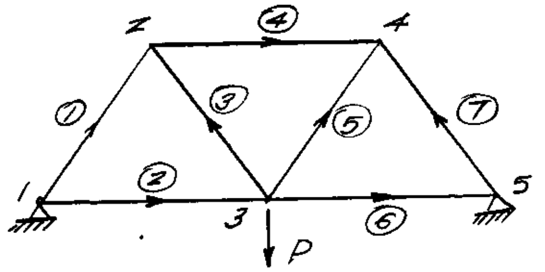

$L_i = L$
$A_i = A$
E
$\therefore K = \frac{AE}{L} = \text{constant}$

1. The stiffness matrix of any truss member i is of the following form:

$$= \left[\begin{array}{cc|cc} K_i\cos^2\alpha_i & K_i\cos\alpha_i\sin\alpha_i & -K_i\cos^2\alpha_i & -K_i\cos\alpha_i\sin\alpha_i \\ K_i\sin\alpha_i\cos\alpha_i & K_i\sin^2\alpha_i & -K_i\sin\alpha_i\cos\alpha_i & -K_i\sin^2\alpha_i \\ \hline -K_i\cos^2\alpha_i & -K_i\cos\alpha_i\sin\alpha_i & K_i\cos^2\alpha_i & K_i\cos\alpha_i\sin\alpha_i \\ -K_i\sin\alpha_i\cos\alpha_i & -K_i\sin^2\alpha_i & K_i\sin\alpha_i\cos\alpha_i & K_i\sin^2\alpha_i \end{array}\right]$$

$$= \left[\begin{array}{c|c} \mathbb{K}_i & -\mathbb{K}_i \\ \hline -\mathbb{K}_i & \mathbb{K}_i \end{array}\right] \quad \text{where} \quad \mathbb{K}_i = \begin{bmatrix} K_i\cos^2\alpha_i & K_i\cos\alpha_i\sin\alpha_i \\ K_i\sin\alpha_i\cos\alpha_i & K_i\sin^2\alpha_i \end{bmatrix}$$

Member	α_i	$\cos\alpha_i$	$\sin\alpha_i$	$\cos^2\alpha_i$	$\sin^2\alpha_i$	$\cos\alpha_i\sin\alpha_i$
1 ∧ 5	$60°$	$1/2$	$\sqrt{3}/2$	$1/4$	$3/4$	$\sqrt{3}/4$
2, 4 ∧ 6	0	1	0	1	0	0
3 ∧ 7	$120°$	$-1/2$	$\sqrt{3}/2$	$1/4$	$3/4$	$-\sqrt{3}/4$

Members 1 ∧ 5: $\mathbb{K}_i = \begin{bmatrix} \frac{1}{4}K & \frac{\sqrt{3}}{4}K \\ \frac{\sqrt{3}}{4}K & \frac{3K}{4} \end{bmatrix}$

Members 2, 4, and 6: $\mathbb{K}_i = \begin{bmatrix} K & 0 \\ 0 & 0 \end{bmatrix}$

Members 3 ∧ 7: $\mathbb{K}_i = \begin{bmatrix} \frac{1}{4}K & -\frac{\sqrt{3}}{4}K \\ -\frac{\sqrt{3}}{4}K & \frac{3}{4}K \end{bmatrix}$

$\therefore$

Members ① ∧ ⑤ :

$$\left[\begin{array}{cc|cc} \frac{1}{4}k & \frac{\sqrt{3}}{4}k & -\frac{1}{4}k & -\frac{\sqrt{3}}{4}k \\ \frac{\sqrt{3}}{4}k & \frac{3}{4}k & -\frac{\sqrt{3}}{4}k & -\frac{3}{4}k \\ \hline -\frac{1}{4}k & -\frac{\sqrt{3}}{4}k & \frac{1}{4}k & \frac{\sqrt{3}}{4}k \\ -\frac{\sqrt{3}}{4}k & -\frac{3}{4}k & \frac{\sqrt{3}}{4}k & \frac{3}{4}k \end{array}\right]$$

Members ②, ④, ∧ ⑥ :

$$\left[\begin{array}{cc|cc} K & 0 & -K & 0 \\ 0 & 0 & 0 & 0 \\ \hline -K & 0 & K & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Members ③ ∧ ⑦ :

$$\left[\begin{array}{cc|cc} \frac{1}{4}k & -\frac{\sqrt{3}}{4}K & -\frac{1}{4}k & +\frac{\sqrt{3}}{4}k \\ -\frac{\sqrt{3}}{4}k & \frac{3}{4}k & \frac{\sqrt{3}}{4}k & -\frac{3}{4}k \\ \hline -\frac{1}{4}k & +\frac{\sqrt{3}}{4}k & \frac{1}{4}k & -\frac{\sqrt{3}}{4}k \\ \frac{\sqrt{3}}{4}k & -\frac{3}{4}k & -\frac{\sqrt{3}}{4}k & \frac{3}{4}k \end{array}\right]$$

