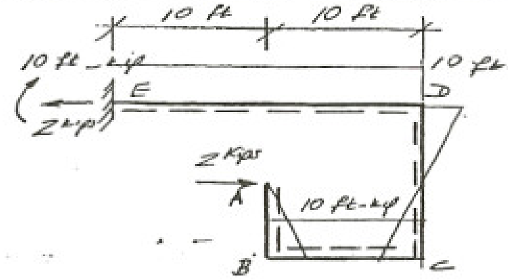
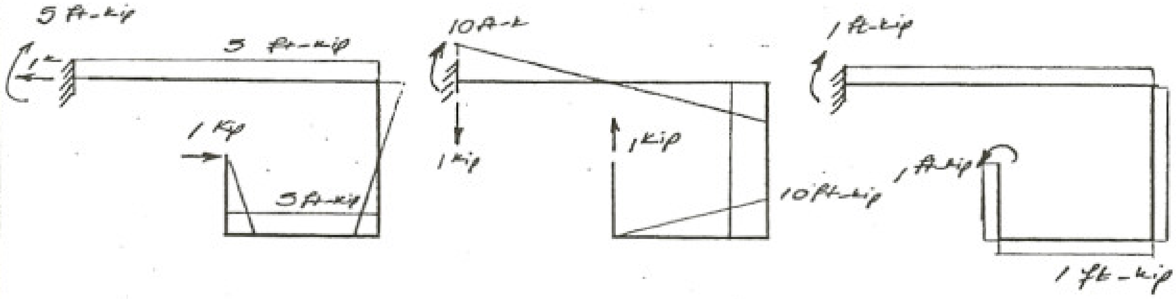
1.



$I = 100 \text{ in}^4$

$E = 30{,}000 \text{ ksi}$



Section	Origin	Range	M_x	m_H	m_V	m_α
AB	A	$0 \to 5'$	$-2x$	$-1.x$	0	$+1$
BC	B	$0 \to 10'$	-10	-5	$-1.x$	$+1$
CD	C	$0 \to 10'$	$-10+2x$	$-5+x$	-10	$+1$
DE	D	$0 \to 20'$	$+10$	$+5$	$-10+x$	$+1$

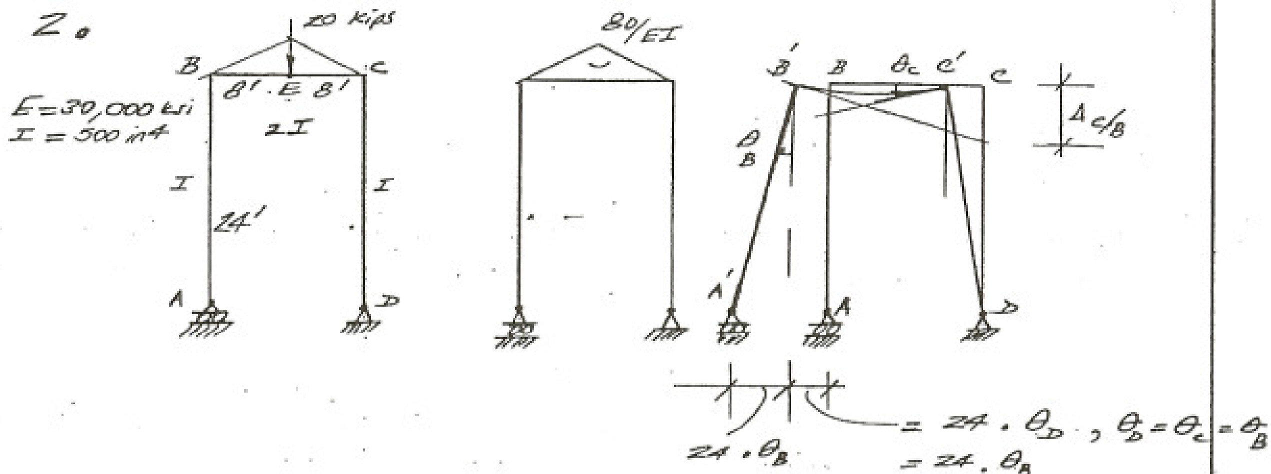
$$u_A = \int \frac{M_x m_H}{EI} dx$$
$$= \frac{1}{EI}\left[\int_0^5 2x^2 dx + \int_0^{10} 50\, dx + \int_0^{10} (2x^2 - 20x + 50) dx + \int_0^{20} 50\, dx\right]$$
$$= \frac{1750}{EI} \cdot (12)^3$$
$$= 1.008 \text{ in} \rightarrow$$

