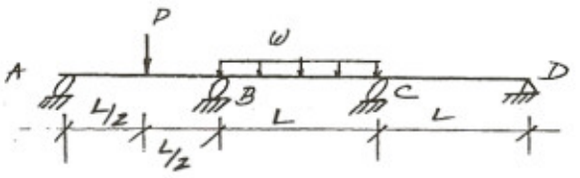
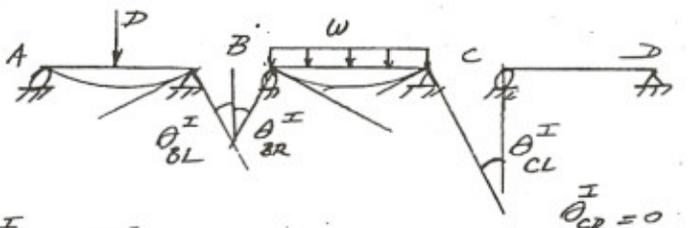
FACULTY OF ENGINEERING & ARCHITECTURE – AMERICAN UNIVERSITY OF BEIRUT

1.



EI = constant



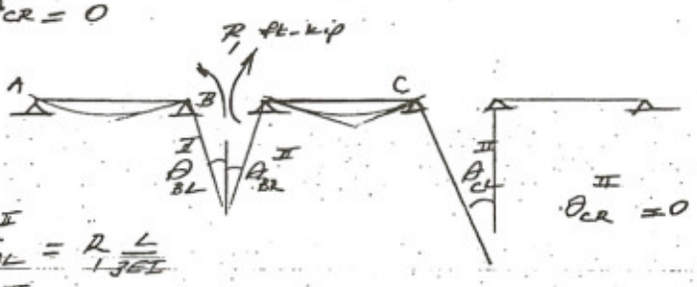
primary structure

$$\theta^I_{BL} = \frac{PL^2}{16EI}$$

$$\theta^I_{BR} = \frac{WL^3}{24EI}$$

$$\theta^I_{CL} = \frac{WL^3}{24EI}$$

$$\theta^I_{CR} = 0$$

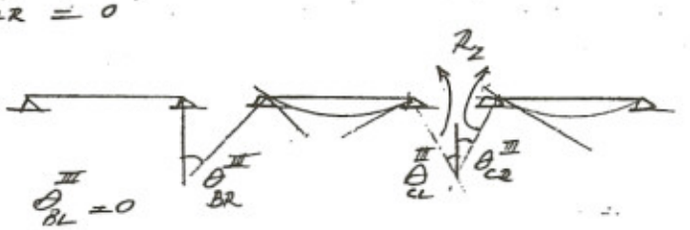


$$\theta^I_{BL} = R_1 \frac{L}{3EI}$$

$$\theta^{II}_{BR} = R_1 \frac{L}{3EI}$$

$$\theta^{III}_{CL} = R_1 \frac{L}{6EI}$$

$$\theta^{II}_{CR} = 0$$



$$\theta^{III}_{BL} = 0$$

$$\theta^{III}_{BR} = R_2 \frac{L}{6EI}$$

$$\theta^{III}_{CL} = \frac{R_2}{2} \frac{L}{3EI}$$

$$\theta^{III}_{CR} = R_2 \frac{L}{3EI}$$

Eng. & Arch. Library

Compatibility Conditions:

At B: $\left(\frac{PL^2}{16EI} + \frac{WL^3}{24EI}\right) + R_1\left(\frac{L}{3EI} + \frac{L}{3EI}\right) + R_2\left(0 + \frac{L}{6EI}\right) = 0$

At C: $\left(\frac{WL^3}{24EI} + 0\right) + R_1\left(\frac{L}{6EI} + 0\right) + R_2\left(\frac{L}{3EI} + \frac{L}{3EI}\right) = 0$

$$\left(\frac{PL}{16}+\frac{\omega L^2}{24}\right)+\frac{2}{3}R_1+\frac{1}{6}R_2=0$$

