American University of Beirut

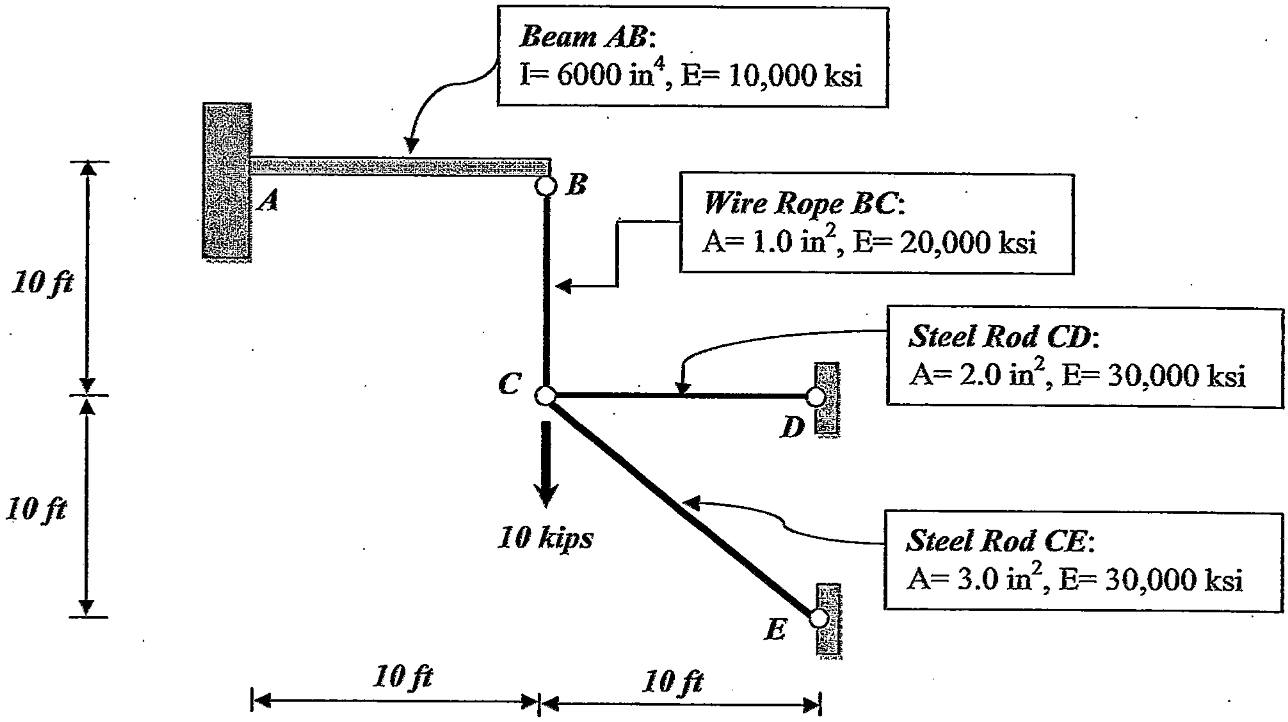
Civil Engineering Dept. | **Faculty of Eng. & Arch.**

Structures II – CIVE410 - | **Homework [5]** | **Spring 2011**

Problem [1]

The structure shown in the accompanying figure consists of a beam ***AB*** connected to the axial members BC, CD, and CE. The composite system is loaded by 10 kips at joint C as shown. Using the Flexibility Method of Analysis, determine the support reactions at *A* and the axial forces in members ***BC, CD***, and ***CE.***

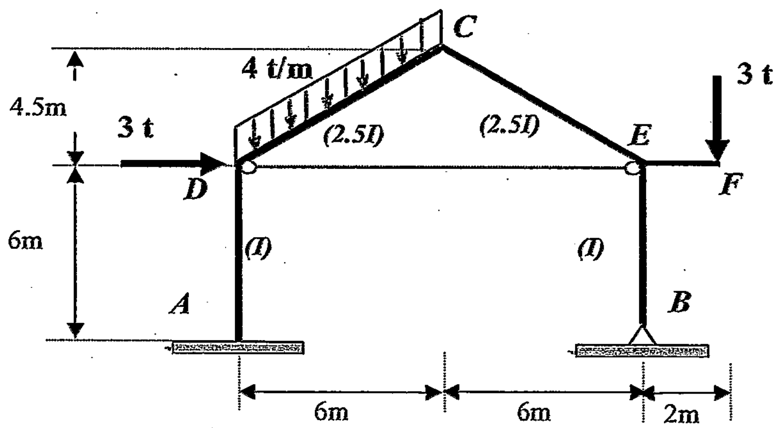
Consider only the effect of bending in ***AB*** (neglect axial and shear deformation), and consider the effect of axial force only in ***BC, CD, and CE*** (neglect flexural deformation).



Problem [2]

The frame ***ADCEB*** shown in the accompanying figure is fixed at A and hinged at B. The frame is strengthened by a tie ***ED*** as shown. Assemble the **flexibility matrix** required for analysis of the frame. Neglect axial deformation in all members except member ***DE***, and Use ***EI= 8000 t.m²*** and ***EA= 20,000 tons*** for member ***DE.***

Use X_B, Y_B and T_{DE} (the force in the Tie DE) as redundants.



