American University of Beirut

Civil Engineering Dept. | **Faculty of Eng. & Arch.**

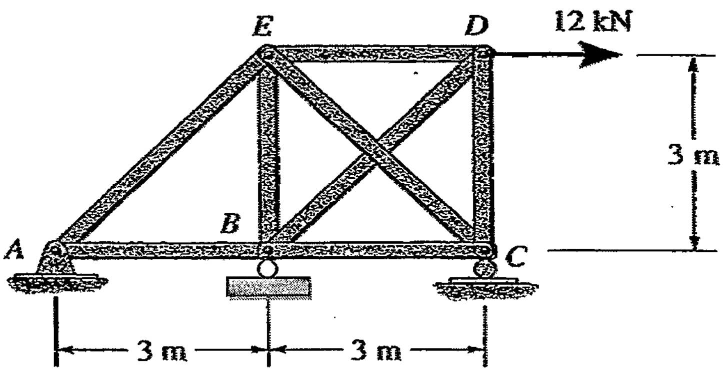
Structures II – CIVE410 - **Homework [4]** **Spring 2011**

Problem [1]

For the truss shown in the accompanying figure, assume the members have a constant EA and that all members are pin connected at their ends. Use $EA = 120{,}000\ kN$.

[a] Determine the force in each member of the truss. Use the reaction at B and the force in member BD as redundants.

[b] Determine the horizontal displacement of point D.



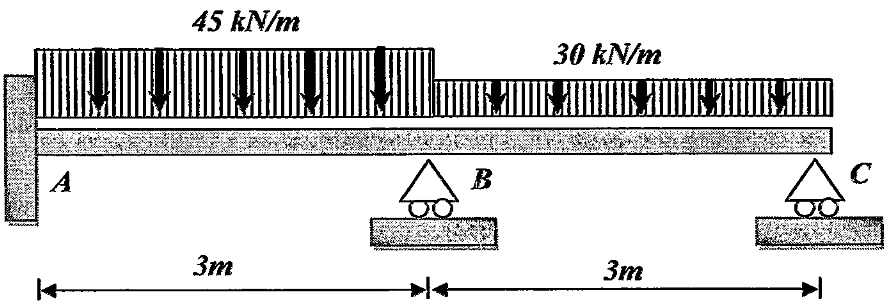
Problem [2]

The beam ABC shown in the accompanying figure has a fixed support at A and roller supports at B and C. Assume E= 200 GPa and $I = 90(10^6)\ mm^4$.

[a] Using the Flexibility Method (Virtual Work Concept) for analysis of indeterminate structure draw the Bending Moment Diagram of the beam. Use Y_B and Y_C as redundants.

[b] Determine the slope at B.

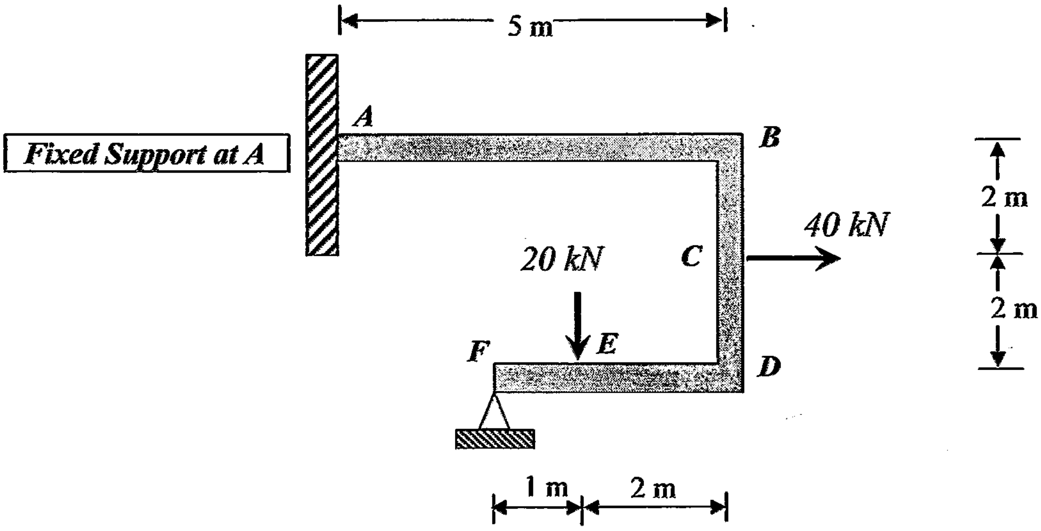
[c] If support B has settled 1.5 cm vertically downwards, determine the supports reactions due to the given loads in addition to the vertical settlement at B.



Problem [3]

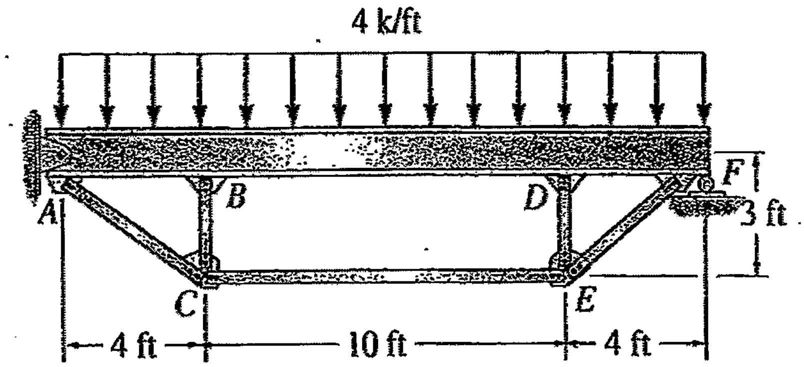
The frame ABDF shown in the accompanying figure is fixed at A and pinned at F. Using the Flexibility Method (Virtual Work Concept) for analysis of indeterminate structure and using reactions at F as redundant determine the bending moment diagram of the frame.

