

Midterm Solutions

1 Problem 7

Suppose you are interested in the growth rates of different countries and assume that for any given country in any given year t , the growth rate g_t is normally distributed with mean μ and standard deviation 0.05. Assume also the growth rates are independent over time. Suppose that half the countries are high growth countries, with mean $\mu_H = 0.1$ and the other half are low growth countries, with mean $\mu_L = 0.01$ but you do not observe whether a country is low growth or high growth.

(a) You randomly select a country. What is the probability that its growth rate is 0.1 or higher?

ANSWER: Half the countries are high growth countries. For a high growth country, the probability of observing growth of 0.1 or higher is 0.5 (since growths are normally distributed with mean 0.1). The other half of countries are low growth countries. For low growth countries, the probability of observing growth of 0.1 or higher is:

$$P(g \geq 0.1 | \text{low growth}) = P\left(Z \geq \frac{0.1 - 0.01}{0.05}\right) = 0.036$$

So the overall probability of randomly selecting a country with growth rate 0.1 or higher is:

$$\begin{aligned} P(g \geq 0.1) &= 0.5 \times P(g \geq 0.1 | \text{high growth}) + 0.5 \times P(g \geq 0.1 | \text{low growth}) \\ P(g \geq 0.1) &= 0.268 \end{aligned}$$

(b) Suppose you are told a given country's growth rate is positive. What is the probability that it is a low growth country?

ANSWER:

$$\begin{aligned} P(\text{low growth} | g \geq 0) &= \frac{P(g \geq 0 | \text{low growth})P(\text{low growth})}{P(g \geq 0 | \text{low growth})P(\text{low growth}) + P(g \geq 0 | \text{high growth})P(\text{high growth})} \\ &= \frac{0.5 \times P\left(Z \geq \frac{0-0.01}{0.05}\right)}{0.5 \times P\left(Z \geq \frac{0-0.01}{0.05}\right) + 0.5 \times P\left(Z \geq \frac{0-0.1}{0.05}\right)} \\ &= 0.37 \end{aligned}$$

(c) Question (b) tells you what the probability is that a country is a low growth country if you observe a positive growth rate. Use your answer to question (b) to calculate the probability

that a country that has a positive growth rate this year will have a negative growth rate next year.

ANSWER: The growth rates over time are independent. The probability that it experiences negative growth next year is:

$$P(g_{t+1} \leq 0) = P(g_{t+1} \leq 0 | \text{low growth}) \times P(\text{low growth}) + P(g_{t+1} \leq 0 | \text{high growth}) \times P(\text{high growth})$$

But the probabilities that the country is high or low growth are not 0.5 in this case, but instead, are updated using the information that $g_t \geq 0$ this year. We calculate in part b the probability that a country is low growth, given that its growth rate is positive this year. So the probability that it is high growth is $1 - 0.37 = 0.63$.

So:

$$P(g_{t+1} \leq 0) = 0.37 \times P(g_{t+1} \leq 0 | \text{low growth}) + 0.63 \times P(g_{t+1} \leq 0 | \text{high growth})$$

$$P(g_{t+1} \leq 0) = 0.37P\left(Z \leq \frac{0 - 0.01}{0.05}\right) + 0.63P\left(Z \leq \frac{0 - 0.1}{0.05}\right)$$
$$P(g_{t+1} \leq 0) = 0.17$$