Problem [3]

A plane view of a grid of floor beams, resting on pillars at A, B, C, D, E, and F is shown in the accompanying Figure [a]. Assuming the beam to have identical flexural rigidity, EI, and zero torsional rigidity:

[1] Using the **Flexibility Method of analysis**, analyze the grid system, given that the segment GH is subjected to a uniformly distributed gravity load of 20 kN/m (Figure [b]). Then, draw the shearing Force and bending moment diagrams of beams CD and AB.

[2] Using the **Flexibility Method of analysis**, analyze the grid system, given that joint G is subjected to a concentrated gravity load of 100 kN (Figure [c]). Then, draw the shearing Force and bending moment diagrams of beams CD and AB.

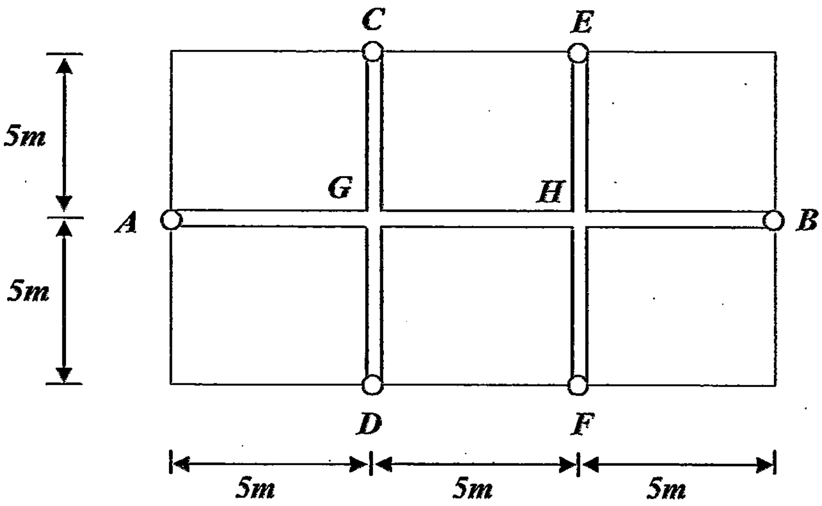


Figure [a]

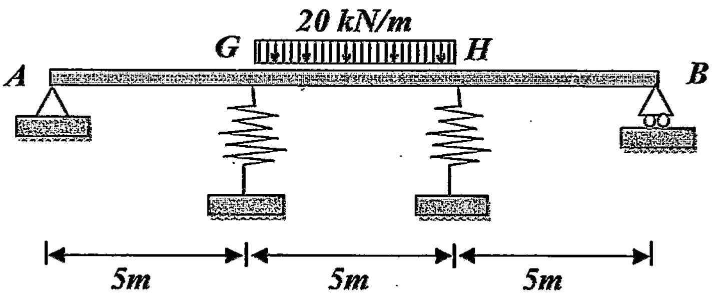


Figure [b]