Neglect axial and shear deformation. Use EI= 2.133×10^5 kN.m^2.



Problem [4]

The queen-post trussed beam is used to support a uniform load of 4 k/ft. Determine the force developed in each of the five struts. Neglect the thickness of the beam and assume the truss members are pin connected to the beam. Also, neglect the effect of axial and shear deformation in the beam ABDF. The cross-sectional area of each strut is 3 in^2 and for the beam I = 600 in^4. Use E= 29,000 ksi.

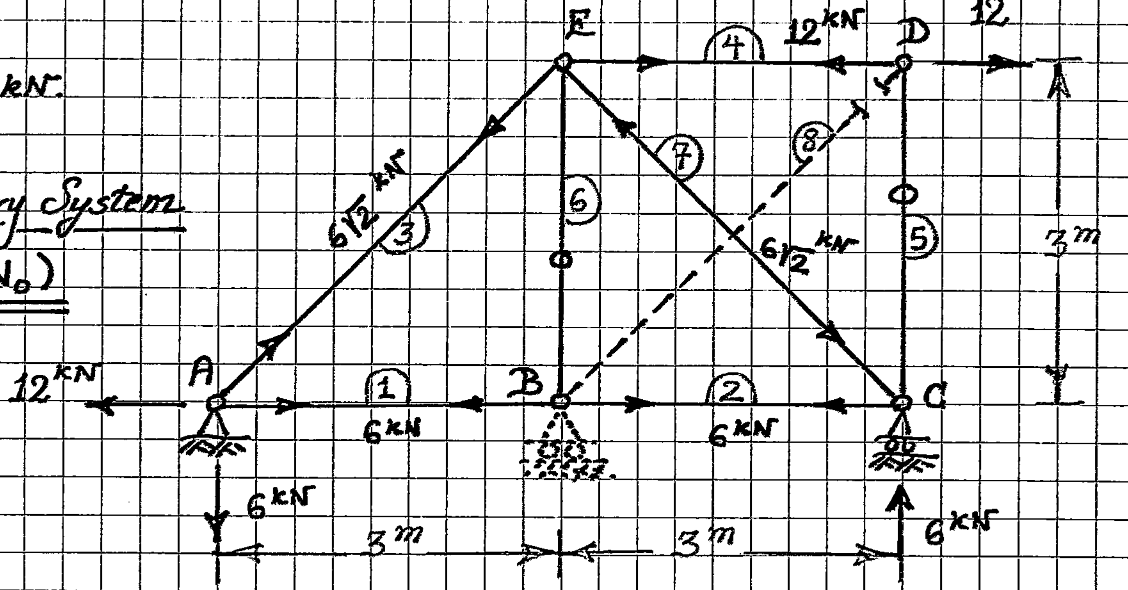


PROBLEM {1}:-

$EA = 120{,}000\ kN.$

Primary System

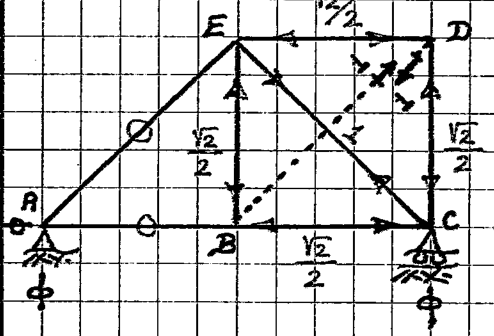
(N_0)



{2}

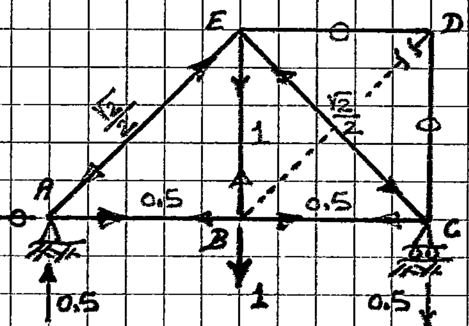
Virtual System [1]

(N_1)



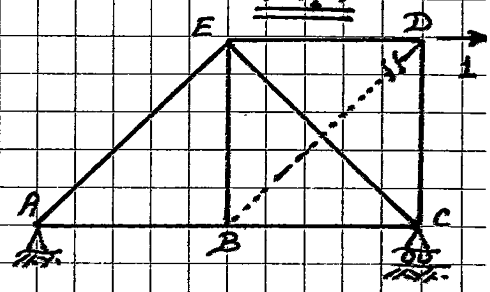
Virtual System [2]

(N_2)



Virtual System For Horz. Displ. of Joint D

(N_1^*)



Member	L (m)	N_0	N_1	N_2	$N_0 N_1 L$	$N_0 N_2 L$	$N_1 N_2 L$	$N_1^2 L$	$N_2^2 L$	$N_F = N_0 + x_1 N_1 + x_2 N_2$	$N_F \cdot N_1^* \cdot L$
1	3	6	0	0.5	0	9	0	0	0.75	7.120	10.68
2	3	6	$-\sqrt{2}/2$	0.5	$-9\sqrt{2}$	9	$-3\sqrt{2}/4$	1.5	0.75	2.822	4.23
3	$3\sqrt{2}$	$6\sqrt{2}$	0	$-\sqrt{2}/2$	0	$-18\sqrt{2}$	0	0	$3\sqrt{2}/2$	6.903	20.71
4	3	12	$-\sqrt{2}/2$	0	$-18\sqrt{2}$	0	0	1.5	0	7.703	23.11
5	3	0	$-\sqrt{2}/2$	0	0	0	0	1.5	0	-4.297	0
6	3	0	$-\sqrt{2}/2$	1	0	0	$-3\sqrt{2}/2$	1.5	3	-2.059	0
7	$3\sqrt{2}$	$-6\sqrt{2}$	1	$-\sqrt{2}/2$	-36	$18\sqrt{2}$	-3	$3\sqrt{2}$	$3\sqrt{2}/2$	-3.991	11.97
8	$3\sqrt{2}$	0	1	0	0	0	0	$3\sqrt{2}$	0	6.077	0
				$\Sigma =$	-74.184	18	-6.182	14.485	8.743	~	70.703