$$\frac{\omega L^2}{24}+\frac{1}{6}R_1+\frac{2}{3}R_2=0$$

$$\rightarrow \quad 4R_1+R_2=-3\frac{PL}{8}-\frac{\omega L^2}{4}$$

$$R_1+4R_2=-\frac{\omega L^2}{4}$$

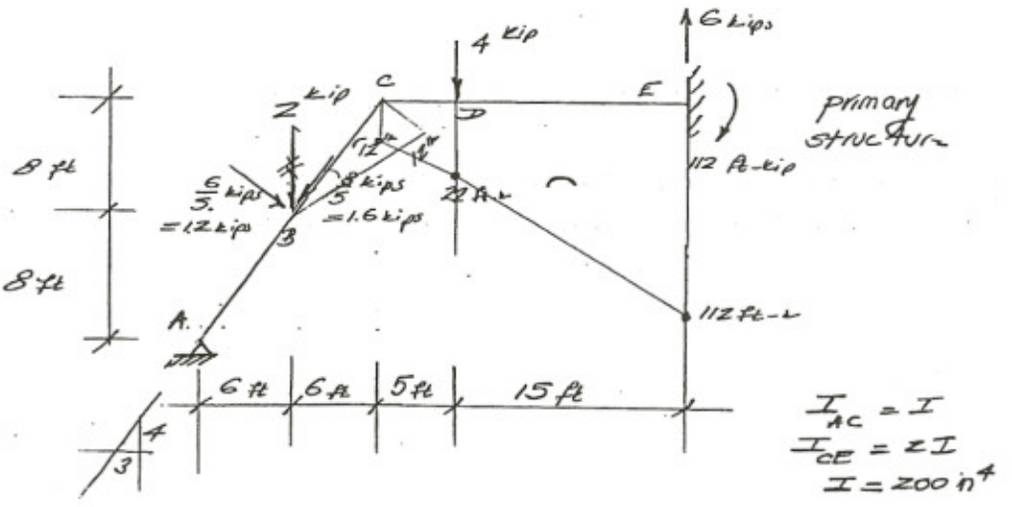
$$\rightarrow \quad R_1=-\frac{1}{20}(\omega L^2+2PL)$$

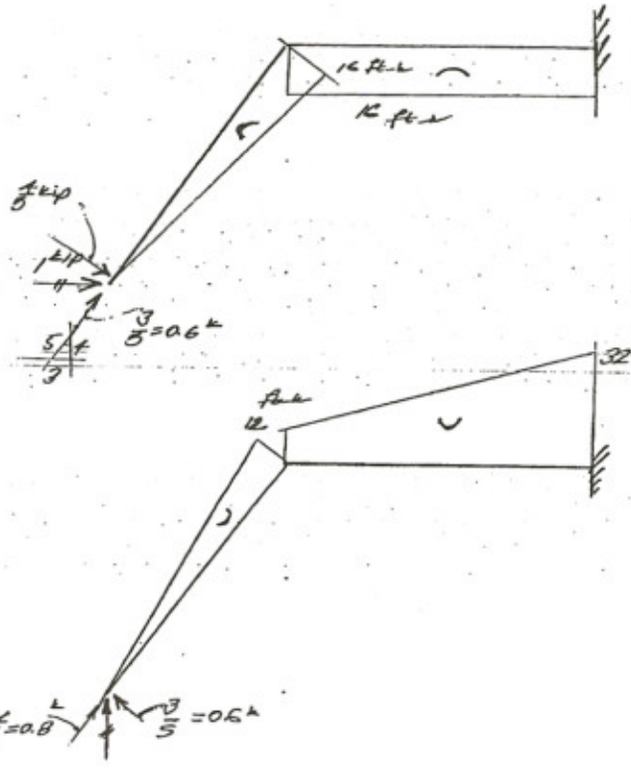
$$R_2=-\frac{1}{40}(2\omega L^2-PL)$$

FACULTY OF ENGINEERING & ARCHITECTURE – AMERICAN UNIVERSITY OF BEIRUT

Eng. & Arch. Library

2.





Section	Origin	Range	M_x	m_M	m_N
AB	A	$0 \to 10'$	0	$-\frac{4}{5}x$	$\frac{3}{5}x$
BC	B	$0 \to 10'$	$-\frac{6}{5}x$	$-8-\frac{4}{5}x$	$+6+\frac{3}{5}x$
CD	C	$0 \to 5'$	$-12-2x$	-16	$+12+x$
DE	D	$0 \to 15'$	$-22-6x$	-16	$17+x$