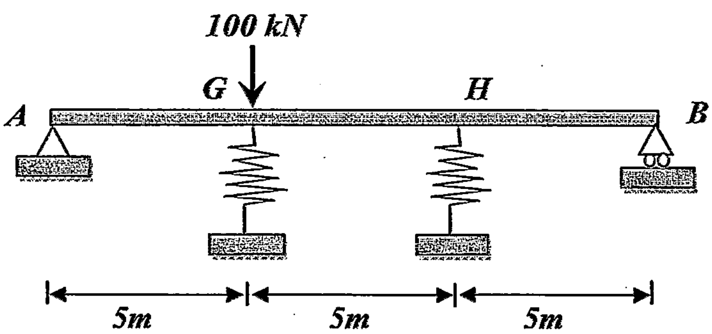


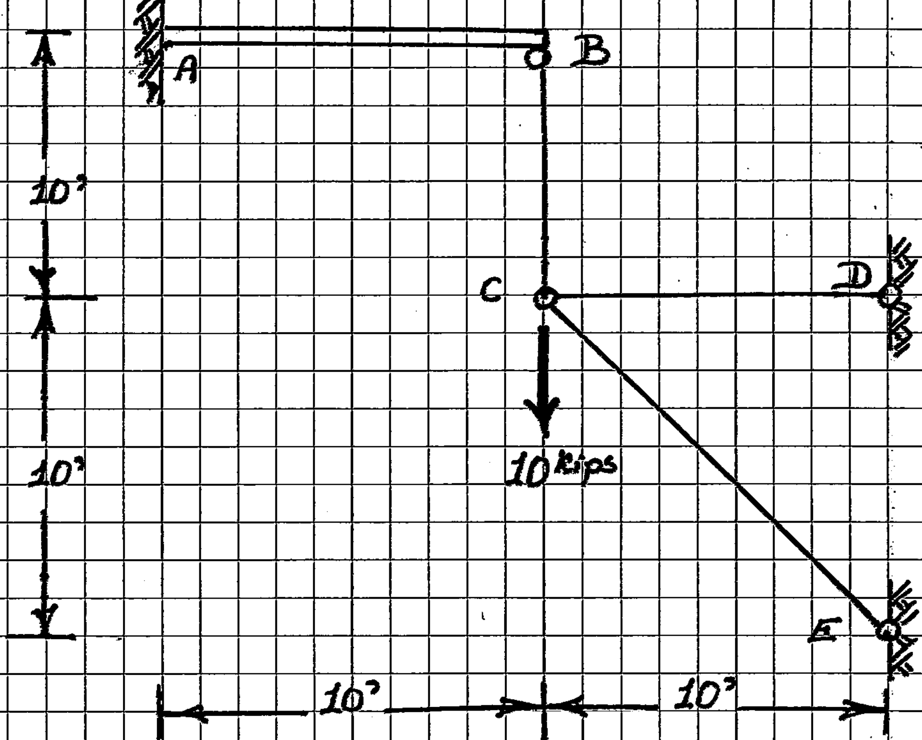
Figure [c]

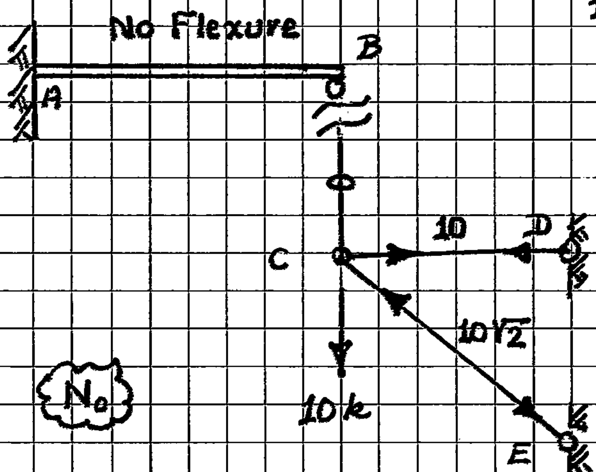
Good Luck & Best Wishes

Dr. Hisham S. Basha, Ph.D.

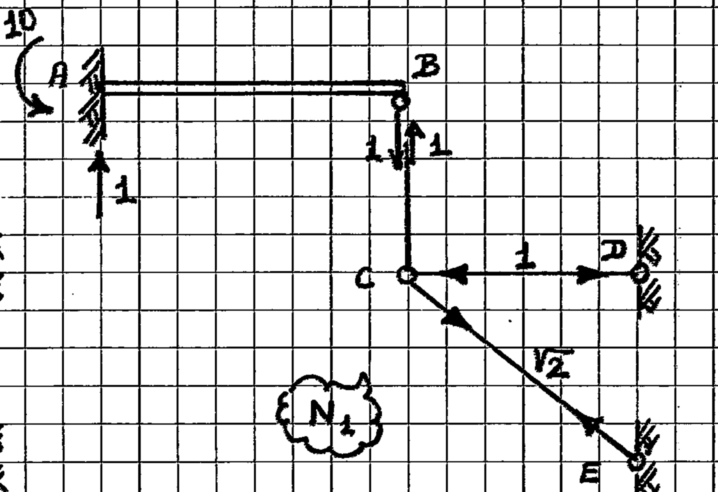
USE FOR ANSWER

PROBLE {1}.

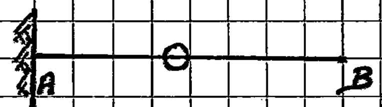




Primary System



Secondary (Virtual) System



M_0 Diagram

M_1 Diagram.

USE FOR ANSWER

$$\boxed{f_{10} + x f_{11} = 0}$$

$$f_{10} = \cancel{\int \frac{M_0 M_1\, dl}{EI}}^{\,0} + \sum \frac{N_0 N_1 L}{EA} = 0 + \frac{(10)(-1)(10*12)}{30{,}000 * 2.0} + \frac{(-10\sqrt{2})(\sqrt{2})(10\sqrt{2})}{30{,}000 * 3.0}$$

$$= -0.02 - 0.0377 = \underline{-0.0577 \text{ in.}}$$

$$f_{11} = \int \frac{M_1 M_1\, dl}{EI} + \sum \frac{N_1 N_1 L}{EA} = \frac{10*12^3}{3EI}(10*10) + \frac{(1)(1)(10*12)}{20{,}000 * 1.0}$$

$$+ \frac{(1)(1)(10*12)}{30{,}000 * 2.0} + \frac{(\sqrt{2})(\sqrt{2})(10\sqrt{2}*12)}{30{,}000 * 3.0}$$

$$\Rightarrow f_{11} = \frac{10*(10*10)*12^3}{3*10{,}000 * 6000} + 0.006 + 0.002 + 0.00377$$

$$= 0.0096 + 0.006 + 0.002 + 0.00377 = \underline{0.02137 \text{ in.}}$$

Now, $x = -\dfrac{f_{10}}{f_{11}} = -\dfrac{-0.0577}{0.02137} = \underline{\underline{+2.70 \text{ k.}}}$

x = Force in Wire Rope BC, the +ve sense indicates force is in the assumed sense of the dummy load (Tension).