$EA = ct$

$$N_F = N_0 + x_1 N_1 + x_2 N_2$$

$$\{F\}_0 + [F]\{X\} = \{0\}.$$

$$\Rightarrow \begin{Bmatrix} f_{10} \\ f_{20} \end{Bmatrix} + \underbrace{\begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}}_{\text{Flexibility Matrix}} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

- $f_{10} = \sum \frac{N_0 N_1 L}{EA} = -\frac{74.184}{EA}$
- $f_{20} = \sum \frac{N_0 N_2 L}{EA} = \frac{18}{EA}$
- $f_{12} = \sum \frac{N_1 N_2 L}{EA} = -\frac{6.182}{EA}$
- $f_{11} = \sum \frac{N_1 N_1 L}{EA} = \frac{14.485}{EA}$
- $f_{22} = \sum \frac{N_2 N_2 L}{EA} = \frac{8.743}{EA}$

OR By System of Eqns:-

$$\Rightarrow f_{10} + x_1 f_{11} + x_2 f_{12} = 0 \quad \text{.... (1)}$$

$$f_{20} + x_1 f_{21} + x_2 f_{22} = 0 \quad \text{.... (2)}$$

Solve for x_1 & x_2:-

$x_1 = +6.0765$ kN and $x_2 = +2.2378$ kN

where X_1 is the redundant axial force in member (8) of the truss; and x_2 is the redundant reaction of support (B). The +ve sign indicates that redundants are in the assumed sense of the dummy load.

{b} Calculate the Horizontal Displacement of Joint (D):-

P.V.W:

$$1 * \delta_{D_{HORIZ}} = \sum \frac{N_F * N_1^* * L}{EA}.$$

where N_1^* is member force in a virtual system subjected a dummy load in the direction of the required displacement, at joint D.

* Note that this system is equivalent to N_0-Diagram when divided by 12.

$$\Rightarrow \delta_{D_{HORIZ}} = +\frac{70.703}{EA} = +0.000589\ m$$

$$\Rightarrow \boxed{\delta_{D_{HORIZ}} = 0.589\ mm \rightarrow}$$

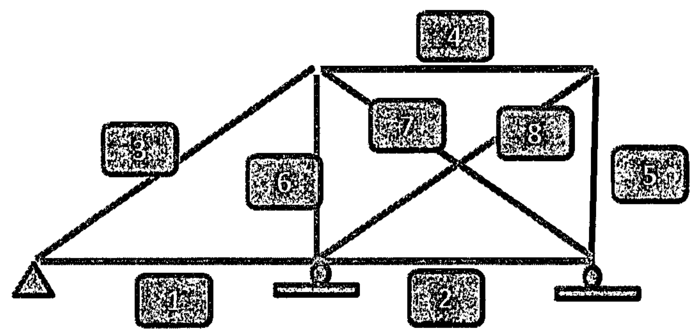
CIVE410- HW[4] - Problem (1)

Member	L	No	N1	N2	N0N1L	NoN2L	N1N2L	N1N1L	N2N2L	Nf	N1'	N1'NfL
1	3.0000	6.0000	0.0000	0.5000	0.0000	9.0000	0.0000	0.0000	0.7500	7.1189	0.5000	10.6784
2	3.0000	6.0000	-0.7071	0.5000	-12.7280	9.0000	-1.0607	1.5000	0.7500	2.8221	0.5000	4.2332
3	4.2426	8.4853	0.0000	-0.7071	0.0000	-25.4557	0.0000	0.0000	2.1213	6.9029	0.7071	20.7085
4	3.0000	12.0000	-0.7071	0.0000	-25.4560	0.0000	0.0000	1.5000	0.0000	7.7032	1.0000	23.1097
5	3.0000	0.0000	-0.7071	0.0000	0.0000	0.0000	0.0000	1.5000	0.0000	-4.2968	0.0000	0.0000
6	3.0000	0.0000	-0.7071	1.0000	0.0000	0.0000	-2.1213	1.5000	3.0000	-2.0590	0.0000	0.0000
7	4.2426	-8.4853	1.0000	-0.7071	-35.9996	25.4557	-3.0000	4.2426	2.1213	-3.9912	-0.7071	11.9733
8	4.2426	0.0000	1.0000	0.0000	0.0000	0.0000	0.0000	4.2426	0.0000	6.0765	0.0000	0.0000
					-74.1836	18.0000	-6.1820	14.4853	8.7426			70.7032

x1=6.0765
x2=2.2378

Nf= No+x1N1+x2N2

Displacement at D = Sum(N1'NfL/EA)



PROBLEM {2}:

[a]. $EI = ct.$

P.V.W.

$$1 * f_{10} = \int \frac{M_0 M_1 \, dl}{EI}$$

$$\Rightarrow f_{10} = \frac{-3}{6EI}\{2(607.5)(3)+0+(135)(3)+0\} + \frac{2}{3EI}(50.625)(3)*1.5$$

$$= -\frac{2025}{EI} + \frac{151.875}{EI} = \boxed{-\frac{1873.125}{EI}}$$

$$f_{11} = \int \frac{M_1 M_1 \, dl}{EI} = \frac{3}{3EI}(3\times3) = \boxed{\frac{9}{EI}}$$