FACULTY OF ENGINEERING & ARCHITECTURE - AMERICAN UNIVERSITY OF BEIRUT

Eng. & Arch. Library

$$\Delta_1 + R_1 f_{11} + R_2 f_{12} = 0$$
$$\Delta_2 + R_1 f_{21} + R_2 f_{22} = 0$$

$$\Delta_1 = \int \frac{M_x m_H dx}{EI_x}$$
$$= \frac{1}{EI}\left[\int_0^{10} \frac{(-\frac{6}{5}x)(-8-\frac{4}{5}x)}{1} dx + \int_0^{5} \frac{(-12-2x)}{2}(-16)dx + \int_0^{15} \frac{(-22-6x)}{2}(-16)dx\right]$$
$$= \frac{9520}{EI}$$

$$\Delta_2 = \int \frac{M_x m_V dx}{EI_x}$$
$$= \frac{1}{EI}\left[\int_0^{10} (-\frac{6}{5}x)(6+\frac{3}{5}x)dx + \int_0^{5} \frac{(-12-2x)}{2}(12+x)dx + \int_0^{15} \frac{(-22-6x)}{2}(17+x)dx\right]$$
$$= -\frac{43145}{3EI}$$

$$f_{11} = \int \frac{m_H^2 dx}{EI_x}$$
$$= \frac{1}{EI}\left[\int_0^{10} (-\frac{4}{5}x)^2 dx + \int_0^{10} (-8-\frac{4}{5}x)^2 dx + \int_0^{5} \frac{(-16)^2}{2} dx + \int_0^{15} \frac{(-16)^2}{2} dx\right]$$
$$= +\frac{12800}{3EI}$$

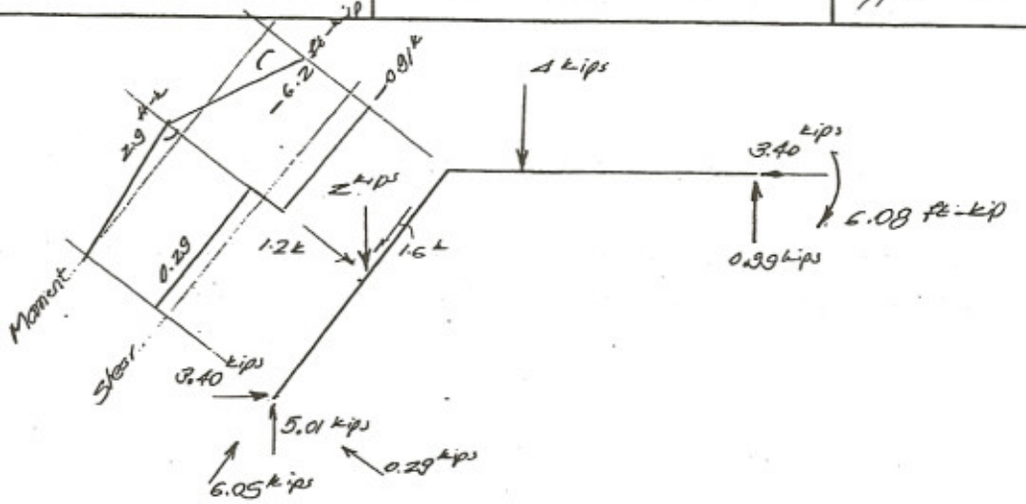
$$f_{12} = f_{21} = \int \frac{m_H m_V}{EI_x} dx$$
$$= \frac{1}{EI}\left[\int_0^{10} (-\frac{4}{5}x)(\frac{3}{5}x)dx + \int_0^{10} (-8-\frac{4}{5}x)(6+\frac{3x}{5})dx + \int_0^{5} \frac{(-16)(12+x)}{2}dx + \int_0^{15} \frac{(-16)(17+x)}{2}dx\right]$$
$$= -\frac{4800}{EI}$$

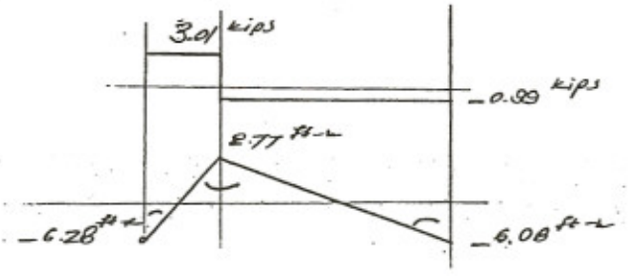
$$f_{22} = \int \frac{m_V^2 dx}{EI_x}$$
$$= \frac{1}{EI}\left[\int_0^{10} (\frac{3}{5}x)^2 dx + \int_0^{10} (6+\frac{3}{5}x)^2 dx + \int_0^{5} \frac{(12+x)^2}{2} dx + \int_0^{15} \frac{(17+x)^2}{2} dx\right]$$
$$= \frac{18400}{3EI}$$

$$\frac{9520}{EI} + R_1 \frac{12800}{3EI} + R_2 \frac{-4800}{EI} = 0$$
$$-\frac{43145}{3EI} + R_1 \frac{-4800}{EI} + R_2 \frac{18400}{3EI} = 0$$

