancel{6}}{\cancel{6}}\left[2(1740)(12)+2(600)(6)+1740(6)+600(12)\right]+\frac{120\times6}{2}(9)\right\}$$
$$+\frac{1}{2EI}\left\{\frac{-\cancel{6}}{\cancel{6}}\left[2(600)(6)+2(\cancel{60})(0)+0+60(6)\right]+\frac{2}{3}(90\times6)\times3\right\}$$
$$=-\frac{63{,}360}{EI}+\frac{3{,}240}{EI}=-\frac{66{,}600}{EI}$$

$$f_{20}={}_A\int^B \frac{M_0 M_2\, dx}{EI}=\frac{1}{EI}\left\{\frac{-6}{6}\left[2(1740)(6)+0+0+600(6)\right]+\frac{120\times6}{2}\times3\right\}$$
$$=\frac{1}{EI}\left\{-24{,}480+1{,}080\right\}=-\frac{23{,}400}{EI}$$

$$f_{12}=f_{21}={}_A\int^B \frac{M_1 M_2\, dx}{EI}+{}_B\int^C \cancel{\frac{M_1 M_2\, dx}{2EI}}^{\,0}=\frac{1}{EI}\left\{\frac{6}{6}\left[2(12\times6)+2(0)+12(0)+6\times6\right]\right\}$$
$$=\frac{180}{EI}$$

$$f_{11}={}_A\int^B \frac{M_1 M_1\, dx}{EI}+{}_B\int^C \frac{M_1 M_1\, dx}{2EI}=\frac{1}{EI}\left\{\frac{6}{6}\left[2(12\times12)+2(6\times6)+2(12\times6)\right]+\frac{6}{3\times2}(6\times6)\right\}$$
$$=\frac{540}{EI}$$

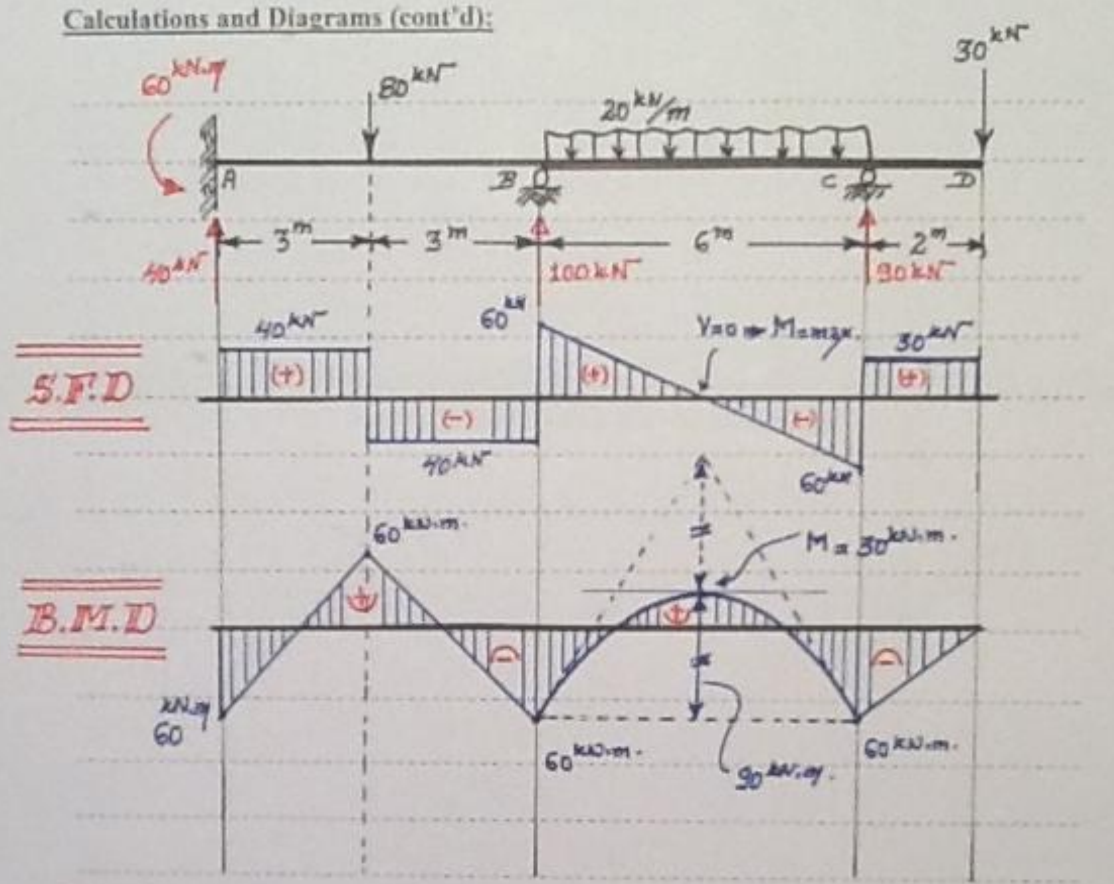
$$f_{22}={}_A\int^B \frac{M_2 M_2\, dx}{EI}=\frac{6}{3EI}(6\times6)=\frac{72}{EI}$$

$$\Rightarrow \frac{1}{EI}\begin{Bmatrix} -66{,}600 \\ -23{,}400 \end{Bmatrix}+\frac{1}{EI}\begin{bmatrix} 540 & 180 \\ 180 & 72 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix}=\begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Solving $\Rightarrow$ $x_1 = 90$ kN and $x_2 = 100$ kN.

$\Rightarrow$ $y_c = 90$ kN & $y_B = 100$ kN.

Calculations and Diagrams (cont'd):



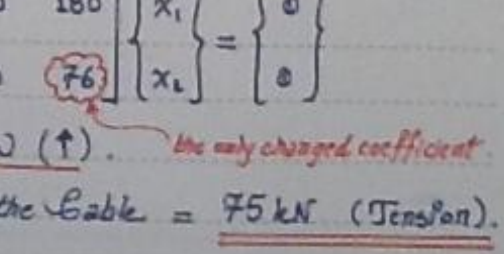
[b] As before; $f_{10} = -\frac{66,600}{EI}$; $f_{20} = -\frac{23,400}{EI}$, $f_{11} = \frac{540}{EI}$

$f_{12} = f_{21} = \frac{180}{EI}$; $f_{22} = \int_A \frac{M_2 M_2}{EI}\,dx + \Sigma\left(\frac{N_1 N_1 L}{EA}\right)_{Tie}$

only changed flexibility coef.

* So, $f_{22} = \frac{72}{EI} + \frac{(1)(1)(5)}{EA}$.

Since $EI = 80,000$ kN.m² and $EA = 100,000$ kN $\Big\}$ ⇒ $EA \simeq 1.25\,EI$.



Virtual System [2]

⇒ $f_{22} = \frac{72}{EI} + \frac{5}{1.25EI} = \frac{76}{EI}$

Then, $\left(\frac{1}{EI}\right)\begin{Bmatrix} -66,600 \\ -23,400 \end{Bmatrix} + \left(\frac{1}{EI}\right)\begin{bmatrix} 540 & 180 \\ 180 & 76 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$

the only changed coefficient

Solving ⇒ $x_1 = y_c = \underline{98.33 \text{ kN}}$ (↑).

and $x_2 = y_B$ = Force in the Cable = $\underline{75 \text{ kN}}$ (Tension).

[10/10]