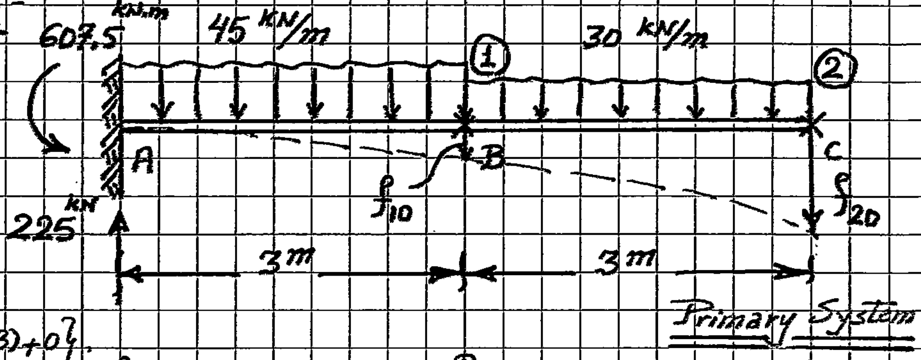
$$f_{20} = \frac{-3}{6EI}\{2(607.5)(6)+2(135)(3)+(607.5)(3)+(135)(6)\} + \frac{1}{EI}\left(\frac{2}{\cancel{3}}\times 50.625\times \cancel{3}\times 4.5\right) - \frac{3}{\cancel{3}EI}(135\times 3) + \frac{1}{EI}\left(\frac{2}{\cancel{3}}\times 33.75\times \cancel{3}\times 1.5\right)$$

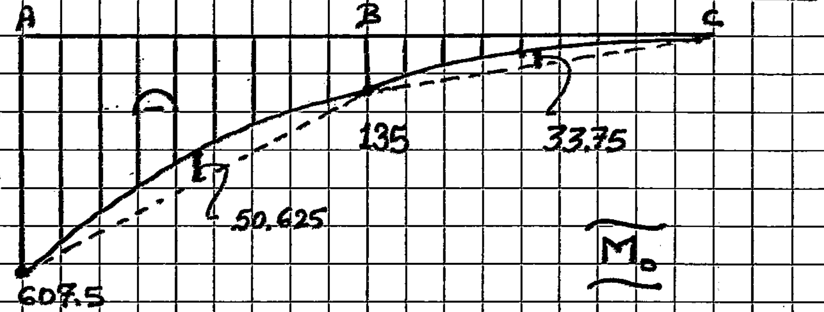
$$= -\frac{5366.25}{EI} + \frac{455.625}{EI} - \frac{405}{EI} + \frac{101.25}{EI}$$

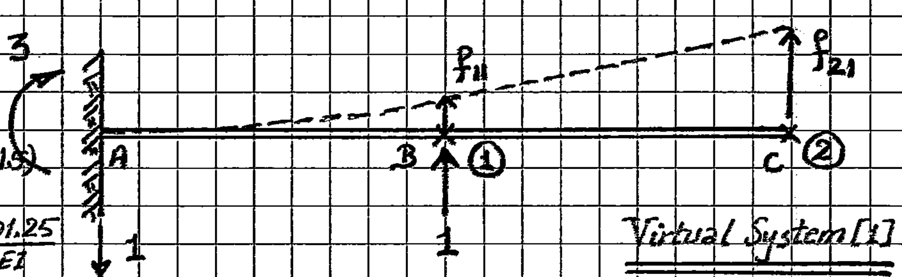
$$= \boxed{-\frac{5214.375}{EI}}$$

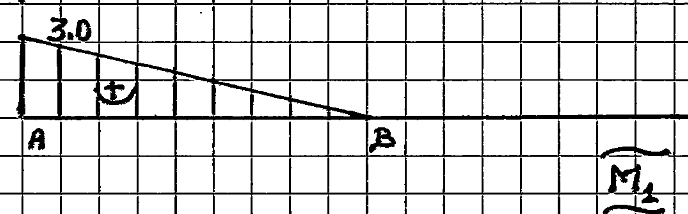
$$f_{22} = \frac{6}{3EI}(6\times6) = \boxed{\frac{72}{EI}}$$

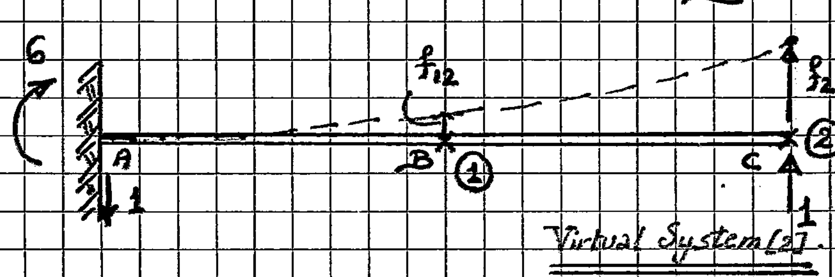
$$f_{12} = f_{21} = \int \frac{M_1 M_2 \, dl}{EI} = \frac{3}{6EI}\{2(3\times6)+0+3\times3+0\} = \boxed{\frac{22.5}{EI}}$$

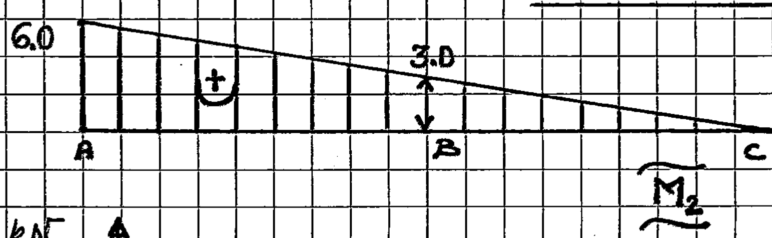












$$\begin{Bmatrix} f_{10} \\ f_{20} \end{Bmatrix} + \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