$$v_A = \int \frac{M_x m_V}{EI} dx = \frac{1}{EI}\left[\int_0^{10} 10x\, dx + \int_0^{10} (100 - 20x) dx + \int_0^{20} (10x - 100)\right]$$
$$= \frac{500}{EI} \cdot (12)^3$$
$$= 0.288 \text{ in.} \uparrow$$

$$\theta_A = \int \frac{M_x m_\alpha}{EI} dx = \frac{1}{EI}\left[\int_0^5 -2x\, dx + \int_0^{10} -10\, dx + \int_0^{10} (-10+2x) dx + \int_0^{20} 10\, dx\right]$$
$$= \frac{75}{EI} \cdot (12)^2$$
$$= 0.0036 \text{ rad} \curvearrowright$$

Eng. & Arch. Library

2.



a- Moment Area Method

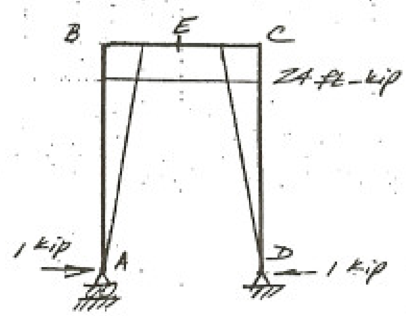
$$u_A = 2(24 \cdot \theta_B) \leftarrow$$

$$\theta_B = (\Delta_{C/B})\frac{1}{16\times12} \text{ rad.} \quad , \quad \Delta_{C/B} = \frac{80 \text{ ft-k}}{2EI} \times \frac{16 \text{ ft}}{2} \cdot 8 \text{ ft} \cdot (12)^3 \frac{\text{in}^3}{\text{ft}^3}$$
$$= 0.2949 \text{ in.}$$

$$\rightarrow \theta_B = 0.00154 \text{ rad.}$$

and $u_A = 2(24\times12)\theta_B = 0.8847 \text{ in.} \leftarrow$

b- Virtual Work Method

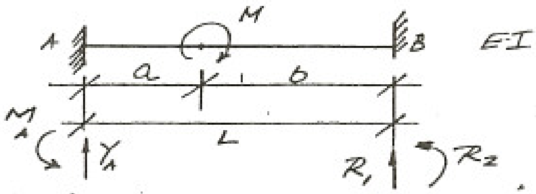


Section	Origin	Range	M_x	m_H
AB	A	0 → 24'	0	− x
BE	B	0 → 8'	10x	− 24
EC	C	0 → 8'	10x	− 24
CD	D	0 → 24	0	− x

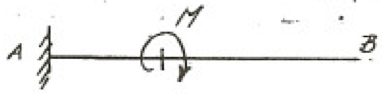
$$u_A = \int \frac{M_x m_H}{EI} dx = \frac{1}{2EI}\left[2\int_0^8 (10x)(-24)\, dx \right]$$
$$= -\frac{15360}{2EI} \cdot (12)^3$$
$$= 0.8847 \text{ in.} \leftarrow$$

3.

