American University of Beirut

Civil Engineering Dept. — Faculty of Eng. & Arch.

Structures II – CIVE410 -

Homework [3]

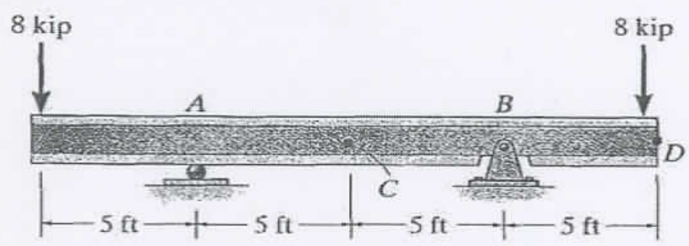
Spring 2011

Problem [1] *(Vers.6: page 377- Prob.9-59, 9-60)*

The steel beam with two overhanging ends, shown in the accompanying figure, has a modulus of elasticity $E=29,000ksi$ and moment of inertia $I=245in^4$. Using the method of virtual work, determine:

[a] the ***vertical displacement of point C***.

[b] the ***slope at point A of the beam***.



Problem [2] *(Vers.7: page 396- Prob.9-60, 9-61)*

The structure shown in the accompanying figure consists of a bar ***ABC*** with rectangular cross-section of *300mm by 100mm* attached to a rod ***DB*** with a diameter of 20mm (***DB*** acts as a truss member).

Consider only the effect of bending in ***ABC*** (neglect axial deformation), and consider the effect of axial force only in ***DB*** (neglect flexural deformation).

Use $E=200GPa$ and the method of virtual work to determine:

[a] the ***vertical displacement of point C***.

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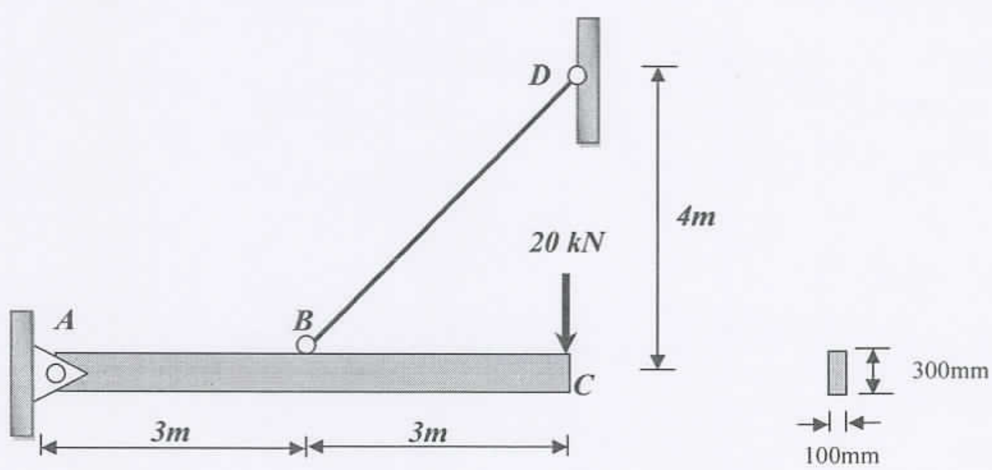
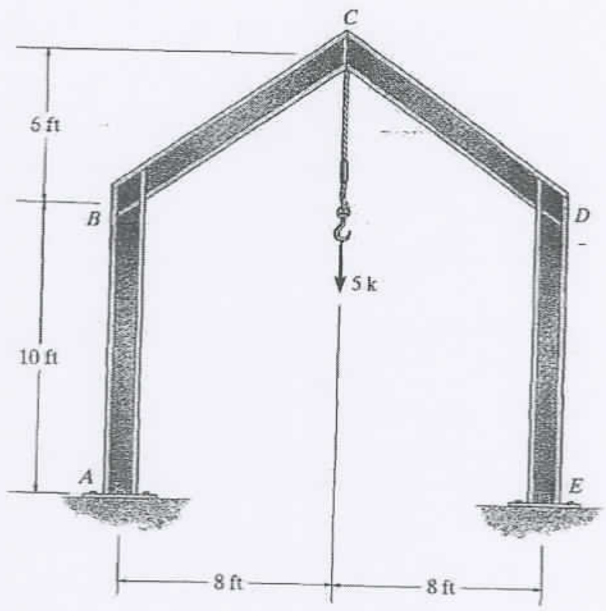


<u>Problem [3]</u> *(Vers6: page 381- Prob.9-97)*

For the three hinged frame shown, use the method of virtual work to determine ***<u>the vertical displacement of point C</u>*** due to the given loads. Assume that the members are pin connected at A, C, and E, and fixed connected at the knee joints B and D. Assume EI is constant. Consider only the effect of bending (neglect effect of axial deformation).