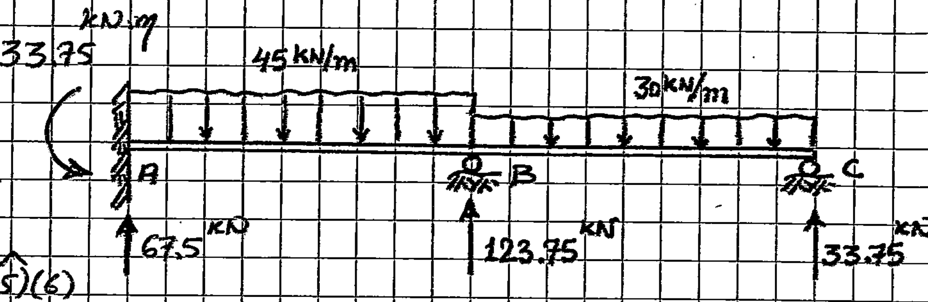
Solving,

$$\Rightarrow X_1 = Y_B = \underline{\underline{123.75\ kN}} \ldots \uparrow$$

$$X_2 = Y_C = \underline{\underline{33.75\ kN}} \ldots \uparrow$$

$M_f = M_0 + X_1 M_1 + X_2 M_2$

$R_f = R_0 + X_1 R_1 + X_2 R_2$.



∴ $M_A = -607.5 + (123.75)(3) + (33.75)(6)$

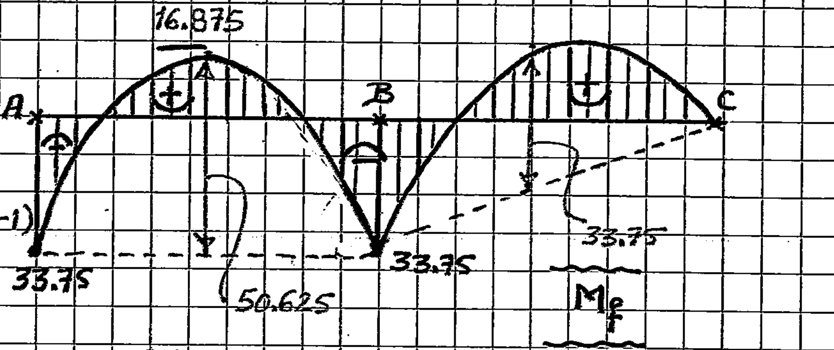
$= -33.75$ kN.m.

$M_B = -135 + 0 + (33.75)(3)$

$= -33.75$ kN.m.

$R_A = 225 + (123.75)(-1) + (33.75)(-1)$

$= 67.5$ kN.

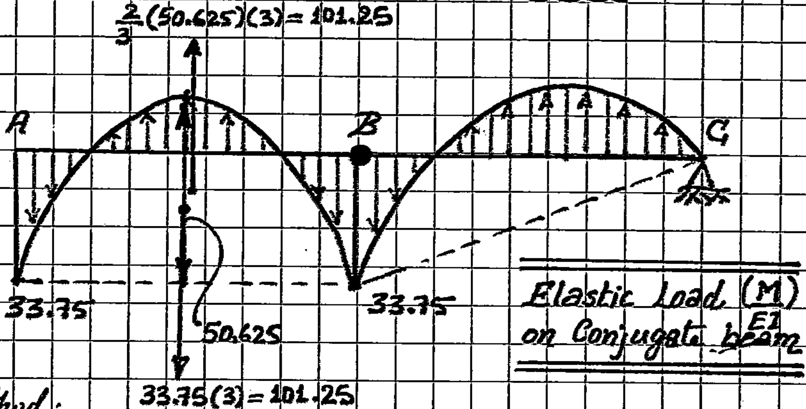


OR simply from Statics.

$\Sigma Y = 0 \Rightarrow R_A = 67.5$ kN.

& $M_A = -33.75$ kNm

$M_B = -33.75$ kN.m.



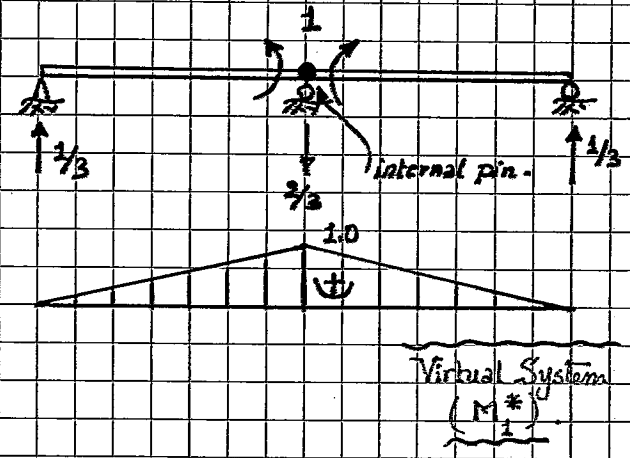
[b]. Slope at B; θ_B = ?.

- Using the Conjugate beam Method:

$\theta_B = \frac{\bar{V}_B}{EI} =$ Zero.

- Using the Concept of Virtual Work.

Choose any determinate and stable system (Virtual System) allowing deformation at and in the direction of the required displacement, and subjected to a dummy load/moment.



P.V.W $\Rightarrow 1 * \theta_B = \int \frac{M_f * M_1^*}{EI} dl$

$\Rightarrow \theta_B = \frac{1}{EI}\left\{-33.75(3)(\frac{1}{2}) + \frac{2}{\cancel{3}}(50.625)(\cancel{3})(\frac{1}{2})\right\}$

$+ \frac{1}{EI}\left\{-\frac{3}{3}(33.75)(1) + \frac{2}{3}(33.75)(3)(\frac{1}{2})\right\}$

$= \frac{1}{EI}\{-50.625 + 50.625\} + \frac{1}{EI}\{-33.75 + 33.75\}$.

= Zero (as before).

[c]. If $\delta_B = 1.5$ cm $\downarrow$.