ⓐ-



EI = constant



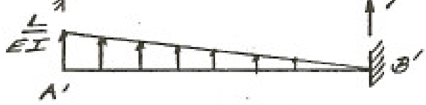
primary structure

conj.

$$\theta_B^I = -\frac{Ma}{EI}$$

$$v_B^I = -\frac{M}{EI}\cdot a \cdot \left(\frac{a}{2}+b\right) = -\frac{Ma}{EI}\left(\frac{a}{2}+b\right)$$

1st redundant structure

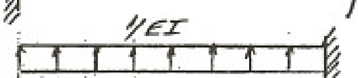


conj:

$$f_{11} = \frac{L}{EI}\cdot\frac{L}{2}\cdot\frac{2L}{3} = \frac{L^3}{3EI}$$

$$f_{21} = \frac{L}{EI}\cdot\frac{L}{2} = \frac{L^2}{2EI}$$

2nd redundant structure



conj.

$$f_{12} = \frac{1}{EI}\cdot L\cdot\frac{L}{2} = \frac{L^2}{2EI}$$

$$f_{22} = \frac{1}{EI}\cdot L = \frac{L}{EI}$$

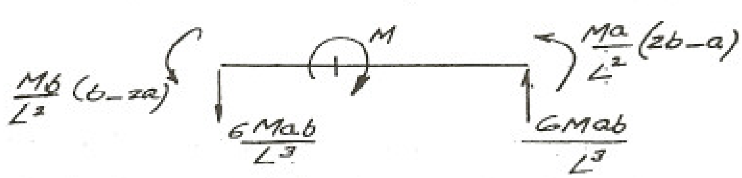
$$v_B^I + f_{11}R_1 + f_{12}R_2 = 0$$

$$\theta_B^I + f_{21}R_1 + f_{22}R_2 = 0$$

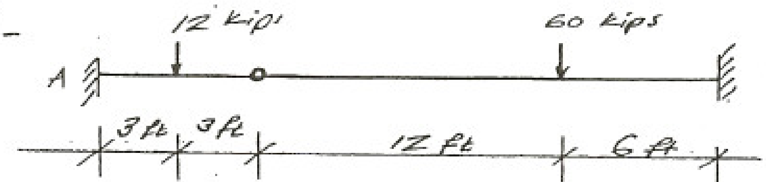
$$\rightarrow \quad -\frac{Ma}{EI}\left(\frac{a}{2}+b\right) + \frac{L^3}{3EI}R_1 + \frac{L^2}{2EI}R_2 = 0$$

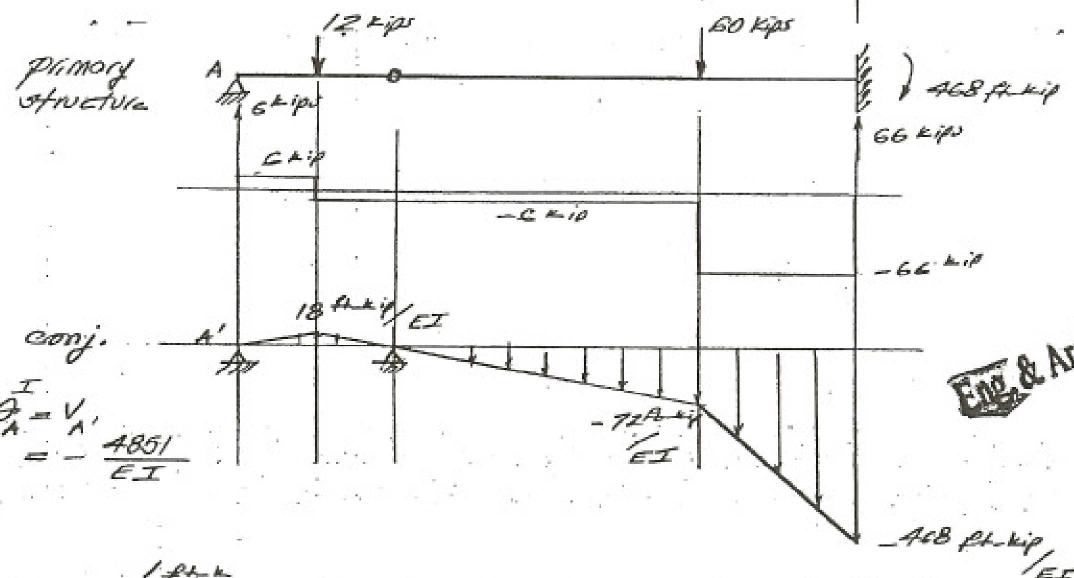
$$-\frac{Ma}{EI} + \frac{L^2}{2EI}R_1 + \frac{L}{EI}R_2 = 0$$