2.

$$
\begin{array}{c}
 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5
\end{array}
\begin{array}{c}
\begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\
\left[\begin{array}{ccccc}
K_1+K_2 & -K_1 & -K_2 & & \\
-K_1 & K_1+K_3+K_4 & -K_3 & -K_4 & \\
-K_2 & -K_3 & K_2+K_3+K_5+K_6 & -K_5 & -K_6 \\
 & -K_4 & -K_5 & K_4+K_5+K_7 & -K_7 \\
 & & -K_6 & -K_7 & K_6+K_7
\end{array}\right]
\end{array}
\left[\begin{array}{c} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{array}\right]
\left[\begin{array}{c} P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \end{array}\right]
$$

or

$$
\begin{array}{c|cc|cc|cc|cc|cc|}
 & \multicolumn{2}{c|}{1} & \multicolumn{2}{c|}{2} & \multicolumn{2}{c|}{3} & \multicolumn{2}{c|}{4} & \multicolumn{2}{c|}{5} \\
\hline
1 & \frac{5}{4}k & \frac{\sqrt{3}}{4}k & -\frac{1}{4}k & -\frac{\sqrt{3}}{4}k & -k & 0 & 0 & 0 & 0 & 0 \\
 & \frac{\sqrt{3}}{4}k & \frac{3}{4}k & -\frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & 0 & 0 & 0 & 0 & 0 \\
\hline
2 & -\frac{1}{4}k & -\frac{\sqrt{3}}{4}k & \frac{3}{2}k & 0 & -\frac{1}{4}k & \frac{\sqrt{3}}{4}k & -k & 0 & 0 & 0 \\
 & -\frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & \frac{3}{2}k & \frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & 0 & 0 & 0 \\
\hline
3 & -k & 0 & -\frac{1}{4}k & \frac{\sqrt{3}}{4}k & \frac{5}{2}k & 0 & -\frac{1}{4}k & -\frac{\sqrt{3}}{4}k & -k & 0 \\
 & 0 & 0 & \frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & \frac{3}{2}k & -\frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & 0 \\
\hline
4 & 0 & 0 & -k & 0 & -\frac{1}{4}k & -\frac{\sqrt{3}}{4}k & \frac{3}{2}k & 0 & -\frac{1}{4}k & \frac{\sqrt{3}}{4}k \\
 & 0 & 0. & 0 & 0 & -\frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & \frac{3}{2}k & \frac{\sqrt{3}}{4}k & -\frac{3}{4}k \\
\hline
5 & 0 & 0 & 0 & 0 & -k & 0 & -\frac{1}{4}k & \frac{\sqrt{3}}{4}k & \frac{5}{4}k & -\frac{\sqrt{3}}{4}k \\
 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{4}k & -\frac{3}{4}k & -\frac{\sqrt{3}}{4}k & \frac{3}{4}k \\
\hline
\end{array}
\left[\begin{array}{c} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \\ v_5 \end{array}\right]
\left[\begin{array}{c} X_1 \\ Y_1 \\ 0 \\ 0 \\ 0 \\ -P \\ 0 \\ 0 \\ X_5 \\ Y_5 \end{array}\right]
$$

Total Stiffness matrix of truss
as a free body (without support conditions)

3.

(a) <u>Partitioning Technique</u>

First step is interchanging the columns:

$$
\begin{bmatrix}
-\frac{1}{4}k & -\frac{\sqrt{3}}{4}k & -k & 0 & 0 & 0 & \frac{5}{4}k & \frac{\sqrt{3}}{4}k & 0 & 0 \\
-\frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{4}k & \frac{3}{4}k & 0 & 0 \\
\frac{3}{2}k & 0 & -\frac{1}{4}k & \frac{\sqrt{3}}{4}k & -k & 0 & -\frac{1}{4}k & -\frac{\sqrt{3}}{4}k & 0 & 0 \\
0 & \frac{3}{2}k & \frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & 0 & -\frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & 0 \\
-\frac{1}{4}k & \frac{\sqrt{3}}{4}k & \frac{5}{2}k & 0 & -\frac{1}{4}k & -\frac{\sqrt{3}}{4}k & -k & 0 & -k & 0 \\
\frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & \frac{3}{2}k & -\frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & 0 & 0 & 0 \\
-k & 0 & -\frac{1}{4}k & -\frac{\sqrt{3}}{4}k & \frac{3}{2}k & 0 & 0 & 0 & -\frac{1}{4}k & \frac{\sqrt{3}}{4}k \\
0 & 0 & -\frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & \frac{3}{2}k & 0 & 0 & \frac{\sqrt{3}}{4}k & -\frac{3}{4}k \\
0 & 0 & -k & 0 & -\frac{1}{4}k & \frac{\sqrt{3}}{4}k & 0 & 0 & \frac{5}{4}k & -\frac{\sqrt{3}}{4}k \\
0 & 0 & 0 & 0 & \frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & 0 & -\frac{\sqrt{3}}{4}k & \frac{3}{4}k
\end{bmatrix}
$$