Good Luck & Best Wishes
Dr. Hisham S. Basha, Ph.D.

Problem [1] *(Vers.6: page 377- Prob.9-59, 9-60)*

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[a] the ***vertical displacement of point C***.

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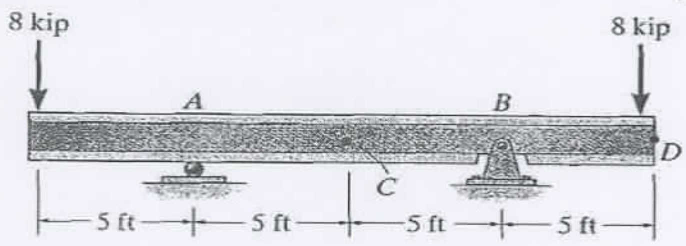
$EI = 7,105,000\ k.in^2$

$\Rightarrow EI = 49,340\ k.ft^2$.



Principle of Virtual Work (P.V.W)

$\Sigma W_{ext} = \Sigma W_{int}$.

Real System

[a] Consider Virtual System (1):

$\Rightarrow 1 * \Delta_{C_V} = \int \frac{M_0 M_1\, dx}{EI}$

$\Rightarrow \Delta_{C_V} = \frac{1}{EI}\left\{\frac{-2.5*10}{2} * 40\right\}$.

$= \ominus \frac{500}{EI}$. (opp. to assumed sense of dummy Load)

$\Rightarrow \boxed{\Delta_{C_V} = 0.01013' = 0.1216'' \uparrow}$

Virtual System (1)

[b] Consider Virtual System (2):-

$1 * \theta_A = \int \frac{M_0 M_2\, dx}{EI}$.

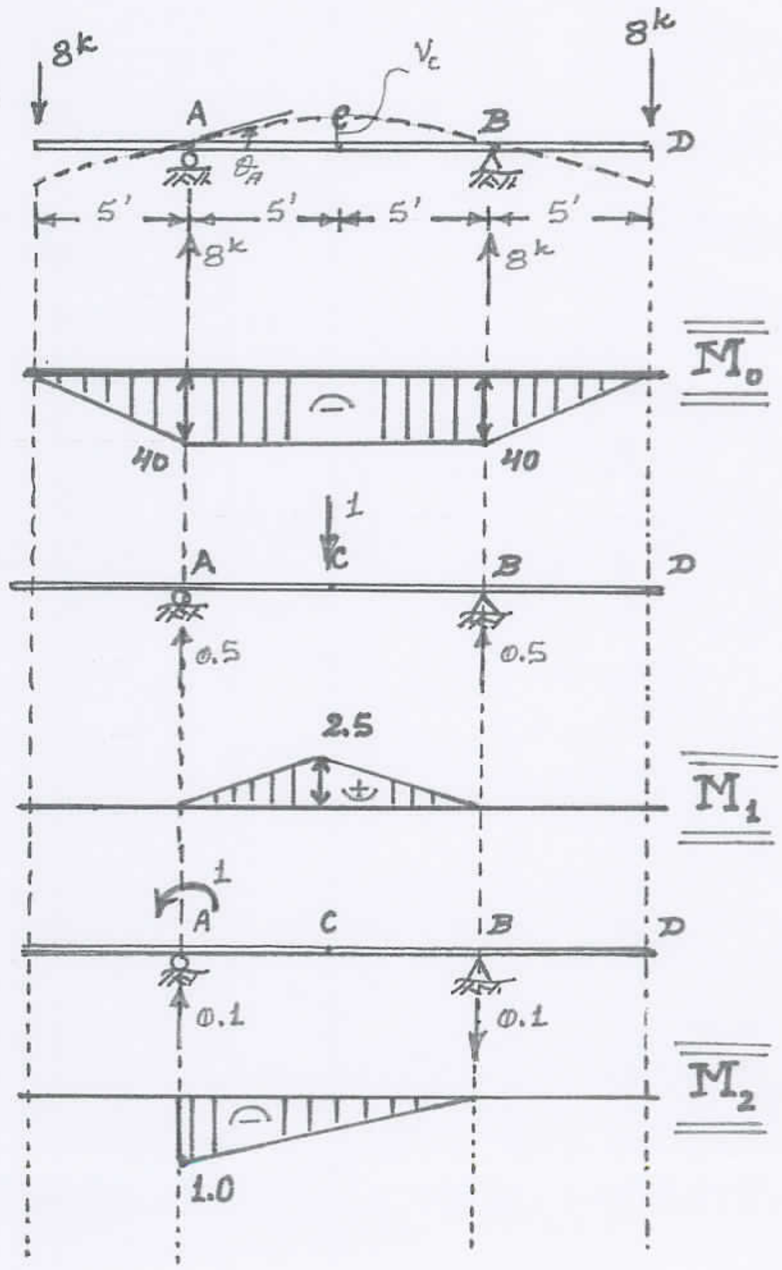
$\Rightarrow \theta_A = \frac{1}{EI}\left\{\frac{1*10}{2} * 40\right\}$.