$$\rightarrow \quad R_1 = \frac{6Mab}{L^3}, \quad R_2 = \frac{Ma}{L^2}(2b-a)$$

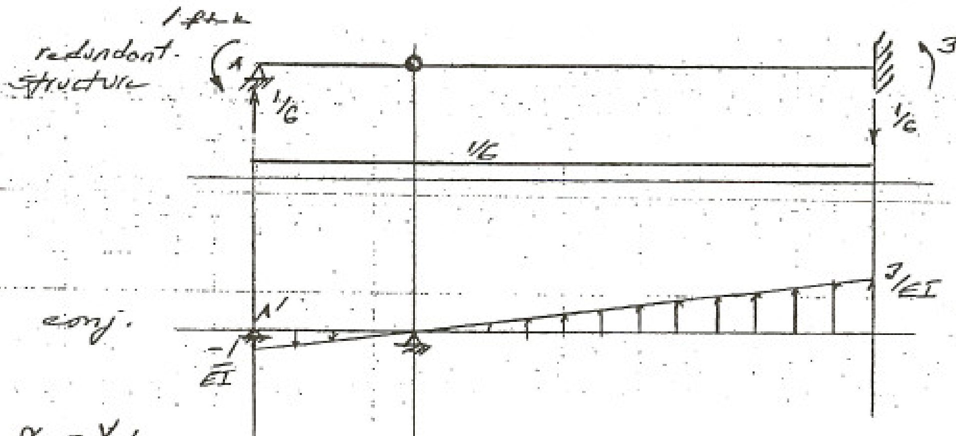


(b) -



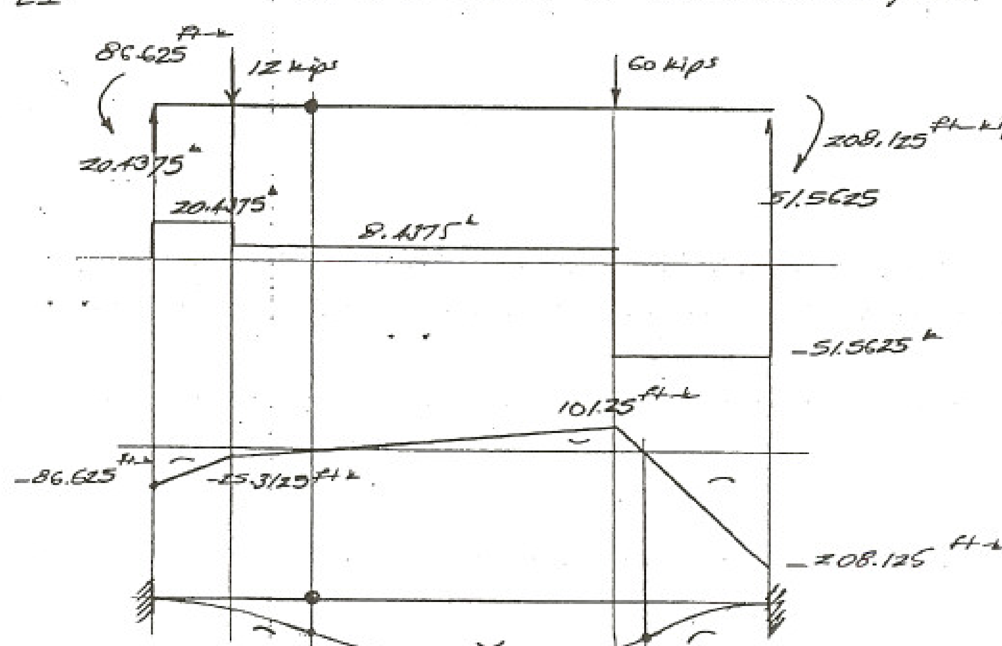


$\theta_A^I = V_{A'} = -\frac{4851}{EI}$

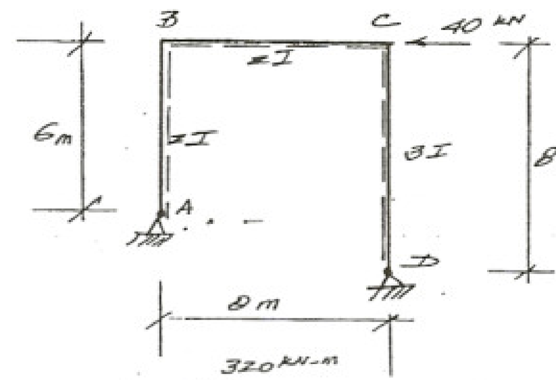


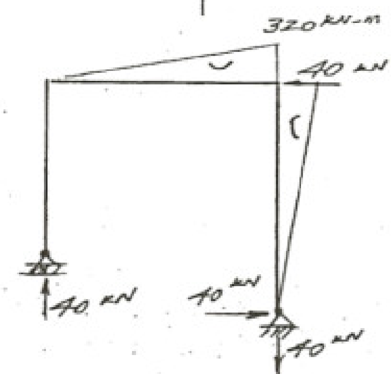
$\alpha_A = V_{A'} = -\frac{56}{EI}$

$\theta_A^I + R\,\alpha_A = 0 \rightarrow R = -86.625$ ft-k

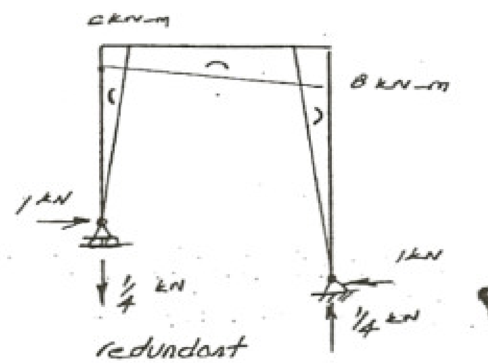


(C)-





primary structure



redundant structure with $R = 1^{kN}$

Eng. & Arch. Library

Section	Origin	Range	M_x	m_x
AB	A	0→6 m	0	$-x$
BC	B	0→8 m	$40x$	$-6-\frac{x}{4}$
CD	D	0→8 m	$40x$	$-x$