$R_1 = X_A = 3.40$ kips →
$R_2 = Y_A = 5.01$ kips ↑

FACULTY OF ENGINEERING & ARCHITECTURE - AMERICAN UNIVERSITY OF BEIRUT
Eng. & Arch. Library



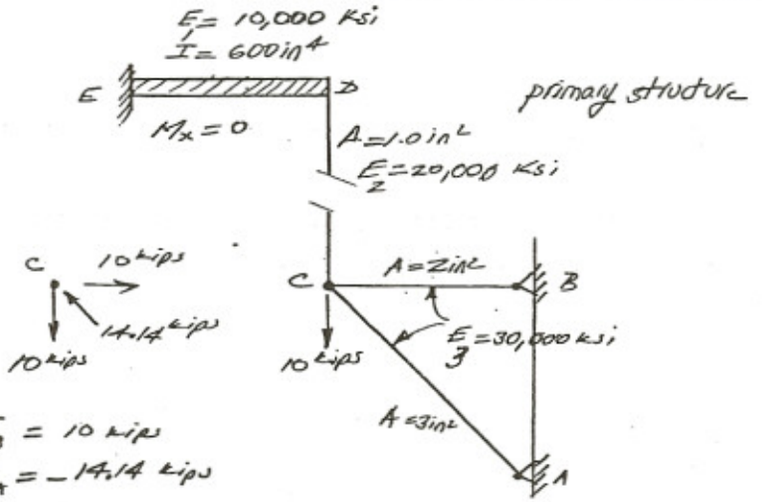


FACULTY OF ENGINEERING & ARCHITECTURE - AMERICAN UNIVERSITY OF BEIRUT

Eng. & Arch. Library

FE 1

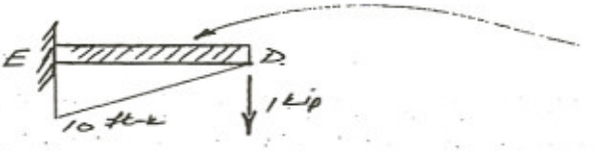
3.



$F_{CB} = 10$ kips

$F_{CA} = -14.14$ kips

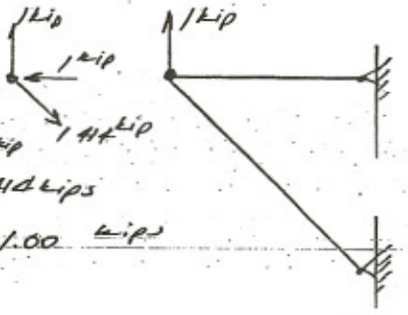
$F_{CD} = 0$



$m_x = -1.0x$

$x: 0 \to 10'$

Redundant structure



$f_{CB} = -1$ kip

$f_{CA} = +1.414$ kips

$f_{CD} = 1.00$ kips

$$\Delta^{I}_{C\downarrow} = \sum \frac{F_i f_i L_i}{A_i E_i} + \int \frac{M_x m_x dx}{EI_x}$$

$$= \frac{(10)(-1)(10\times 12)}{2\times 30{,}000} + \frac{(-14.14)(1.414)(10\sqrt{2})\times 12}{3\times 30{,}000}$$

$$= -0.0577$$

$$\Delta^{II}_{C\downarrow} = R\left(\sum \frac{f_i^2 L_i}{A_i E_i} + \int \frac{m_x^2 dx}{EI}\right)$$

$$= R\left(\frac{(1)^2\times(10\times 12)}{2\times 30{,}000} + \frac{(1.414)^2\times(10\sqrt{2}\times 12)}{3\times 30{,}000} + \frac{(1)^2\cdot x\,(10\times 12)}{1.0\times 20{,}000} + \int_0^{10} \frac{(-x)^2 dx}{(10{,}000)(600)}\times 12\right)$$

$$= 0.10777\,R$$

$$\Delta^{I}_{C\downarrow} + \Delta^{II}_{C\downarrow} = 0 \Rightarrow R = .0.54 \text{ kips} \downarrow$$

FACULTY OF ENGINEERING & ARCHITECTURE - AMERICAN UNIVERSITY OF BEIRUT

Eng. & Arch. Library