Now, $\boxed{R_{final} = R_0 + x R_1}$

$\Rightarrow$ $F_{CD} = 10 + (2.70)(-1) = \underline{\underline{+7.30 \text{ k}}}$ ... (Tension).

$F_{CE} = -10\sqrt{2} + (2.70)(\sqrt{2}) = \underline{\underline{-10.32^k}}$ ... (Compression).

$y_A = 0 + (2.70)(1) = \underline{\underline{2.70 \text{ k}}}$ (↑).

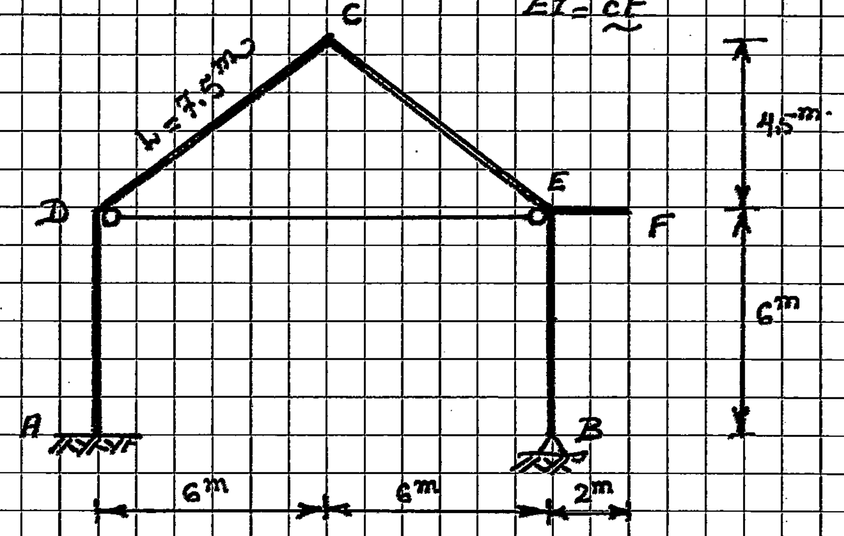
$M_A = 0 + (2.70)(-10) = \underline{\underline{-27 \text{ k.ft}}}$ A

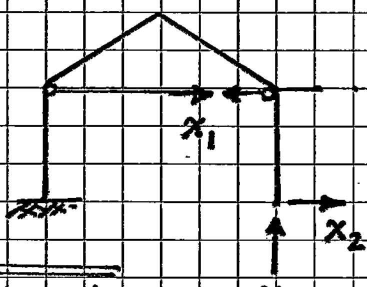
PROBLEM {2}.

System is indet. to the third degree.

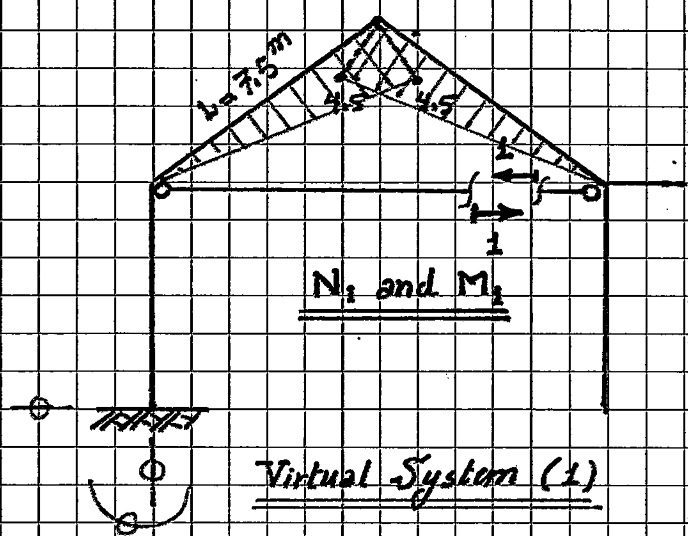
$\rightarrow [F]_{3\times3}$.