2^{nd} step is interchanging the rows:

stiffness matrix of the free joints

$$
\left[\begin{array}{cccccc|cccc}
\frac{3}{2}k & 0 & -\frac{1}{4}k & \frac{\sqrt{3}}{4}k & -k & 0 & -\frac{1}{4}k & -\frac{\sqrt{3}}{4}k & 0 & 0 \\
0 & \frac{3}{2}k & \frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & 0 & -\frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & 0 \\
-\frac{1}{4}k & \frac{\sqrt{3}}{4}k & \frac{5}{2}k & 0 & -\frac{1}{4}k & -\frac{\sqrt{3}}{4}k & -k & 0 & -k & 0 \\
\frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & \frac{3}{2}k & -\frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & 0 & 0 & 0 \\
-k & 0 & -\frac{1}{4}k & -\frac{\sqrt{3}}{4}k & \frac{3}{2}k & 0 & 0 & 0 & -\frac{1}{4}k & \frac{\sqrt{3}}{4}k \\
0 & 0 & -\frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & \frac{3}{2}k & 0 & 0 & \frac{\sqrt{3}}{4}k & -\frac{3}{4}k \\
\hline
-\frac{1}{4}k & -\frac{\sqrt{3}}{4}k & -k & 0 & 0 & 0 & \frac{5}{4}k & \frac{\sqrt{3}}{4}k & 0 & 0 \\
-\frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & 0 & 0 & 0 & \frac{\sqrt{3}}{4}k & \frac{3}{4}k & 0 & 0 \\
0 & 0 & -k & 0 & -\frac{1}{4}k & \frac{\sqrt{3}}{4}k & 0 & 0 & \frac{5}{4}k & -\frac{\sqrt{3}}{4}k \\
0 & 0 & 0 & 0 & \frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & 0 & -\frac{\sqrt{3}}{4}k & \frac{3}{4}k
\end{array}\right]
\left[\begin{array}{c}
u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ \hline u_1 \\ v_1 \\ u_5 \\ v_5
\end{array}\right]
=
\left[\begin{array}{c}
0 \\ 0 \\ 0 \\ -P \\ 0 \\ 0 \\ \hline X_1 \\ Y_1 \\ X_5 \\ Y_5
\end{array}\right]
$$

(b) MIT Method

The degrees of freedom u_1, v_1, u_5, and v_5 are zero's.

∴ Referring back to the stiffness matrix at the bottom of page 2:

- Make all terms of the 1st, 2nd, 9th and 10th rows = 0
- Make all terms of the 1st, 2nd, 9th, and 10th columns = 0
- Make the terms (1,1), (2,2), (9,9), (10,10) = 1
- Make the 1st, 2nd, 9th, and 10th terms of the load vector = 0

$$
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{3}{2}k & 0 & -\frac{1}{4}k & \frac{\sqrt{3}}{4}k & -k & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{3}{2}k & \frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{4}k & \frac{\sqrt{3}}{4}k & \frac{5}{2}k & 0 & -\frac{1}{4}k & -\frac{\sqrt{3}}{4}k & 0 & 0 \\
0 & 0 & \frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & \frac{3}{2}k & -\frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & 0 \\
0 & 0 & -k & 0 & -\frac{1}{4}k & -\frac{\sqrt{3}}{4}k & \frac{3}{2}k & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{\sqrt{3}}{4}k & -\frac{3}{4}k & 0 & \frac{3}{2}k & 0 & \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} u_1 \\ v_1 \\ u_2 \\ v_2 \\ u_3 \\ v_3 \\ u_4 \\ v_4 \\ u_5 \\ v_5 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

same as in method (a); this is the stiffness matrix for the free joints.

(c) Berkeley Method

In the stiffness matrix at the bottom of page (2) apply the following modifications:

- Add 10^{10} to the terms (1,1), (2,2), (9,9), and (10,10)
- Make 0 the 1st, 2nd, 9th, and 10th terms of the load vector