Economic Statistics I
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1 Problem 1

(15 points)

The English diarist Samuel Pepys asked his friend Sir Isaac Newton the following question: is it more likely to get at least one 6 when 6 dice are rolled, at least two 6's when 12 dice are rolled, or at least three 6's when 18 dice are rolled? After considerable correpsondence, Newton convinced Pepys that the first event is the most likely. Compute the three probabilities.

2 Problem 2

(25 points)

An urn contains four balls numbered 1 to 4. Two balls are drawn simultaneously.

(a) Let X be the larger of the two numbers drawn. Find the probability distribution function of X.

(b) Let V be the sum of the two numbers drawn. Find the probability distribution function of V.

(c) What are the expected value and variance of V?

(d) What is the distribution function of V conditional on $X = 3$?

(e) What is the joint probability of X and V? Are they independent?

3 Problem 3

(6 points)

Condosleezza and Hobballah have been in serious conflict lately. Trying to bully Hobballah into accepting her terms, Condosleezza sent a threatening letter to Hobballah through the Servant of the Holy Moly. Three weeks have passed and Condosleezza has still not heard back. Assuming that (1) Hobballah has a 70% chance of responding if, in fact, he received the letter, and that (2) the Servant of the Holy Moly has a one in fifty chance of losing the letter, what is the probability that Hobballah never received the threatening letter?

4 Problem 4

(8 points)

Suppose that an aluminium pipes manufacturer faces demand y, which follows an exponential distribution $f(y) = 6e^{-6y}$ for $y > 0$.

(a) What is the median quantity demanded?

(b) The firm's profit Q is a function of demand:

$$Q(y) = 2(1 - e^{-2y})$$

What is the firm's expected profit?

5 Problem 5

(12 points)

(a) What is the probability that a normally distributed random variable with mean 1 and variance 100 exceeds 10?

Suppose $X_1, X_2, ..., X_{120}$ are 120 independent random variables, each following a normal distribution with mean 1 and variance 100.

(b) You calculated, in question (a) the probability that any one of these variables exceeds 10. What is the probability that exactly 40 of the X_i's exceed 10?

(c) What is the mean of the random variable $Y = X_1 + X_2 + X_3 + ... + X_{120}$?

(d) What is the variance of Y?

6 Problem 6

(12 points)

According to a study that appeared in 2004 in The Lancet, a prestigious British medical journal, the death rate in Iraq in 2002 before the US-led invasion in 2003 had a mean of 5 deaths per 1,000 persons per year and a standard deviation of 2.65. In 2004, after the US-led invasion, the death rate is calculated by the researchers at 12.3 per thousand per year.

(a) What is the maximum probability that we observe a death rate of 12.3 per thousand per year or more if the mean is 5 per thousand per year and the standard deviation 2.65?

(b) Assume for question (b) that the death rate is normally distributed. Now what is the probability of observing a death rate of 12.3 per thousand per year or more if the mean death rate is 5 per thousand per year and the standard deviation is 2.65?

(c) Some members of the US Administration have discredited the study and tried to defend the US by claiming that the US intervention in Iraq has not led to a change in the distribution of deaths since 2002. According to your answers to questions (a) and (b) how likely is this claim to be true?

7 Problem 7

(22 points)

Suppose you are interested in the growth rates of different countries and assume that for any given country in any given year t, the growth rate g_t is normally distributed with mean μ and standard deviation 0.05. Assumealso that the growth rates are independent over time (so that this year's growth is independent of next year's growth). Suppose that half the countries are high growth countries, with mean $\mu_H = 0.1$ and the other half are low growth countries, with mean $\mu_L = 0.01$ but you do not observe whether a country is low growth or high growth.

(a) You randomly select a country. What is the probability that its growth rate is 0.1 or higher?

(b) Suppose you are told a given country's growth rate is positive. What is the probability that it is a low growth country?

(c) Question (b) tells you what the probability is that a country is a low growth country if you observe a positive growth rate. Use your answer to question (b) to calculate the probability that a country that has a positive growth rate this year will have a negative growth rate next year.