$$[F] = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}_{3\times3}$$

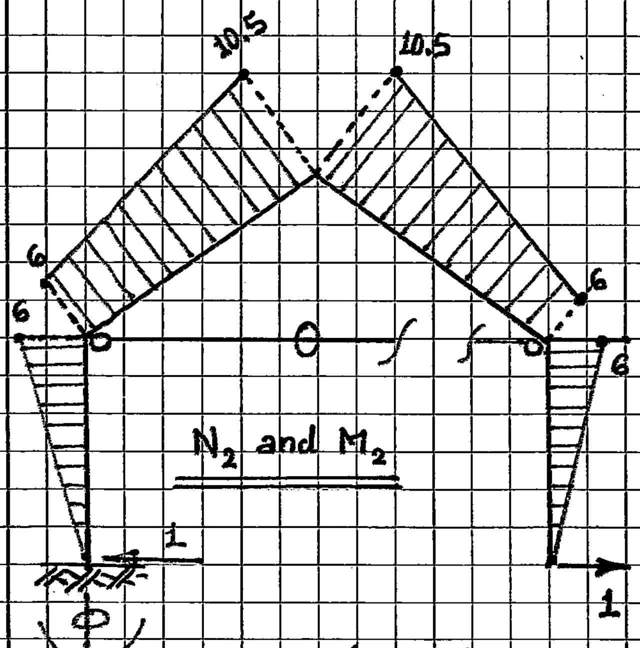




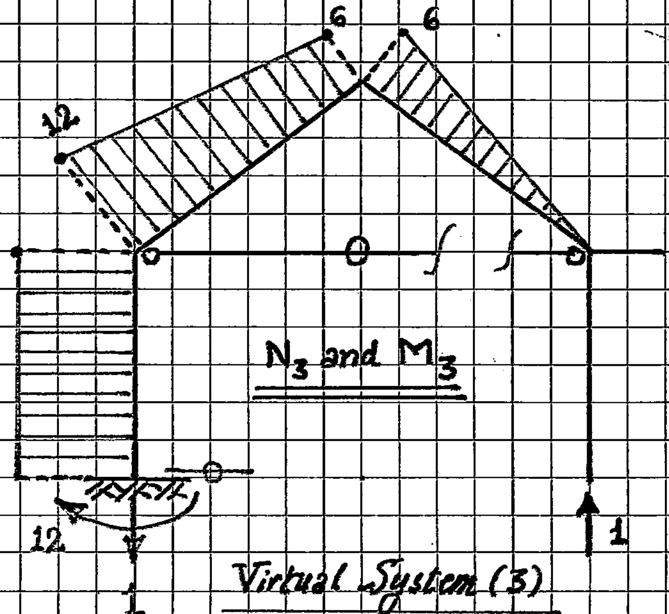
Released System.



Virtual System (1)



Virtual System (2)



Virtual System (3)

Coefficients of the Flexibility Matrix.

$$f_{11} = \int \frac{M_1 M_1\, dl}{EI} + \Sigma \frac{N_1 N_1 L}{EA} \qquad f_{21} = f_{12} = \checkmark$$

$$f_{12} = \int \frac{M_1 M_2\, dl}{EI} + \Sigma \cancel{\frac{N_1 N_2 L}{EA}} \to 0 \qquad f_{31} = f_{13} = \checkmark$$

$$f_{13} = \int \frac{M_1 M_3\, dl}{EI} + \Sigma \cancel{\frac{N_1 N_3 L}{EA}} \to 0 \qquad f_{23} = f_{32} = \checkmark$$

$$f_{22} = \int \frac{M_2 M_2\, dl}{EI} + \Sigma \cancel{\frac{N_2 N_2 L}{EA}} \to 0$$

$$f_{23} = \int \frac{M_2 M_3\, dl}{EI} + \Sigma \cancel{\frac{N_2 N_3 L}{EA}} \to 0$$

$$f_{33} = \int \frac{M_3 M_3\, dl}{EI} + \Sigma \cancel{\frac{N_3 N_3 L}{EA}} \to 0$$

Note: $EI = 8000\ t.m^2$
$EA = 20{,}000\ t$
$\Rightarrow EA \approx 2.5\, EI$

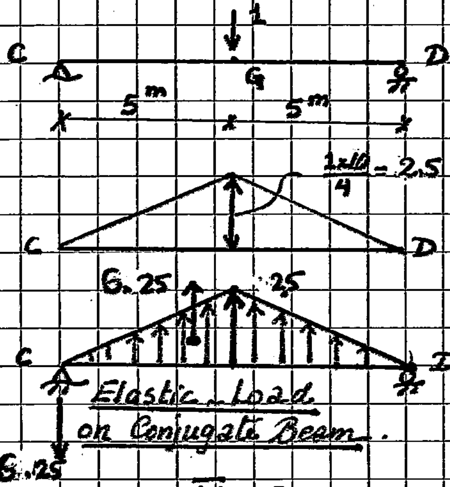
- $f_{11} = \frac{7.5}{3EI}\left[4.5 * 4.5\right] * 2 + \frac{(1)(1)(12)}{EA} = \boxed{\frac{106.05}{EI}}$
- $f_{12} = \frac{7.5}{6EI}\left\{0 + 2(10.5)(-4.5) + (6)(-4.5) + 0\right\} * 2 = \boxed{-\frac{303.75}{EI}}$
- $f_{13} = \frac{7.5}{6EI}\left\{0 + 2(6)(-4.5) + (12)(-4.5) + 0\right\} + \frac{7.5}{3EI}(-6)(4.5) = \boxed{-\frac{202.5}{EI}}$
- $f_{22} = 2 * \left\{\frac{7.5}{6EI}\left[2(6\times6) + 2(10.5)(10.5) + (6)(10.5) + (10.5)(6)\right] + \frac{6}{3EI}(6\times6)\right\} = \frac{1.1}{E}$
- $f_{23} = \frac{1}{EI}(12\times6\times3) + \frac{7.5}{6EI}\left[2(6\times12) + 2(10.5\times6) + (6\times6) + (12\times10.5)\right] +$
  $\frac{7.5}{6EI}\left[0 + 2(6\times10.5) + 0 + (6\times6)\right] = \boxed{\frac{958.5}{EI}}$
- $f_{33} = \frac{1}{EI}(12\times6\times12) + \frac{7.5}{6EI}\left[2(12)(12) + 2(6\times6) + 12\times6 + 6\times12\right] + \frac{7.5}{3EI}(6\times6)$
  $= \boxed{\frac{1584}{EI}}$