Real Displacement at ①
-ve since opp. to dummy load.

$$\underbrace{\begin{Bmatrix} f_{10} \\ f_{20} \end{Bmatrix}}_{\text{as before}} + \underbrace{\begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}}_{\text{as before}} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} -1.5/100 \\ 0 \end{Bmatrix}$$

$$f_{1_{final}} = f_{10} + x_1 f_{11} + x_2 f_{12} = -\frac{1.5}{100} \quad \text{---} \quad (1)$$

$$f_{2_{final}} = f_{20} + x_1 f_{21} + x_2 f_{22} = 0 \quad \text{---} \quad (2)$$

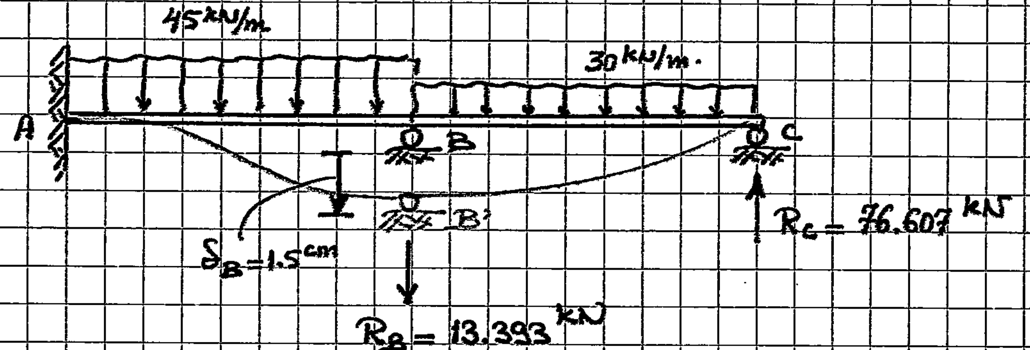
$$\Rightarrow \quad -\frac{1873.125}{EI} + \frac{9}{EI} x_1 + \frac{22.5}{EI} x_2 = -\frac{1.5}{100} \quad \text{---} \quad (1)$$

$$-\frac{5214.375}{EI} + \frac{22.5}{EI} x_1 + \frac{72}{EI} x_2 = 0 \quad \text{---} \quad (2)$$

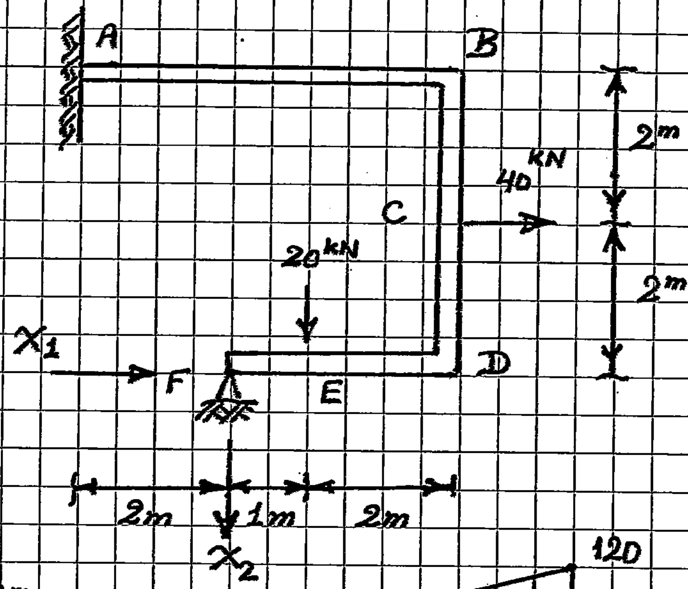
Using $EI = 200 \times 10^6 \frac{kN}{m^2} * 90 \times 10^6 \times 10^{-12} m^4 = 18000 \; kN.m^2$.

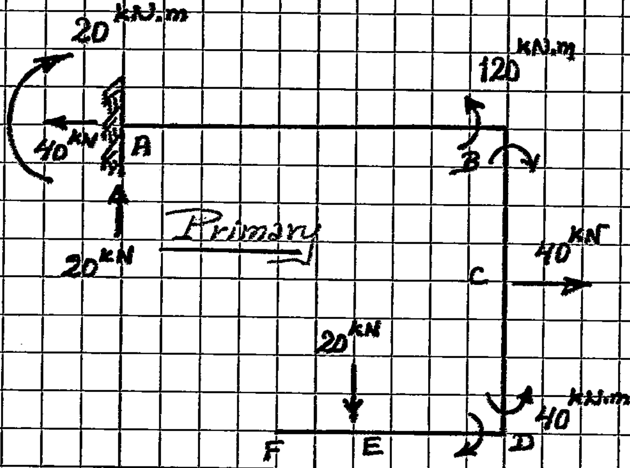
Solving $\Rightarrow$ $x_1 = -13.393$ kN, $x_2 = 76.607$ kN $\Rightarrow$ $\boxed{R_B = 13.393 \text{ kN} \downarrow}$, $\boxed{R_C = 76.607 \text{ kN} \uparrow}$

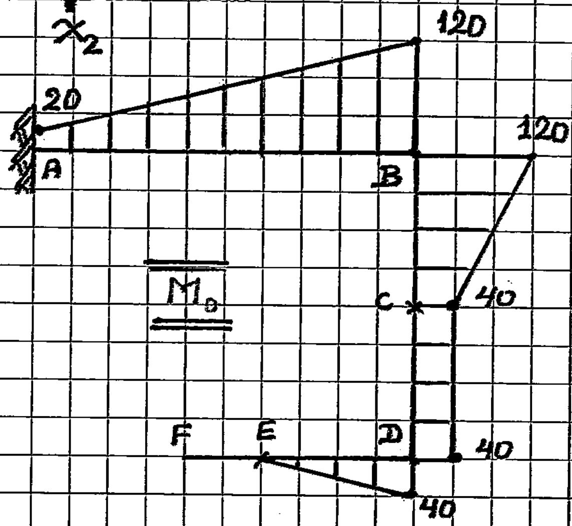
opposite to assumed sense of the dummy load.