$= \oplus \frac{200}{EI} = 4.05\times 10^{-3}$

(in the assumed sense).

Virtual System (2)

$\Rightarrow \boxed{\theta_A = 4.05\times 10^{-3}\ rd\ \ CCW}$



Problem [2] *(Vers.7: page 396- Prob.9-60, 9-61)*

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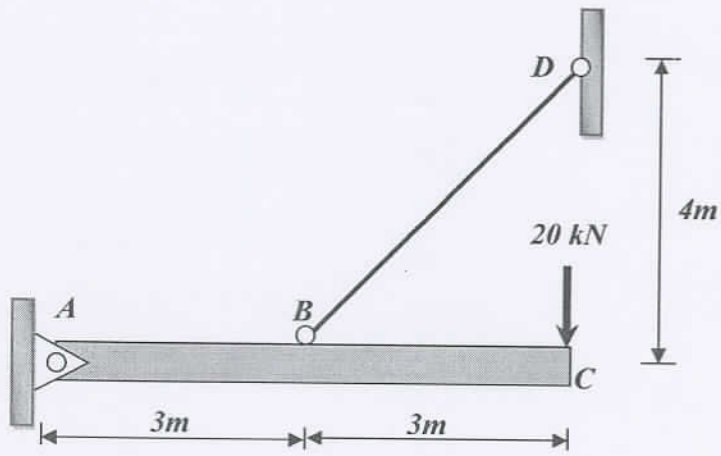


Beam ABC:-

$$I = \frac{(0.1)(0.3)^3}{12} = 2.25 \times 10^{-4} \; m^4$$

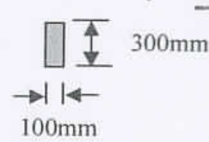
$$E = 200 \; GPa = 200 \times 10^6 \; \frac{kN}{m^2}$$