$$\Rightarrow \left[\mathcal{F}\right]_{3\times3} = \begin{bmatrix} 106.05 & -303.75 & -202.5 \\ -303.75 & 1190.25 & 958.5 \\ -202.5 & 958.5 & 1584 \end{bmatrix} \times \frac{1}{EI} = \begin{bmatrix} 0.01325 & -0.038 & -0.02 \\ & 0.1488 & 0.1198 \\ & & 0.198 \end{bmatrix}$$

PROBLEM {3}

[1].

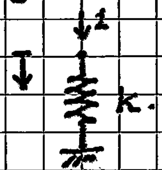
Beams CD and EF have identical stiffnesses. (K).



$$v_G = \frac{\overline{M}}{EI} = -\left[6.25(5) + 6.25\left(\frac{5}{3}\right)\right] \frac{1}{EI}$$

$$\boxed{v_G = -\frac{20.833}{EI} \quad (\downarrow)}$$

$\Rightarrow$ Equiv. linear Spring of Axial Stiffness (k).



Hooke's Law:

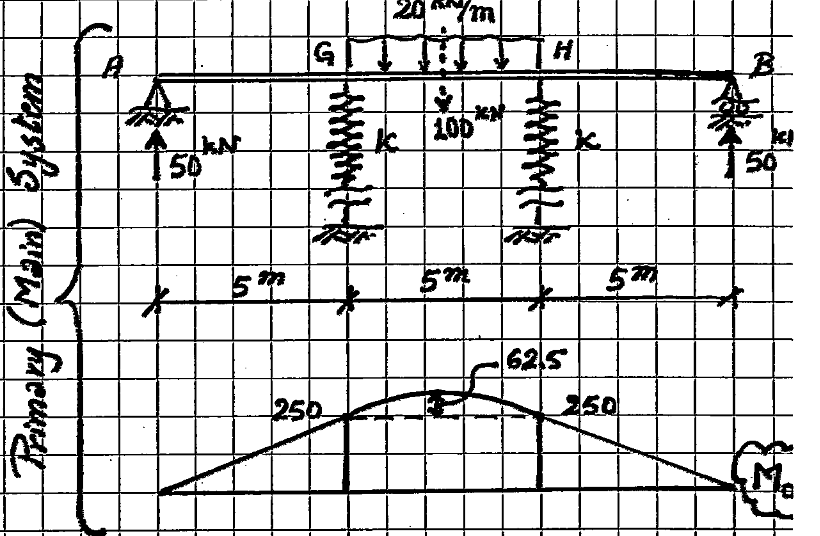
$\Rightarrow 1 = K \cdot v_G$

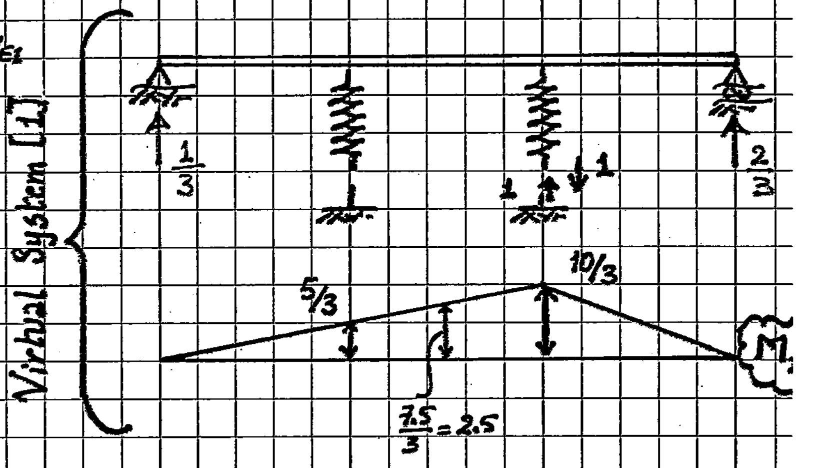
$$\Rightarrow K = \frac{1}{v_G} = \frac{EI}{20.833}.$$