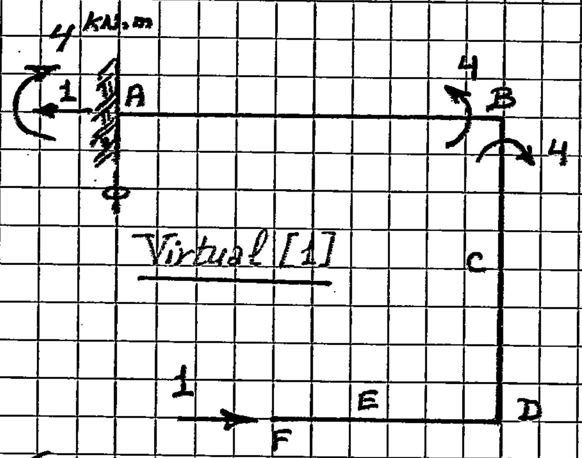


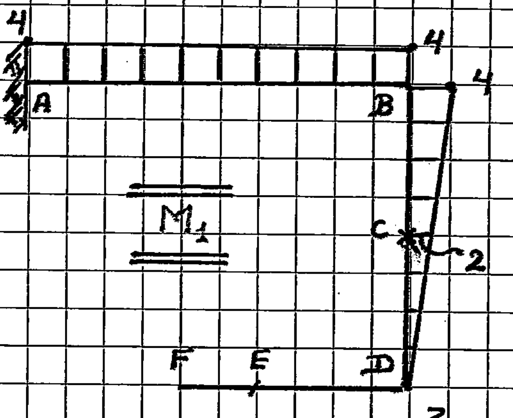
PROBLEM {3}:~

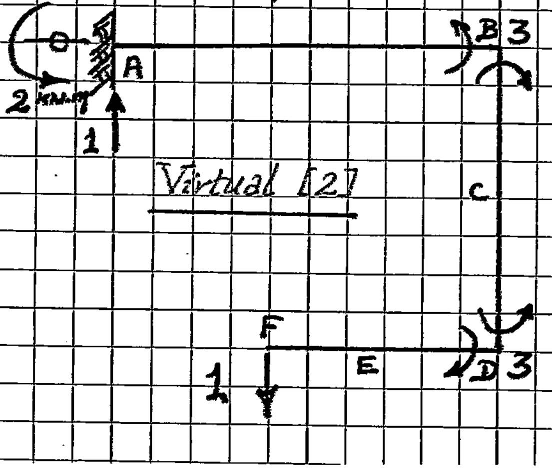

A
B
C
D
E
F
40 kN
20 kN
X_1
X_2
2m
2m
1m
2m
2m
2m

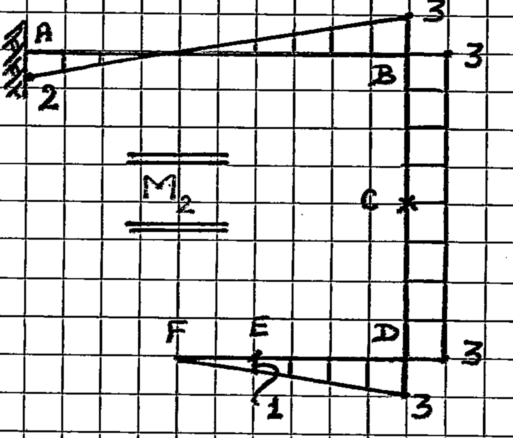

20 kN.m
40 kN
A
20 kN
120 kN.m
B
Primary
C
40 kN
20 kN
40 kN.m
F
E
D


120
20
A
B
120
M_0
C
40
F
E
D
40
40


4 kN.m
1
A
4
B
4
Virtual [1]
C
1
E
F
D


4
A
4
B
4
M_1
C
2
F
E
D


0
A
2 kN.m
1
B 3
Virtual [2]
C
F
E
D 3
1


A
3
2
B
3
M_2
C
F
E
D
3
1
3

$$\begin{Bmatrix} f_{10} \\ f_{20} \end{Bmatrix} + \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \; ; \quad \begin{aligned} & f_{10} + x_1 f_{11} + x_2 f_{12} = 0 \;\ldots(1) \\ & f_{20} + x_1 f_{21} + x_2 f_{22} = 0 \;\ldots(2) \end{aligned}$$

- $f_{10} = \int \frac{M_0 M_1 \, dl}{EI} = \frac{1}{EI}\left\{\left(\frac{20+120}{2}\right) * 5 * 4\right\} + \frac{2}{6EI}\left\{2(120)4 + 2(40)(2) + 120(2) + 40(4)\right\}$
$+ \frac{1}{EI}\left\{(40\times 2) * 1\right\}.$

$= \frac{1}{EI}\left\{1400 + 506.667 + 80\right\} = \boxed{\frac{1986.67}{EI}}$

- $f_{20} = \int \frac{M_0 M_2 \, dl}{EI} = \frac{5}{6EI}\left\{2(20)(-2) + 2(120)(3) + 20(3) + 120(-2)\right\} +$
$+ \frac{1}{EI}\left\{40 \times 4 \times 3 + \left(\frac{80\times 2}{2}\right) * 3\right\} + \frac{2}{6EI}\left\{0 + 2(40)(3) + 0 + 40\times\right.$