$$u_A^{I} = \int \frac{M_x m_x}{EI_x}dx = \frac{1}{EI}\left[\int_0^8 \frac{(40x)(-6-x/4)}{2}dx + \int_0^8 \frac{(40x)(-x)}{3}dx\right]$$

$$= \frac{1}{EI}(-4693.33 - 2275.56)$$

$$= -\frac{6968.89}{EI}$$

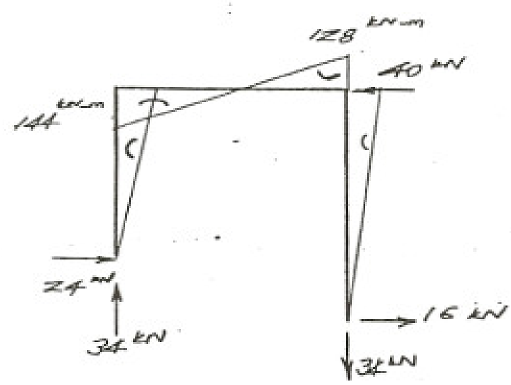
$$u_A^{II} = R\int \frac{m_x^2 dx}{EI_x} = \frac{R}{EI}\left[\int_0^6 \frac{x^2}{2}dx + \int_0^8 \frac{(-6-x/4)^2}{2}dx + \int_0^8 \frac{x^2 dx}{3}\right]$$

$$= \frac{R}{EI}(36 + 197.33 + 56.89)$$

$$= \frac{290.22}{EI}$$

$$u_A^{I} + u_A^{II} = 0 \rightarrow -\frac{6968.89}{EI} + R\frac{290.22}{EI} = 0 \rightarrow R = 24.01\ kN$$

$$\simeq 24\ kN$$


128 kN-m
40 kN
144 kN-m
24 kN
34 kN
16 kN
34 kN