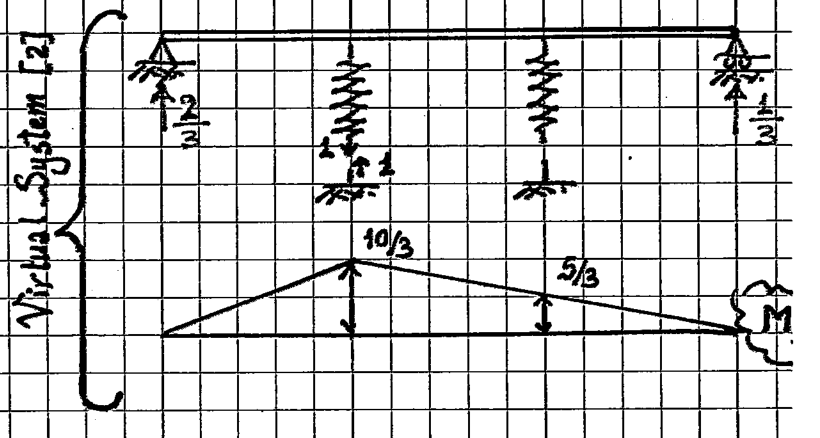
OR $\boxed{\Delta_c = \frac{PL^3}{48EI}} \Rightarrow K = \frac{P}{\Delta} = \frac{48EI}{L^3}$

$$\Rightarrow k = \frac{EI}{20.833} \quad \text{(same)}.$$

$$\begin{Bmatrix} f_{10} \\ f_{20} \end{Bmatrix} + \underbrace{\begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}}_{\text{Flex. Matrix}} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$







- $f_{10} = \int \frac{M_0 M_1 \, dl}{EI} + \Sigma \frac{N_0 N_1 L}{EA} = \int_B \frac{M_0 M_1 \, dl}{EI} + \Sigma \left(N_0 N_1 * \frac{1}{k}\right)$.

$$= \int_A^G \frac{M_0 M_1 \, dl}{EI} + \int_G^H \frac{M_0 M_1 \, dl}{EI} + \int_H^B \frac{M_0 M_1 \, dl}{EI} + \Sigma \left(N_0 N_1 * \frac{1}{k}\right)$$

$$= \frac{5}{3EI}\left(\frac{5}{3} \times 250\right) + \frac{1}{EI}\left[(250 \times 5)(2.5) + \frac{2}{3}(62.5)(5)(2.5)\right] + \frac{5}{3EI}\left(\frac{10}{3} \times 250\right) + \text{Zero}.$$

$$= (694.44 + 3,645.83 + 1,388.89)\frac{1}{EI} = \boxed{\frac{5,729.16}{EI}}$$

- $f_{20} = f_{10}$ ---- Due to Symmetry.

- $f_{11} = \frac{10}{3EI}\left(\frac{10}{3} \times \frac{10}{3}\right) + \frac{5}{3EI}\left(\frac{10}{3} \times \frac{10}{3}\right) + (1)(1)\left(\frac{20.833}{EI}\right) = \boxed{\frac{76.388}{EI}}$.

- $f_{22} = f_{11}$ ---- Due to Symmetry.

- $f_{12} = f_{21} = \int \frac{M_1 M_2 \, dl}{EI} + \Sigma \left(N_1 N_2 * \frac{1}{k}\right)$

$$= \frac{5}{3EI}\left(\frac{10}{3} \times \frac{5}{3}\right) \times 2 + \frac{5}{6EI}\left[2(5/3)(10/3) + 2(10/3)(5/3) + (5/3)(5/3) + (10/3)(\right.$$

$$+ \text{Zero} + \text{Zer}$$

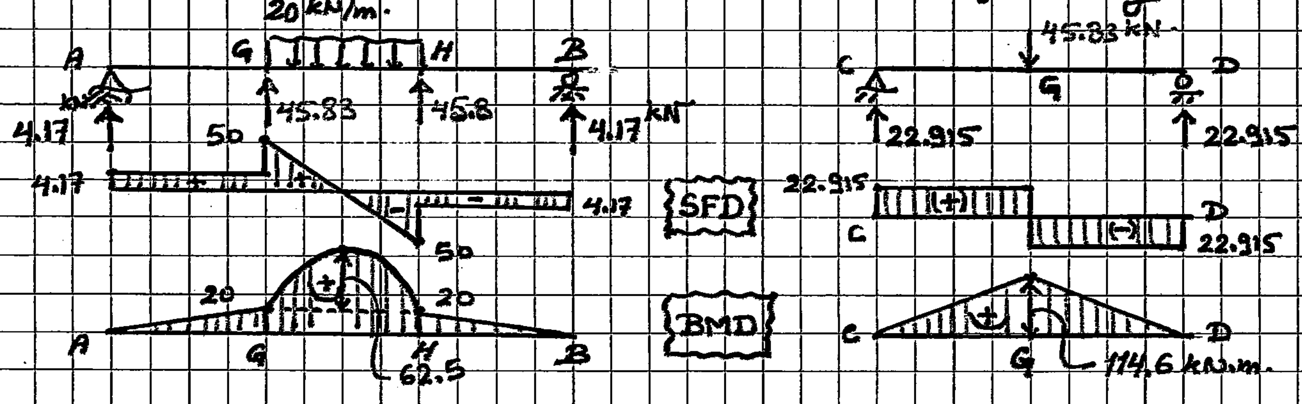
$$= \frac{18.5185}{EI} + \frac{30.093}{EI} = \boxed{\frac{48.611}{EI}}.$$

$$\Rightarrow \frac{1}{EI}\begin{Bmatrix} 5,729.16 \\ 5,729.16 \end{Bmatrix} + \frac{1}{EI}\begin{bmatrix} 76.388 & 48.611 \\ 48.611 & 76.388 \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}.$$