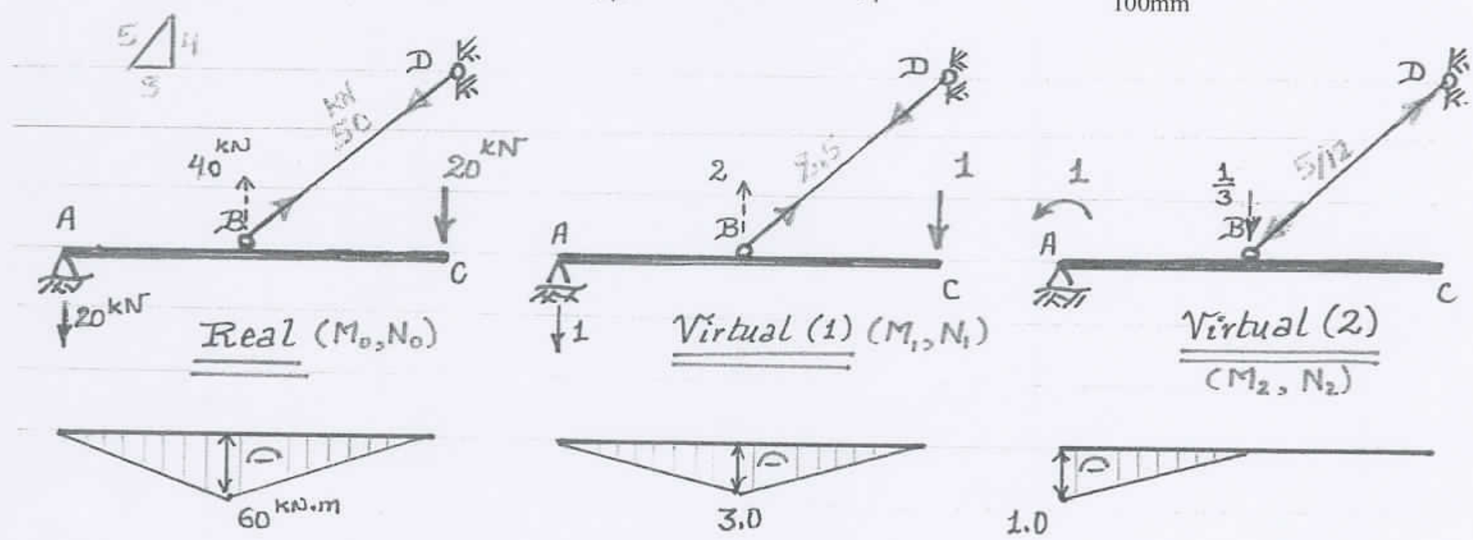
$$\Rightarrow EI = 45000 \; kN.m^2.$$

Truss BD:~

$$A = \frac{\pi (0.02)^2}{4} = 3.1416 \times 10^{-4} \; m^2$$

$$\Rightarrow EA = 62,832 \; kN$$





P.V.W ⇒ Consider Virtual System (1):~ $\Sigma W_{ext} = \Sigma W_{int}$.

$$\Rightarrow 1 * \Delta_{c_v} = \int \frac{M_0 M_1 \, dx}{EI} + \Sigma \frac{N_0 N_1 L}{EA} = \frac{2}{EI}\left[\frac{60 \times 3}{2} * \frac{2}{3} \times 3\right] + \frac{(50)(2.5)(5)}{EA}$$

$$\Rightarrow \Delta_{c_v} = \frac{360}{EI} + \frac{625}{EA}$$ where $EI = 45,000 \; kN.m^2$ & $EA = 62,832 \; kN$.

$$\Rightarrow \Delta_{c_v} = +0.01795 \; m = \boxed{+17.95 \; mm \downarrow}$$

P.V.W ⇒ Consider Virtual System (2):~ $\Sigma W_{ext} = \Sigma W_{int}$.

$$\Rightarrow 1 * \theta_A = \int \frac{M_0 M_2 \, dx}{EI} + \Sigma \frac{N_0 N_2 L}{EA} = \frac{1}{EI}\left[\frac{60 \times 3}{2} * \frac{1}{3}\right] + \frac{(50)(-5/12)(5)}{EA}$$

$$\Rightarrow \theta_A = \frac{30}{EI} - \frac{104.167}{EA} = -0.9912 \; rd * 10^{-3}$$

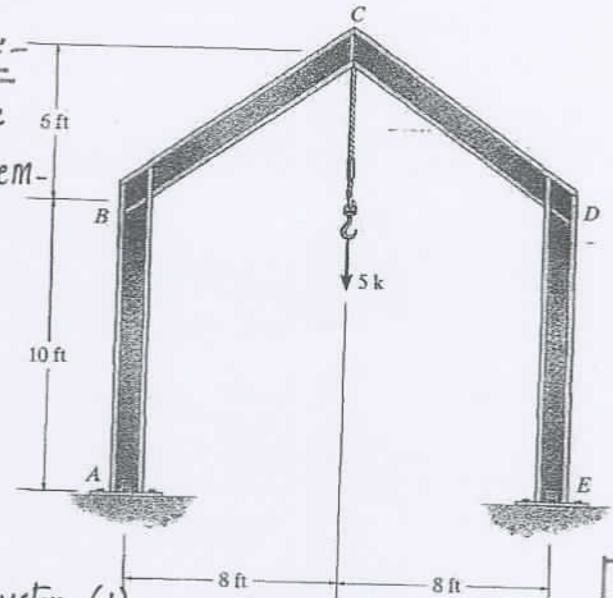
opp. to assumed sense of dummy moment. ⇒ $\boxed{\theta_A = 0.9912 * 10^{-3} \; rd \;\; C.W.}$

Problem [3] *(Vers6: page 381- Prob.9-97)*

For the three hinged frame shown, use the method of virtual work to determine ***the vertical displacement of point C*** due to the given loads. Assume that the members are pin connected at A, C, and E, and fixed connected at the knee joints B and D. Assume EI is constant. Consider only the effect of bending (neglect effect of axial deformation).

Extra Question:-

Calculate also the horizontal displacement of point D.



EI = constant for all members of the frame.

