Physics 205L Final exam
Name: $\qquad$
Section number: $\qquad$
Instructor's Name: $\qquad$
ID number: $\qquad$ DO NOT START THE EXAM BEFORE YOU ARE TOLD TO BEGIN

Grading

| I |  |
| :---: | :--- |
| II-1 |  |
| II-2 |  |
| II-3 |  |
| TOTAL |  |

The duration of this exam is 60 minutes.
No notes or books allowed.

Scientific calculators are allowed
All results should be given with the exact number of significant figures.
I. (55\%) The dielectric constant $k$ of the material that fills the space between the two conductors of a coaxial cable is to be determined. Cables of different outer diameters $b$ were used and the capacitance of each was measured. All cables have an inner diameter of $a=0.037 \mathrm{~cm}$ and a length $L=1+/-0.001 \mathrm{~m}$. The permittivity of free space is $\varepsilon_{0}=8.85 \times 10^{-12} \mathrm{C}^{2} / \mathrm{Nm}^{2}$
The results below were obtained:

| $b[\mathrm{~cm}]$ | $C(\mathrm{pF})$ |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0.110 | 102.10 |  |  |  |  |
| 0.231 | 60.72 |  |  |  |  |
| 0.320 | 51.55 |  |  |  |  |
| 0.415 | 46.01 |  |  |  |  |
| 0.517 | 42.17 |  |  |  |  |
|  |  |  |  |  |  |

You may find the formulae on page 5 useful.
a- Write down the necessary relation between $C$ and $b$.
b- Choose your variables $x$ and $y$ such that you obtain a linear relationship between them.
c- Use linear regression to find the slope of the line along with its error.
d- Determine the dielectric constant $k$ along with its error.

## Linear Regression

The method of least squares is used to fit a curve (find a theoretical equation) to a set of experimental data. First assume that a linear relation exists between $y$ and $x$

$$
\begin{equation*}
y=A x+B \tag{1}
\end{equation*}
$$

Substitution of $x=x_{i}$ will in general not give the value of $y_{i}$. The "errors" will be

$$
\begin{equation*}
\mathrm{e}_{\mathrm{i}}=y-y_{\mathrm{i}}=A x_{\mathrm{i}}+B-y_{\mathrm{i}} \tag{2}
\end{equation*}
$$

To determine the best straight line that fits the N, sets of data, A and B have to be chosen so that the sum of the squares of the "errors" is minimized. This means that the simultaneous equations, obtained by equating the partial derivatives of $\left(y-y_{i}\right)^{2}$ with respect to $A$ and $B$ to zero, should be solved. This condition leads then to the following results

$$
\begin{equation*}
A=\frac{N \sum\left(x_{i} y_{i}\right)-\sum x_{i} \sum y_{i}}{\Delta} \tag{3}
\end{equation*}
$$

and

$$
\begin{equation*}
B=\frac{\sum x_{i}^{2} \sum y_{i}-\sum x_{i} \sum\left(x_{i} y_{i}\right)}{\Delta} \tag{4}
\end{equation*}
$$

where

$$
\begin{equation*}
\Delta=N \sum x_{i}^{2}-\left(\sum x_{i}\right)^{2} \tag{5}
\end{equation*}
$$

The correlation coefficient $r$ provides an indicator of how good a fit the best straight line is. This coefficient is defined as

$$
\begin{equation*}
r=\frac{\sum\left(x_{i}-\bar{x}\right)\left(y_{i}-\bar{y}\right)}{\sqrt{\sum\left(x_{i}-\bar{x}\right)^{2} \sum\left(y_{i}-\bar{y}\right)^{2}}} \tag{6}
\end{equation*}
$$

For $r=0$, the values of x and y are independent of one another and there is no linear correlation. The closer $r$ is to +1 or to -1 , the better the linear correlation is.
Finally, the error in A is given by:

$$
\sigma_{A}^{2}=\frac{N}{N-2} \frac{\sum e_{i}^{2}}{\Delta}
$$

## II. Questions

1. In the "Planck's Constant Experiment" (15\%)
a- We used the equation $V_{S}=\left(\frac{h}{e}\right) f-\frac{\phi}{e}$ in order to determine h and $\phi$. Explain how.(10\%)
b- Define the photoelectric effect. (5\%)
2. In the e/m experiment, why did we tilt the Helmholtz coils and kept them at that orientation throughout the experiment? (10\%)
3. In "Ohm's Law" experiment, explain how the resistivity of metal rods were determined. Include necessary equations. (20\%)