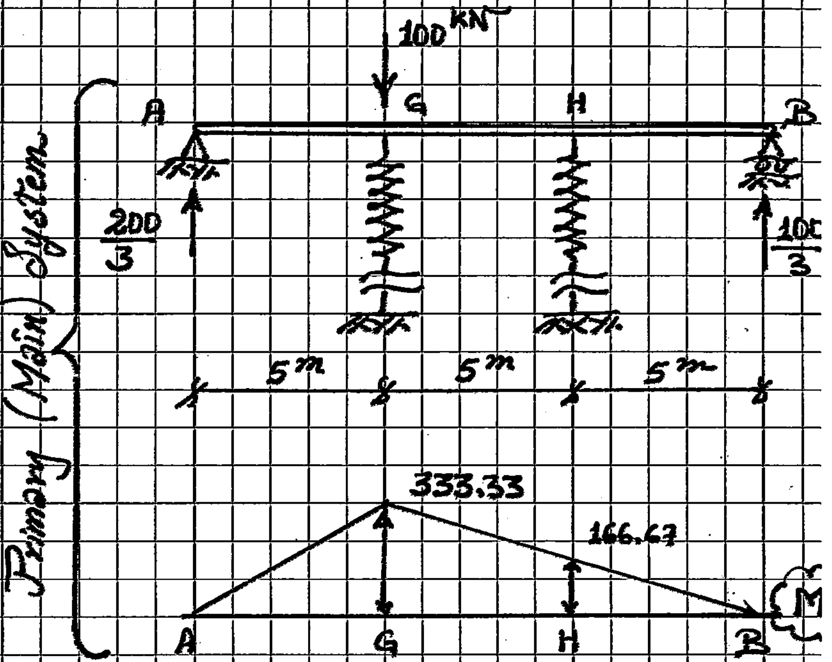
Due to symmetry $X_1 = X_2 = X$ (Force in the spring).

$$\Rightarrow X = -\frac{5729.16}{125} = -45.83 \text{ kN}$$ (-ve → comp. in the spring.)

opp. to assumed sense of the dummy load.



[2].



$$\begin{Bmatrix} f_{10} \\ f_{20} \end{Bmatrix} = ?$$

- $f_{10} = \int \frac{M_0 M_1\, dl}{EI} + \cancel{\Sigma \left(N_0 N_1 * \frac{1}{k}\right)}^{\,0}$.
- $f_{20} = \int \frac{M_0 M_2\, dl}{EI} + \cancel{\Sigma \left(N_0 N_2 * \frac{1}{k}\right)}^{\,0}$

$$\Rightarrow f_{10} = \frac{5}{3EI}\left(333.33 \times \frac{5}{3}\right) + \frac{5}{6EI}\Big[2(333.33)\left(\frac{5}{3}\right) + 2(166.67)\left(\frac{10}{3}\right) + (333.33)\left(\frac{10}{3}\right) + (166.67)\left(\frac{5}{3}\right)\Big] + \frac{5}{3EI}\left(\frac{10}{3}\right)(166.67) = \frac{1}{EI}\left(925.92 + 3009.26 + 925.94\right).$$

$$= \boxed{\frac{4{,}861.12}{EI}}$$

$$f_{20} = \frac{5}{3EI}\left(\frac{10}{3} \times 333.33\right) + \frac{5}{6EI}\Big[2\left(\frac{10}{3}\right)(333.33) + 2(166.67)\left(\frac{5}{3}\right) + \left(\frac{10}{3}\right)(166.67) + \left(\frac{5}{3}\right)(333.33)\Big] + \frac{5}{3EI}\left(\frac{5}{3}\right)(166.67).$$

$$= (1851.83 + 3240.74 + 462.97)\frac{1}{EI}.$$

$$= \boxed{\frac{5555.6}{EI}}$$

Note:

Virtual System [1] $\rightarrow$ As before

Virtual System [2] $\rightarrow$ As before

$\Rightarrow [F]_{2\times 2}$ as before.

$$\Rightarrow \frac{1}{\cancel{EI}}\begin{Bmatrix} 4861.12 \\ 5555.6 \end{Bmatrix} + \begin{bmatrix} \frac{76.388}{\cancel{EI}} & \frac{48.611}{\cancel{EI}} \\ \frac{48.611}{\cancel{EI}} & \frac{76.388}{\cancel{EI}} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Solving $\Rightarrow$ $X_1 = \underbrace{-29.17 \text{ kN}}_{\text{comp. in the spring}}$ and $X_2 = -54.17$ kN.

Plot S.F.D & BMD as usual ....