Consider Virtual System (1):-

Moments are plotted From Compression Side.

$$1*\Delta_{C_V} = \int \frac{M_0 M_1\, dx}{EI}.$$

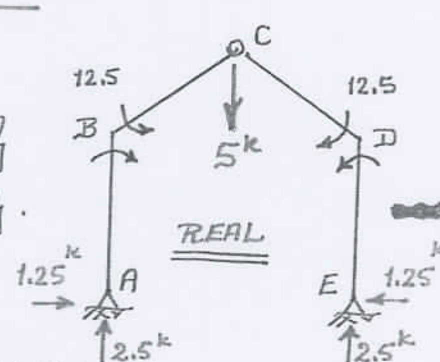
$$\Rightarrow \Delta_{C_V} = \frac{2}{EI}\left\{\frac{12.5*10}{2}*\frac{2}{3}\times 2.5\right\} + \frac{2}{EI}\left\{\frac{12.5\times 10}{2}*\frac{2}{3}\times 2.5\right\}.$$

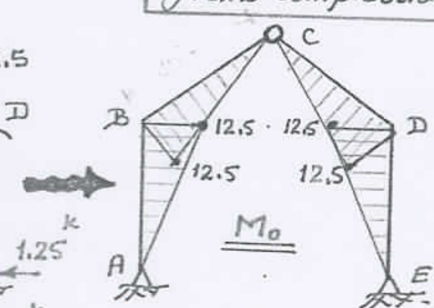
$$= +\frac{416.67}{EI}.$$

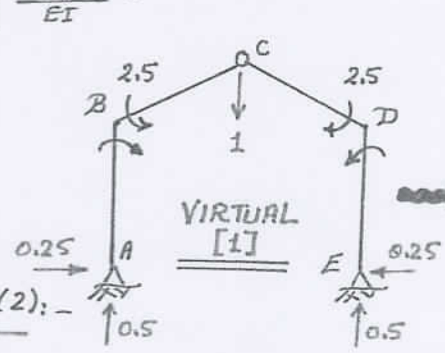
Note that:

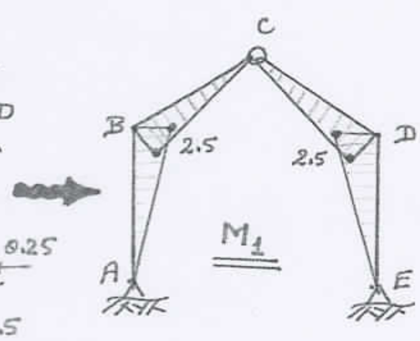
$$1\times\Delta_{C_V} = 2\int_B^C \frac{M_0 M_1\, dx}{EI} + 2\int_A^B \frac{M_0 M_1\, dx}{EI}.$$

$$\Rightarrow \boxed{\Delta_{C_V} = \frac{416.67}{EI}\downarrow}$$









Extra Problem:-

Consider Virtual System (2):-

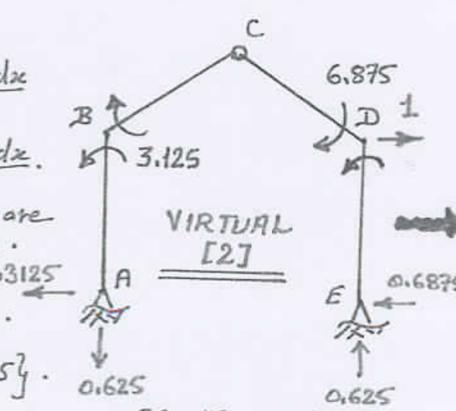
$$1*\Delta_{D_H} = \int \frac{M_0 M_2\, dx}{EI},$$

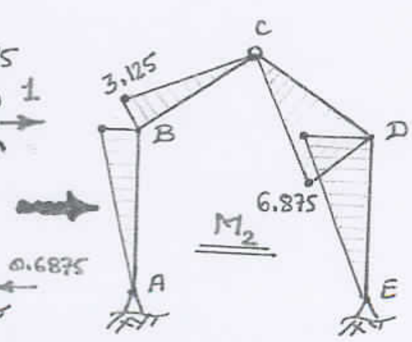
$$\Rightarrow \Delta_{D_H} = \int_A^B \frac{M_0 M_2\, dx}{EI} + \int_B^C \frac{M_0 M_2\, dx}{EI} + \int_C^D \frac{M_0 M_2\, dx}{EI} + \int_D^E \frac{M_0 M_2\, dx}{EI}.$$

* Note that the first two integrals are equal, and so are the last two.

$$\Rightarrow \Delta_{D_H} = \frac{-2}{EI}\left\{\frac{12.5\times 10}{2}*\frac{2}{3}*3.125\right\} + \frac{2}{EI}\left\{\frac{12.5\times 10}{2}*\frac{2}{3}*6.875\right\}.$$

diagrams are in opp sides





$$\Rightarrow \Delta_{D_H} = -\frac{260.42}{EI} + \frac{572.92}{EI} = \boxed{+\frac{312.5}{EI}\rightarrow}$$