$= \frac{383.33}{EI} + \frac{720}{EI} + \frac{93.33}{EI}$

$= \boxed{\frac{1196.67}{EI}}$

- $f_{12} = f_{21} = \int \frac{M_1 M_2 \, dl}{EI} = \frac{5}{6EI}\left\{2(2)(-4) + 2(3)(4) + (-2)(4) + (3)(4)\right\} + \frac{1}{EI}\left\{3\times 4\times 2\right\} + 0$

$= \frac{1}{EI}\left\{10 + 24 + 0\right\} = \boxed{\frac{34}{EI}}.$

- $f_{11} = \int \frac{M_1 M_1 \, dl}{EI} = \frac{1}{EI}\left\{4\times 5\times 4\right\} + \frac{4}{3EI}\left\{(4)(4)\right\} = \frac{1}{EI}\left\{80 + 21.333\right\}$

$= \boxed{\frac{101.33}{EI}}$

- $f_{22} = \int \frac{M_2 M_2 \, dl}{EI} = \frac{5}{6EI}\left\{2(2)(2) + 2(3)(3) + 2(-3) + (-2)(3)\right\} + \frac{1}{EI}\left\{3\times 4\times 3\right\} +$
$+ \frac{3}{3EI}\left\{(3)(3)\right\} = \frac{11.667}{EI} + \frac{36}{EI} + \frac{9}{EI}$

$= \boxed{\frac{56.667}{EI}}$

Now, $\{\delta\}_0 + [F]\{X\} = \{\delta\}_{Real} \Rightarrow \{X\} = \{\delta_{Real} - \delta_0\} * [F]^{-1}$

where $[F]^{-1} = \frac{\text{Reciprocal of } [F]}{\text{Determinate of } [F]}$

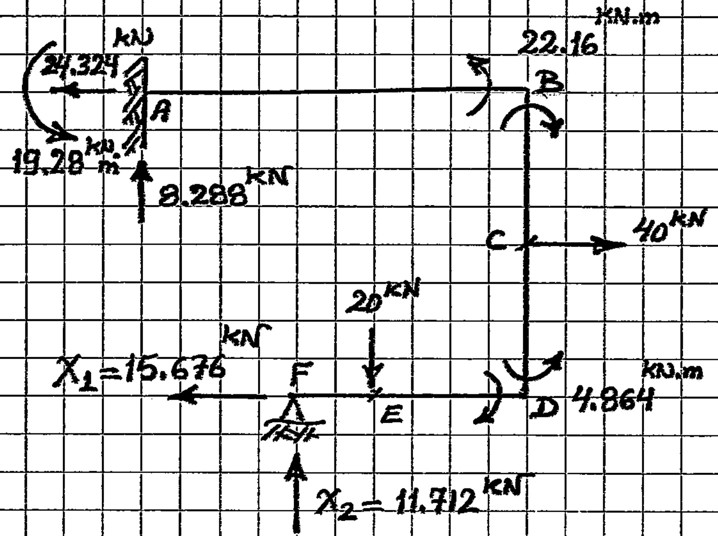
$$\left(\frac{1}{\cancel{EI}}\right)\begin{Bmatrix} 1986.67 \\ 1196.67 \end{Bmatrix} + \left(\frac{1}{\cancel{EI}}\right)\begin{bmatrix} 101.33 & 34 \\ 34 & 56.667 \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

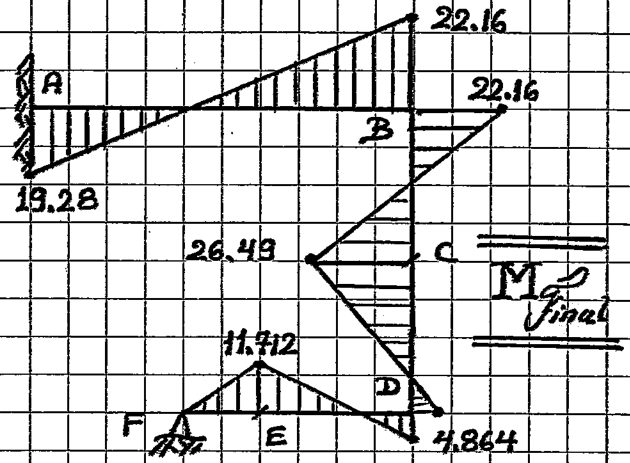
$$\Rightarrow \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \frac{1}{4{,}586.07}\begin{bmatrix} 56.667 & -34 \\ -34 & 101.33 \end{bmatrix}\begin{Bmatrix} -1986.67 \\ -1196.67 \end{Bmatrix} \Rightarrow \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} -15.676 \text{ kN} \\ -11.712 \text{ kN} \end{Bmatrix}$$

$$\Rightarrow \begin{cases} X_1 = X_F = \ominus 15.676^{\,kN} \\ X_2 = Y_F = -11.712^{\,kN} \end{cases}$$

opp. to assumed sense of the dummy load.

$$M_f = M_0 + X_1 M_1 + X_2 M_2 \quad .... \quad \text{OR BY STATICS.}$$





All Moments are plotted

From Compression Side

$$\Rightarrow X = -\frac{f_{10}}{f_{11}} = -\frac{(\delta_{EE'})_0}{(\delta_{EE'})_1} = -\frac{\left(-\frac{5320}{EI}\right)}{\left(\frac{114}{EI} + \frac{29}{EA}\right)}$$

Substitute for EI & EA.

$$\Rightarrow X = \frac{0.044028}{0.00127678} = \underline{\underline{+34.5\ k}}.$$

$\Rightarrow$ Axial Force in Member $\underline{\underline{CE = 34.5\ k}}$ in the assumed sense of the dummy load (Tension), ...

$$F_{Final} = F_0 + X F_1 \Rightarrow F_{EF} = F_{AC} = 0 + (34.5)(+1.25) = \underline{\underline{43.125^k\ (T)}}.$$

$$F_{DE} = F_{BC} = 0 + (34.5)(-0.75) = \underline{\underline{-25.875\ (C)}}.$$